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THE
CIVIL SERVICE ARITHMETIC.

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CIVIL SERVICE ARITHMETIC.

BY

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THE CIVIL SERVICE

ARITHMETIC.

1. QUANTITY is a term applied to anything susceptible of being increased or diminished.

2. A UNIT is one, or an individual thing of the quantity under consideration.

3. The term NUMBER may be applied to a unit or an assemblage of units of any magnitude whatever. An *abstract number* is not applied to any particular object. A *concrete number* is applied to some particular thing, as, two crowns, five apples.

4. ARITHMETIC is the science of numbers, and is either theoretical or practical. *Theoretical Arithmetic* explains the nature, properties, and relations of numbers: *Practical Arithmetic* applies those principles to the various purposes of life.

5. All numbers are expressed by the following ten characters or figures, either *singly* or in *combination,* 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, read, *nought* or *cypher, one, two, three, four, five, six, seven, eight, nine.* The nine characters, 1, 2, 3, &c., are called *significant figures;* and sometimes the ten figures are called *digits.*

remove it *two* places to the left, we make its value *one hundred times* greater, and so on: thus 600 is equal to one hundred times 6. On the contrary, if we remove a figure *one place* to the right, we decrease its value *ten-fold;* if we remove it *two places* to the right, we decrease its value *one hundred-fold.* In the one case, the *units* become *tens,* the *tens, hundreds,* &c.: in the other the *hundreds* become *tens,* the *tens, units.*

A NOUGHT placed to the *left* of a figure or number has no *effect,* since it does not alter the position with respect to the *units' place.*

Write in figures the following numbers:

1. Five hundred and ninety.
2. One thousand four hundred and three.
3. Two thousand and seventy-seven.
4. Four thousand five hundred and seventy.
5. Eight thousand and nine.
6. Nine thousand and ninety-seven.
7. Four hundred and sixty-one thousand and one.
8. Two millions five thousand and eighty-seven.
9. Twenty-seven millions seven hundred and six.
10. Eight hundred and forty thousand and twenty.
11. Ten millions four thousand and eight.
12. Eleven hundred and two millions seventy thousand and fifty.
13. Ninety thousand and forty.
14. Three hundred and four millions forty thousand and four.

Proposed at various Examinations.

15. Six hundred and three thousand and fifty.
16. Seven millions three thousand and forty.
17. Eight thousand millions one thousand and two.
18. Five hundred thousand four hundred and one.
19. Thirty millions seven thousand and forty.
20. Five hundred and six millions and twenty.
21. Four millions five hundred and five.
22. Eight thousand millions six hundred.
23. Ten millions twenty thousand and one.

24. Four hundred millions four thousand and forty.
25. Nine thousand and fifty millions sixty thousand and four.
26. Eight hundred and five thousand millions and ninety-nine.*

The system of Notation just explained, though used in France, Canada, and the United States of America, has not yet been universally adopted in Great Britain. As will be perceived from glancing at the following table, what is called the English system consists in dividing into periods of *six figures* each.

ENGLISH NUMERATION TABLE.

&c., &c.	Hundreds of thousands of billions.	Tens of thousands of billions.	Thousands of billions.	Hundreds of billions.	Tens of Billions.	Billions.	Hundreds of thousands of millions.	Tens of thousands of millions.	Thousands of millions.	Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
	0	0	0	0	0	0,	0	0	0	0	0	0,	0	0	0	0	0	0
	Billions' period.						Millions' period.						Units' period.					

ROMAN NOTATION TABLE.

The ancient Romans expressed numbers in the following manner:—

1 ... I.	7 ... VII.	20 ... XX.	100 ... C.
2 ... II.	9 ... IX.	29 ... XXIX.	500 ... D.
3 ... III.	10 ... X.	40 ... XL.	560 ... DLX.
4 ... IV.	11 ... XI.	53 ... LIII.	820 ... DCCCXX.
5 ... V.	14 ... XIV.	89 ... LXXXIX.	910 ... DCCCCX.
6 ... VI.	18 ... XVIII.	90 ... XC.	1000 ... M.

That the learner may have some idea of what a million is, it is well to state that it would require a man between 16 and 17 days, counting 100 sovereigns per minute, 10 hours every day in order to count a million of sovereigns. To count a billion at the same rate would require him to work more than forty-five thousand years. Had he commenced at the creation of the world, he would not yet have one-seventh of his task completed.

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* The learner is requested to test his ability to read numbers by trying to read the answers of the above examples. The Examiners have hitherto used the English system of reading numbers.

SIMPLE ADDITION.

12. ADDITION is the process by which we find one number equal to two or more numbers when taken together. The result or answer is called the *Sum.* The numbers to be added are called *Addends.*

The addition of abstract numbers or of applicate numbers of one denomination, is called SIMPLE ADDITION.

The *sign* of addition is $+$; thus $4 + 6$, read, 4 *plus* or more 6, and means that 4 and 6 are to be added together.

It is also necessary to explain the use of another sign, namely, $=$, read, '*equal to*;' thus $4 + 6 = 10$, read, *four plus six, equal to ten*

The sign or abbreviation ∴ is frequently used in works on arithmetic for *therefore.*

13. RULE FOR SIMPLE ADDITION.

1°. Write the *addends* under each other, so that *units* may stand under *units, tens* under *tens,* &c.

2°. Add up the figures in the *units' place:* set down the *units* of the sum and carry the tens, if any, to the next or tens' column; proceed in the same manner with the remaining columns.

3°. Having added the last column, set down its sum in full.

EXAMPLE 1. Find the sum of 424, 134, 327, and 26.

The numbers are written as in the annexed operation. Then commencing at the right or units' place, we say, 6 and 7 are 13; 13 and 4 are 17; 17 and 4 are 21 *units* or *ones,* which are = 2 tens and 1 unit; we set down the 1 *unit;* and *carrying* the 2 tens to the next column, we say, 2 and 2 are 4; 4 and 2 are 6; 6 and 3 are 9; 9 and 2 are 11 = 1 hundred and 1 ten; we set down the 1 *ten,* and *carry* the 1 hundred to the next column, the sum of which is 9; we set down the 9. The sum or answer is 911.

$$\begin{array}{rl} \left.\begin{array}{r}424\\134\\327\\26\end{array}\right\} & \textit{addends.}\\ \hline 911 & \textit{Sum.}\end{array}$$

14. REASON OF THE RULE.—The rule for addition is obvious from the *principles of Notation* already explained.

1°. If we place units under units, tens must be under tens, hundreds under hundreds, &c. We write the numbers in this order that we may have those figures of the *same kind* or *local value* under each other, since we cannot add *units* to *tens*, or *tens* to *any other column* except its own.

2°. We commence at the *units' place* for the convenience of *carrying* to the higher places. We set down the number of units in the sum of this column, and carry the tens to the tens' column, which is the same kind as what we carry, and so on with the other columns.

3°. We always write the sum of the *left hand* column in full, because we have nothing more to add to it.

15. PROOF.—I. The most convenient mode of proving ADDITION, that is, of testing the accuracy of the operation, is to begin at the top and count it down: if the two results agree, the work is *most probably* correct.

II. Another method of proof, depending upon the principle, "The whole is equal to the sum of all its parts," will be understood from the following example:—

In writing the example in order to prove it we *omit the first addend* 9864. Having found the sum of the remaining addends, 19123, we add the *neglected addend* to it, and thus find the same result, 28987. It is plain that the omission of any other addend would answer equally well. To avoid the necessity of writing the example *twice* as in the above explanation, it will be sufficient to draw a line under the *first addend*, and omit this addend in the addition intended as proof.

Example.	*Proof.*
9864	4769
4769	7907
7907	5869
5869	578
578	
	19123
28987	9864
	28987

II.

Find the sum of :—

1.	2.	3.
39704	760987	9786549897
88888	859378	7654398799
76543	999999	6547998887
9799	75867	6437896467
3787	937899	7776666698

4. 4059 + 760786 + 439 + 986407 + 408067.
5. 54 + 960483 + 197 + 546078 + 88067 + 478609.
6. 980 + 43609 + 9908 + 78078 + 11011 + 990573.
7. 777 + 439 + 709053 + 54806 + 655073 + 864 + 1998.

8. Add, one hundred and seventy-nine; nine thousand four hundred and eighty-nine; six hundred and twelve thousand seven hundred and ninety-six; nine hundred and sixteen thousand seven hundred and forty; sixty-nine thousand seven hundred and eighty-seven.

9. Add, three hundred and sixty-five; four hundred and seventy-nine; six hundred and ninety-four; seven hundred and nine; nine hundred and seventy-eight; forty-eight; nineteen.

10. Find the sum of, four hundred and nine thousand seven hundred and forty-nine; six hundred and seventy-eight thousand and seventy-five; eight hundred and sixty-nine thousand and fifty; nine thousand eight hundred and seventy-six; five hundred and seventy-four.

11. Add, thirty-nine millions six hundred and ninety-four thousand four hundred and ten; forty millions forty thousand and forty; ninety-seven millions ninety-seven thousand and ninety-seven; fifty-six thousand and seven; three hundred and seventy-nine; five hundred and eight.

12. Adam lived 930 years; Seth, 912; Enos, 905; Cainan, 910; Mahaleel, 895; Jared, 962; Enoch, 365; Methusaleh, 969; Lamech, 777; Noah, 950; Shem, 600; Arphaxad, 438; Salah, 433; Heber, 464; Peleg, 239; Reu, 239; Serug, 230; Nahor, 148; Terah, 205; Abraham, 175; Isaac, 180; Jacob, 147; Joseph, 110; Moses, 120; Joshua, 110. What is the sum of all their ages?

Proposed at various Examinations.

13.	14.	15.
64935039	576987	64935039
3945684	66989	3945684
59712593	755777	59712593
78916376	89788	80643978
12863121	778876	12863121
83752463	79997	83752463

16.	17.	18.
34689104	9076787	34689104
83674823	789766	83674823
97615307	79798	97615307
41923584	875666	41923584
76325918	744455	13895193
67431297	988888	67431297
93206140	77777	93206140
72169837	99999	72169837

19.
650350497
37290154
728236525
59843157
381402388
948635749
751298797
445737587
983176679

20.
789725
333333
979797
868631
704093
217479
579068
89787
6679

21.
7836744364
795242165
677167113
68259617
4992429
83485354
7768432
32483547
91998796

22.
97679017795
7505307944
34451556
447789667
8139004579
7896341297
654788578

23.
789786
17932
904766
788777
867999
754444
989895

24.
879678996
640350397
39456846
597125937
7891637676
128631219
837524638

25.
77777777
98914566
9773988
7463966
950466
98890978

26.
445678
979779
86547
345607
93478
45555

27.
66666666
78982377
77721040
98684397
6752589
97876679

28.
6778275
690343
8771527
9478698
798667497
999999999

29.
789787
77896
78964
83245
79427
88888

30.
79361377
975086
67745366
9370979
77871897
88888888

The learner should be accustomed to add in the following manner until he can do so with accuracy and despatch: Because 4 and 7 are 11, 14 and 7 are 21, 24 and 7 are 31. Again, 16 and 8 are 24; say, "because 6 and 8 are 14," 26 and 8 are 34, 36 and 8 are 44, and so on. It is necessary to point out to him that 11, 21, 31, &c., all end in the same figure

31. 7968543
9678897
6756666
776988
57777
99997
85359
77706
55555
43456
987654
789475

34. 6789786
736589
766565
4474747
89896
59478
3686
5968
97865
878787
65430
77777
98765

37. 73456078
64440789
93232456
4578903
787894
3604509
7865867
9678796
8432109
4730345
5786547
3433249
9654327
7777777

32. 8987654
778965
897789
954378
878679
545454
373737
666665
890787
678976
477896
887887

35. 9678543
778780
698765
543042
417197
39898
87736
96453
9798
6954
79876
36789
965368

38. 54997865
3786501
7321047
9895678
8907865
7756786
317891
978888
866545
754323
647986
447986
754973
555555

33. 7366789
787878
565656
737373
457897
8054
97866
88788
79794
59529
7868
999999

36. 8756789
656543
57004
734343
629273
96789
87869
78762
76798
45454
797878
886764
9999

39. 97986541
33333333
44444444
99999999
88888888
57575754
68686868
97979797
43434343
84210787
45543656
59898656
74747474
88888888

SUBTRACTION.

16. SUBTRACTION is the process of finding the difference between any two numbers.

The number to be subtracted is called the *Subtrahend*; the number from which the *Subtrahend* is to be taken is called the *Minuend*. The result or answer is called the *Remainder* in commercial arithmetic, and the *Difference* in mathematical science.

SIMPLE SUBTRACTION, like SIMPLE ADDITION, is confined to *abstract* numbers or *applicate* numbers of but one denomination.

17. RULE FOR SIMPLE SUBTRACTION.

1°. Write the smaller number under the *greater*, just as two addends are written.

2°. Take *each* figure in the smaller number from the figure *corresponding* to it in the larger, commencing at the units' place.

3°. If the *lower* figure be greater than the figure *over* it, increase the *latter* by 10, and subtract. After setting down the remainder, *carry* 1 to the next figure to be subtracted, and so on.

Ex. 1. From 739 take 434.

In this example we say, 4 from 9 and 5 remain; 3 from 3 and nothing remains; 4 from 7 and 3 remain.

$$\begin{array}{r} 739 \\ 434 \\ \hline 305 \end{array} \textit{ Ans.}$$

Ex. 2. What is the difference between 7372 and 3454?

We cannot take 4 units from 2 units, then we say 4 from 12 (10 + 2) and 8 remain. *Carry* 1 to 5, 6 now, 6 from 7 and 1 remains; 4 from 3, we cannot, but 4 from 13 and 9 remain; carry 1 to 3 make 4, 4 from 7 and 3.

$$\begin{array}{r} 7372 \\ 3454 \\ \hline 3918 \end{array} \textit{ Ans.}$$

18. REASON.—The reason of the rule is obvious, and depends on the fact, that the difference of the *corresponding parts* of two numbers is equal to the *difference* of the numbers themselves. If we wish to take 59 from 161, we can say $161 = 150 + 11$ and $59 = 50 + 9$, hence we have

Explanation.

161	=	150 + 11
59	=	50 + 9
102		100 + 2 = 102

19. Proof.—I. Add the remainder to the smaller number, and the sum will be equal to the greater.

II. Subtract the answer from the greater number, and the remainder will be equal to the less.

The sign of subtraction is –, thus, 10 – 6 = 4; read 10 minus or less 6 = 4; and denotes that the number after the sign is to be taken from the number which precedes it.

III.

	(1)	(2)	(3)	(4)
From	78069	86004	496736	69086
Take	16736	19363	279687	47979

5. 100100 – 3656.
6. 59064 – 56899.
7. 100300 – 7786.
8. 709600 – 46867.
9. 63712 – 16019.
10. 58634 – 57016.

Various Examinations.

11. *Subtract* 868704368 *from* 7648200069.
12. *From* 6101507601061 *take* 3806790989.
13. *From* 683216432 *take* 89532179.
14. *From* 4723698007 *take* 494789127.
15. *From* 4723698007 *take* 493298127.
16. *From* 8054010657811 *take* 76909748598.
17. *From* 84331306 *take* 56421808.
18. *From* 998765432 *take* 123456789.
19. *From* 256714894 *take* 93553760.
20. *From* 501505010678 *take* 794090589.
21. *From* 97040341061 *take* 933701697.

SIMPLE MULTIPLICATION.

20. Multiplication is a *short method* of *Addition* when the addends are all alike.

When the addends are all the same it is only necessary to set down one of them, now called the *Multiplicand*, or number to be multiplied.

The number showing how many addends are to be taken is called the *Multiplier*. The answer, or what is called the *Sum* in Addition, is called, in Multiplication, the *Product*.

The following example will sufficiently illustrate these definitions.

Ex. 1. Multiply 7 by 4, in other words, add 7 four times.

By Addition, 7 + 7 + 7 + 7 = 28 *Ans.*

By Multiplication, 7 × 4 (read, 7 multiplied by 4) = 28. 7 is *the multiplicand*, 4 the *multiplier*, and 28 the *product*.

Proof.

. . . .
. . . .
. . . .
. . . .

The numbers 7 and 4 which are multiplied together are also called *factors*, in reference to the product.

The *sign* × implies that the numbers between which it is placed are to be multiplied together.

The multiplier and multiplicand may *change* places, without affecting the product, since 7 times 4 = 4 times 7.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

SIMPLE MULTIPLICATION is confined to the same kind of quantities as Simple Addition.

The multiplier must be invariably an abstract number, as it always indicates the number of times the multiplicand is to be taken.

CASE I.

21. *When the multiplier is not greater than* 12.

RULE.—Multiply each figure of the multiplicand by the multiplier, by means of the *Multiplication Table*, setting down and carrying, just as in Addition.

Ex. 1. Multiply 3798 by 9.

$$\begin{array}{r} 3798 \\ 9 \\ \hline 34182 \end{array}\ \textit{Ans.}$$

In the annexed example, we say, 9 times 8 = 72, set down 2, and *carry* 7; 9 times 9 = 81, and 7 = 88, write down 8, and *carry* 8, and so on. This case may be conveniently *proved* by Addition.

IV.

1. 98677 × 7.
2. 78069 × 9.
3. 86099 × 11.
4. 97645 × 12.
5. 70869 × 8.
6. 790063 × 11.

CASE II.

22. *When the multiplier is greater than* 12.

RULE.—1°. Set down the multiplier under the multiplicand, just as two addends are written.

2°. Commencing with the units' figure, multiply every figure of the multiplicand by every figure of the multiplier, *taking care* to write the *first figure* of each product *as far* from the *units' place* as the *figure* of the multiplier which produces it.

3°. Add all these products, and their sum will be the complete product.

Ex. 2. Multiply 429 by 45, and 3736 by 127.

We first multiply by 5, as in Case I. Next we multiply by 4, placing the first figure, 6, of its product under the tens' place, as directed by the rule.

429
45
2145 = *prod. by* 5
17160 = *prod. by* 40
19305 = *prod. by* 45

3736
127
26152 = *prod. by* 7
7472 = *prod. by* 20
3736 = *prod. by* 100
474472 = *prod. by* 127

Ex. 3. Multiply 897063 by 504017.

897063
504017
6279441 = *product by* 7
897063 = *product by* 10
3588252 = *product by* 4000
4485315 = *product by* 500000
452135002071 = *Total product, by* 504017.

REASON.—

1°. This is a matter of convenience.

2°. Since we cannot multiply by the whole multiplier at once, we divide it into parts by taking each of its figures separately.

3°. This is self-evident.

PROOFS.—I. Multiply the multiplier by the multiplicand and the same result will be obtained.

II. The explanation given (**20**) is the simplest proof.

III. By *Addition*, as in (**20**).

IV. The most natural method of proof is by division; but this would be anticipating that rule, of which the learner is supposed to be ignorant.

V. If we commence with the left-hand figure of the multiplier and fall back to the right, instead of to the left, it will afford a method of proof. This is the eastern mode of multiplying.

VI. Cast the nines out of the sum of the figures of the multiplier and set down the remainder; cast the nines also out of the sum of the multiplicand and set down the remainder; multiply these two remainders together and cast the nines out of the product, this last remainder is the same as the remainder obtained by casting the nines out of the product if the work be correct.

In the second example, we find the sum of the figures of the multiplier, 4+5=9=*one* nine without any remainder; we write 0 above, as in the margin. Again, 4+2+9=15=*one* nine and 6 remainder; set down 6 opposite the first remainder 0. Now multiply these remainders, which gives 0. Lastly the sum of the figures in the product, 1+9+3+0+5=18=*two* nines, and leaving no remainder as before; we write 0 opposite the last figure written.

0
0 0
6

IV.

7. 8679 × 23, and by 28.
8. 76979 × 48, „ 56.
9. 99706 × 73, „ 85.
10. 860478 × 95, „ 102.
11. 860978 × 127, „ 135.
12. 98653 × 557, „ 629.
13. 77869 × 560, „ 978.
14. 75060 × 914, „ 970.
15. 937860 × 9600, and by 908.
16. 207019 × 19307, „ 390.
17. 20136720 × 215670.
18. 40707900 × 6048070.
19. 9940806 × 9773060.
20. 7048960 × 7040980.
21. 9607587 × 948060.

CONTRACTIONS.

23. *When one or both of the factors end in cyphers.*

RULE.—Multiply as if there were *none*, and add so many cyphers to the result as have been neglected.

Ex. 4. 7200 × 3600. Here 72 × 36 = 2592, but we neglected *four* cyphers; therefore the *true* product is 25920000. Any cyphers in the multiplier except at the end are always neglected in the operation.

22. 86400 × 8700, and by 1700. 23. 19900 × 7600, and by 6500.

24. *To multiply by a composite number.*

RULE.—Multiply *successively* by its factors.

Ex. 5. 9648 × 42.

It is evident that 6 times the product by 7, will be the product by 42.

$$\begin{array}{rl} 9648 & \\ 7 & \\ \hline 67536 & = \textit{product by } 7 \\ 6 & \\ \hline 405216 & = \textit{product by } 42 \end{array}$$

24. 86736 × 64, and by 72. 25. 87697 × 121, and by 132.

25. *To multiply by any number consisting of nines.*

RULE.—Add as many cyphers to the multiplicand as there are nines in the multiplier, and subtract the *multiplicand* from the result. Thus, 864 × 99 = 86400 − 864 = 85536 *Ans.*

REASON.—When we add as many cyphers as there are nines we multiply by a number 1 *too great*, hence we must subtract the multiplicand. For, 100 = 99 + 1; 1000 = 999 + 1; and 10 = 9 + 1, &c.

26. Always in multiplying take the *most convenient* number as multiplier. Thus, 7 × 112, say, 7 times 2, &c. This *hint* will be found of considerable utility in many subsequent operations.

IV.

Post Office.

26. Multiply 60538067 by 83.
27 ,, 64059008 by 720106.
28. ,, 60538067 by 94.
29. ,, 64059008 by 807609.
30. ,, 46054 by 78.
31. ,, 247003 by 6020.
32. ,, 46054 by 68.
33. ,, 247003 by 4050.

Dockyards.

34. ,, 376546 by 407091.
35. ,, 96072015 by 2603.

Various Departments.

36. Multiply 98703542 by 700706.
37. ,, 7001009 by 7007867.
38. ,, 978015632 by 9738.

When more than two factors are employed in forming a product, it is called a *Continued Product.* Thus, 3 × 4 × 6 = 72, the continued product of 3, 4, and 6.

NOTES.—1. The method of *proof* (vi.) depends on a property of the number 9, viz.:—'any number divided by 9 will leave the same remainder as the sum of its figures divided by 9.'

2. The explanation of the annexed example in multiplication will be found useful.

Having multiplied by 1 (written the multiplicand), multiply *this product* by the next figure 2, because it is more directly under the eye than the multiplicand. The product by 4 should be found by taking twice the product by 2; the product by 8, by taking twice this last product. For the product by 4, the left-hand figure of the multiplier, write the product by 4 already found, or, take one-half of the product by 8.

```
     763479
      48421
     ------
     763479
    1526958
   3053916
  6107832
 3053916
-----------
36968416659
```

3. When the figures of the multiplicand are fewer than these of the multiplier, or afford facilities for multiplying, like those pointed out above (the multiplier not having similar advantages), multiply as in **26.**

DIVISION.

27. Division is the process of finding how often one number is contained in another. The number to be divided is called the *Dividend*; the number by which the *dividend* is to be divided, is called the *Divisor*; and the result or answer the *Quotient.*

When the dividend does not contain the divisor an *exact* number of times, there is a *Remainder*, which is always less than the divisor.

Division enables us to find the *third* part, or *any* part, of a number; or, it shows how to find one of the factors of a given product, when the other is known.

As this rule shows how often one number must be taken to make up another, it is plain that the number of times we can *subtract* a less number from a greater, is the number of times the greater contains the less. Therefore Division is a *contracted* mode of *Subtraction*, and bears a relation to it somewhat *similar* to that which *Multiplication* has to *Addition.* For instance, if we require to know how often 5 is contained in 30, we may say, $30-5=25$, and $25-5=20$, and so on, until the remainder becomes 0, or *less* than 5; the number of times we subtract is the *Quotient.* But we see at once that 30 contains 5, 6 times by the Multiplication Table.

The *sign* of Division is ÷, and indicates, when placed between two numbers, that the number which precedes is to be divided by that which follows it. Division is also expressed by writing the divisor *under* the dividend, separated by a line. Thus $40 \div 10$, and $\frac{40}{10}$, mean the *same* thing; read, 40 divided by 10.—$(40 \div 10 = 4)$.

Case I.

28. ***When the divisor does not exceed* 12.**

Rule.—1°. Set down the divisor to the *left* of the dividend, separating them by a *curve* line.

2°. Find by the Multiplication Table how often the divisor is contained in the *first* figure, or, if necessary, the number expressed by the first *two* or the first *three* figures of the dividend; set down the figure denoting the number of times as the first figure of the quotient. Subtract *mentally*, the product of this quotient figure and the divisor, from the number expressed by the figures of the dividend already used.

3°. To the remainder annex the next figure of the dividend and proceed as before, until all the figures of the dividend have been considered.

4°. Should there still be a remainder it must be written after the quotient and separated from it by +.

Ex. 1. Divide 3484 by 4.

$$\begin{array}{r} 4\overline{)3484} \\ \hline 871 \text{ Ans.} \end{array}$$

Here we see that 4 will not be contained in 3, that is, we cannot take 4 from 3; therefore, we say, 4 into 34, 8 times and 2 over; we say 2 before 8 makes 28, 4 in 28, 7 times; and 4 into 4, once. Therefore the quotient is 871.

Reason.—1°. Writing the dividend and divisor as directed is a matter of convenience.

2°. In the example, we require to take the *fourth* part of 3484: we cannot take the 4th-part part of 3000, since 3 will not contain 4; therefore we consider 3000 as 30 hundreds, to which when 400 is added it makes 3400; 4 will be contained in 34, 8 times, that is, 800 times, and leaves 2, that is 200. This last we consider not as 200, but as its equivalent 20 tens, to which we add 8 tens, making 28 tens, 4 into 28 tens, 7 tens times. Lastly, 4 into 4 once.

$$\begin{array}{rrl} 4) & 3484 & \\ & 3200 & 800 \\ \hline 4) & 284 & 70 \\ & 280 & \\ \hline 4) & 4 & 1 \\ & 4 & \\ \hline & & 871 \text{ Ans.} \end{array}$$

Since 3484 = 3000 + 400 + 80 + 4, the work in full will be as in margin.

V.

1. 98656 ÷ 3, and by 5. 2. 83295 ÷ 7, and by 8

3. 9349344 ÷ 9, and by 12.

LONG DIVISION.

29. *When the divisor is greater than* 12.

RULE.—Instead of multiplying and subtracting *mentally* as in *Short Division*, we must write down the *whole* of the work as in the following example.

Ex. 2. 86560÷11.

As this is an example coming under *Short Division*, the best manner to understand the *reason* and *nature* of the process is to work the example both ways, and compare the different steps of each operation. We just say, 11 into 86, as in the former case, 7, then 7 × 11 = 77, which we write *under* 86 and subtract; to the remainder 9 we *bring down* 5, the next figure of the dividend, marking it by a dot to show that it has been *used*, and so on. After *all* the figures of the dividend have been brought down, there is a remainder 9, which is written as before When any dividend, formed of a remainder and a figure brought down, is *less* than the divisor, we must place a 0 in the quotient and bring down another figure.

```
11)86568
   77···
   ──── (7869 + 9 Rem.
    95
    88
    ──
     76
     66
     ──
     108
      99
     ───
       9
```

Ex. 3. 1104654+327.

Here, as 3 will not be contained in 1, we must say 3 in 11, 3 times; we write 3 in the quotient and multiply. After subtracting, we say 3 in 12, 4 times, but 4 × 2 (the second figure of the divisor) = 8, which would be *too great*; therefore the proper quotient figure is 3. Again, 3 into 25, 8 times, but 8 × 2 = 16, which would also be *too great*, then we must put 7 in the quotient, &c.

```
327)1104654
    981···
    ────(3378 + 48 Rem.
    1236
     981
    ────
     2555
     2289
     ────
      2664
      2616
      ────
        48
```

PROOF.—I. Multiply the quotient by the divisor and the product + the remainder (if any), will be equal to the dividend.

II.—Subtract the remainder from the dividend, and divide what is left by the quotient; the new result or quotient will be the same as the given divisor.

V.

4. 70633 ÷ 23, and by 24.
5. 96802 ÷ 29, „ 37.
6. 186879 ÷ 63, „ 73.
7. 897608 ÷ 98, „ 106.
8. 2760865 ÷ 179, and by 224.
9. 6960508 ÷ 586, „ 625.
10. 864739 ÷ 907, „ 943.

Post Office, Customs, &c.—Junior Appointments.

11. Divide 7461968 by 8.
12. „ 7461968 by 9.
13. „ 434589760126 by 7.

Education Office.

14. Divide 106397320524 by 12.

III.—Add every *alternate* line of figures just as they stand in the operation, their *sum* should be equal to the dividend.

IV. Cast the *nines* out of the divisor and quotient, multiply the remainders and cast the nines out of their product; to this last remainder add the remainder found by casting the nines out of the remainder (if any) in the example. This sum should be equal to the remainder found by casting the nines out of the dividend.

Reason of Proofs.—I. This follows at once from the definition of Division.

II. This is merely *reversing* the process.

III. This proof depends on the principle, 'The whole is equal to the sum of all its parts.'

IV. This depends upon a property of the number 9 already referred to.

30. As we are unable to see at once how often the divisor is contained in the dividend, we actually separate the latter into *parts* during the process, and find how often the divisor is contained in each part. The quotient is the *sum* of the number of times the divisor is contained in each part of the dividend. The dividend is divided in the following manner during the process:—*one part* is the product of the divisor, and the first quotient figure having its local value; the *other part* is the difference between this product and the whole dividend, and so on.

Whenever a figure is written in the quotient the same as one already placed there, write down the product of the former figure and divisor. If 3 be placed in the quotient after 6, take *one-half* of the product by 6; and so on.

31. The following *axioms* have been found useful, and may be employed with considerable advantage whenever they are applicable to an example: indeed, almost all the contractions in arithmetic are derived from these two simple principles.

I. The product of any two numbers is the *same* as *one-half* of one number multiplied by *twice* the other; or, *one-fourth* of one multiplied by *four times* the other, and so on. For instance, 380 yds. at 7½*d.* per yard = 190 at 1*s.* 3*d.* (7½*d.* × 2) = 95 at 2*s.* 6*d.*, then take ⅛ = £11 17*s.* 6*d.* *Ans.*

Again 7½ yds. at 3*s.* 6*d.* = 15 at 1*s.* 9*d.* = 30 at 10½*d.* = 315*d.* = £1 6*s.* 3*d.* *Ans.* Also, 7 cwts. 2 qrs. 14 lbs. at £4 10*s.* 8*d.* = 15 cwts. 1 qr. at £2 5*s.* 4*d.* = 61 cwts. at 11*s.* 4*d.* = £34 11*s.* 4*d.* *Ans.*

II. The quotient of any two numbers is the same as *one-half* of one, divided by *one-half* the other, or *one-fourth* of one, divided by *one-fourth* of the other. It is also the same as *twice* one divided by *twice* the other, or any *multiple* or *part* of one, divided by the *same multiple, or a like part* of the other. For example, 18 ÷ 4½ = 36 ÷ 9 = 4; and 23⅓ ÷ 3⅓ = 70 ÷ 10 = 7; again, 1000 ÷ 68 = 500 ÷ 34 = 250 ÷ 17 = 14$\frac{12}{17}$.

Candidates who are to know 'Elementary Arithmetic' are proposed two questions, such as the two following:—

15. Divide 846123998776 by 7.
16. „ 630762540981 by 652.

Admiralty (Dockyards).

17. Divide 750324098176 by 826.

Various Departments.

18. Divide 2370056 by 93.
19. „ 327840937575 by 716.
20. „ 5946784379 by 492.
21. „ 2370056 by 79.
22. „ 32780937575 by 589.

32. *To divide by a composite number.*

Rule.—1°. Divide first by one factor, as in Short Division, then divide this quotient by the other factor.

2°. Multiply the *last remainder* by the *first divisor*, and to the *product* add the *first remainder* for the *true* remainder.

Ex. 4. 86497 ÷ 36.

When we divide by the second factor 6, we have a remainder 4; this 4 is not 4 units but 4 *sixes*. Then 6 × 4 + 1 = 25 the true remainder.

$$36\left\{\begin{array}{l}6)\underline{86497}\\ 6)\underline{14416}+1\end{array}\right.$$

2402 + 4 × 6 + 1 = 25. *Ans.*

V.

23. 486906 ÷ 42, and by 81. 24. 786456 ÷ 108, and by 121.
25. 5604060 ÷ 385 and by 48.
26. 7936497 ÷ 648 and by 49.
27. 8695687 ÷ 792. and by 108.

33. *When the divisor ends in cyphers.*

Rule.—1°. Neglect the cyphers, and cut off as many figures from the right of the dividend.

2°. Divide with the remaining figures of the divisor, and annex the figures cut off from the dividend to the remainder.

Ex. 5. 96846 ÷ 1200 = 968,46 ÷ 12,00 = 80 + 846 *Rem.*

Hence, to divide by any number consisting of 1 followed by any number of cyphers, we have merely to cut off from the right of the dividend as many figures as there are cyphers: the other figures of the dividend form the quotient, and those cut off, the remainder.

Ex. 6. 18767 ÷ 100 = 187,67 = 187 + 67 *Ans.*

Ex. 7. 96186 ÷ 7300.

73,00)961,86
73
—— (13 + 1286, *Rem.*
231
219
——
1286

V.

28. 684379 ÷ 1400, and by 1000. 29. 469786 ÷ 100, and by 6300.
30. 9860400 ÷ 1700, and by 1860.

CONTRACTIONS.

34. The work of division may be considerably *contracted* by multiplying and subtracting *mentally* at the same time, *omitting* the product of the divisor and each quotient figure, and writing the remainder only. This method should not be introduced until the learner can work any example in the rule with the *greatest* ease; otherwise his progress will be *retarded.* The work will stand as below. Compare both methods, and the reason will be evident.

Usual Method.
173)6847329
519····
—— (39579
1657
1557
——
1003
865
——
1382
1211
——
1719
1557
——
162

Contracted Method.
173)6847329
—— ···
1657 (39579
——
1003
——
1382
——
1719
——
162

35. *To divide by* 25.

RULE.—Multiply the dividend by 4; cut off *two* figures to the right; one fourth of the number expressed by these two figures will be the *true* remainder.

Ex. 8. 7324÷25=7324 × 4=29296, and $292\frac{96}{4}$=292+24 *Ans.*

36. *To multiply by* 25.

RULE.—Annex 00 and divide by 4.

Ex. 9. 375 × 25=37500÷4=9375 *Ans.*

37. *To divide by* 35, 45, 55, *&c*

RULE.—Divide *twice* the dividend by 70, 90, 110, &c. Take *half* the remainder for the *true* one.

Ex. 10. 69475÷35=138950÷70=1985 *Ans.*; and 785675÷45 =785675 × 2÷90=1571350÷90=17459+40=17459+20 *Ans.*

38. *To multiply by* 125.

RULE.—Annex 000 and divide by 8.

Ex. 11. 116 × 125=116000÷8=14500 *Ans.*

39. *To divide by* 125.

RULE.—Divide 8 times the dividend by 1000, and take *one-eighth* of the remainder for the *true* one.

Ex. 12. 2125÷125=2125 × 8÷1000=17000÷1000=17 *Ans.*

40. *To multiply by* 35, 45, 55, *&c.*

RULE.—Multiply by 70, 90, 110, &c., *respectively*, and take *one-half* of the product.

Ex. 13. 735 × 35=735 × 70÷2=51450÷2=25725 *Ans.*

V.

31. 968645÷25, and by 35.
32. 897895÷60, and by 125.
33. 978685÷25, and by 45.
34. 996846÷25, and by 45.

41. *To multiply when the multiplier contains* ONE-HALF *or* QUARTERS.

RULE.—For ½, add *one-half* of the multiplicand to the result obtained by the rule; for ¼, add *one-fourth* of it; and for ¾, add the result obtained by multiplying the multiplicand by 3 and dividing the product by 4.

Ex. 14. Multiply 168 by 7½.

Here we multiply 168 by 7 in the *usual* way; we then say 2 into 16, 8 times; 2 into 8, 4 times, and add the 84 to the product by 7.

Reason.—When we multiply any number by 1, we get that number itself as product, and of course when we multiply by ½, that is ½ of 1, we will get *one-half* the number as product.

```
 168
  7½
────
1176
  84
────
1260 Ans.
────
```

V.

35. 97688 × 19½, and by 36½. | 36. 86408 × 46¼, and by 86¾.

42. *To divide by any number containing* ONE-HALF *or* QUARTERS.

Rule.—1°. Reduce both dividend and divisor to *halves* or *quarters*; then use *these* as dividend and divisor.

2°. The remainder will be in *halves*, if we reduce to halves; in *quarters*, if we reduce to quarters.

Ex. 15. 444 ÷ 18½.

Here we reduce both to *halves*, and then divide.

```
18½)444
 2    2
 ─   ──
37) 888
    74
    ──(24 Ans.
    148
    148
    ───
```

37. 86976 ÷ 9½, and by 25½. | 38. 79684¼ ÷ 6¾, and by 29¼.

Note.—In any example in division we may divide the divisor and dividend by the same number, and use the quotients instead of the original numbers. Thus, 96 ÷ 20 = 24 ÷ 5 = 4⅘.

In dividing by several factors the following rule should be observed to find the *true* remainder:—

Multiply the last remainder by the previous divisor, or divisors if more than one has been used, and to the product add the previous remainder (if any) for the *true* remainder.

COMPOUND ARITHMETIC.

WEIGHTS, MEASURES, AND TABLES.

I. MONEY.[*]

4 farthings	=	1 penny
12 pence	=	1 shilling
24*d.* or 2*s.*	=	1 florin
30*d.*	=	1 half-crown
60*d.* or 2 h. crs.	=	1 crown
8h. crs. or 10f. or 20*s.*	=	£1
21 shillings	=	1 guinea

The *symbols* for pounds, shillings, and pence, are, *£*, *s*, *d*.

II. AVOIRDUPOIS.[†]

16 drachms	=	1 ounce
16 ounces	=	1 pound
14 pounds	=	1 stone
28 pounds or 2 st.	=	1 quarter
4 quarters or 8 st. or 112 lbs.	=	1 hundredwt.
20 hundreds or 160 st.	=	1 ton

The *symbols* respectively are, dr., oz., lb., st., qr., cwt., t.

III. TROY.

24 grains	=	1 pennywt.
20 pennywts.	=	1 ounce
12 ounces	=	1 pound

The *symbols* are, gr., dwt., oz., lb. This weight is used for *gold*, *silver*, and *jewellery*; and in *philosophical experiments*. The pound *troy* contains 5760 grains, the pound *avoirdupois* 7000 grains.

IV. APOTHECARIES.

20 grains	=	1 scruple
3 scruples	=	1 drachm
8 drachms	=	1 ounce
12 ounces	=	1 pound

The *symbols* are, gr., sc., dr., oz. Apothecaries use it for *mixing* their medicines. The *grain*, *ounce*, and *pound*, are the *same* as in TROY.

---

[*] Our *standard* gold coins consist of 22 parts pure gold, and 2 parts copper or alloy; that is, $\frac{11}{12}$ pure gold and $\frac{1}{12}$ copper. The value of gold, from which the coins are made, called gold of the *mint fineness*, is £3 17*s.* 10½*d.* per ounce. From every pound Troy of standard gold, $46\frac{29}{40}$ sovereigns are made; hence every sovereign weighs $123\frac{171}{823}$ grains. Gold is a *legal tender*, without limitation, and is the *general standard* of value.

Our *standard* silver coins are made of a metal consisting of 37 parts of pure silver and 3 parts copper. The *mint price* of standard silver is 5*s.* 6*d.* per ounce. Every pound Troy of silver is coined into 66*s.*, consequently, a shilling weighs $87\frac{3}{11}$ grains. Silver is a *legal tender* for any sum not exceeding £2.

Copper is now coined at the rate of 48 pence to the pound Avoirdupois; hence a penny weighs ⅓ of an ounce, and 12 pennies ¼ of a pound. Copper is a *legal tender* only to the amount of one shilling.

[†] The *imperial pound*, Avoirdupois, is the unit of weight for all heavy goods. An *imperial gallon* of distilled water weighs 10lb., and consists of 277·274 cubic inches. A

V. Cloth.

2¼ inches = 1 nail
4 nails = 1 quarter
4 quarters = 1 yard
5 quarters = 1 ell, English
3 quarters = 1 Flemish ell
6 quarters = 1 French ell

VI. Length.

12 inches = 1 foot
3 feet = 1 yard
5½ yards = 1 pole or perch
40 poles = 1 furlong
8 furlongs = 1 mile
3 miles = 1 league

VII. Grain.

6 stones = 1 barrel of *bran*
12 „ = 1 „ of *malt*
14 „ = 1 „ of *oats*
16 „ = 1 „ of *bere*
20 „ = 1 „ of *peas* or *wheat*
16 „ = 1 *bag* of flour
20 „ = 1 *sack* of flour
2 barrels = 1 *quarter* of wheat or malt

VIII. Surface.[*]

144 *square* in. = 1 *square* foot
9 *square* feet = 1 *square* yard
30¼ *sq.* yds. = 1 *square* rod, pole, or perch
40 perches = 1 rood
4 roods = 1 acre

IX. Capacity.

4 gills or naggins = 1 pint
2 pints = 1 quart
2 quarts = 1 pottle
2 pottles = 1 gallon
2 gallons = 1 peck
4 pecks = 1 bushel
8 bushels = 1 quarter
5 quarters = 1 load
63 gallons = 1 hogshead
36 gallons = 1 barrel of beer
2 hogsheads = 1 pipe or butt
2 pipes = 1 tun

In some places a barrel of beer has 32, in others, 34 gallons.

X. Time.

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
4 weeks = 1 month
12 months = 1 year
365 days = 1 year
366 days = 1 *leap year*

A *leap-year* occurs every fourth year. If the number expressing the year can be divided by 4 without leaving a remainder, it is a leap-year, except it be an exact century.

XI. Solidity.

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard
40 cubic ft. rough timber } 1 ton
50 cubic ft. hewn timber } or load
42 cubic ft. = 1 ton of shipping

---

pound, *Avoirdupois*, is = 14 oz. 11 dwt. 16 gr. *Troy*: therefore an ounce Troy, *exceeds* an ounce Avoirdupois by 1 dwt. 18½ gr. 144 lbs. Avoirdupois = 175 lbs. Troy.

[*] The *difference* between Irish and English measure is in the *perch*: 5½ yds. make 1 p. English, and 7 yds. 1 p. Irish; hence 11 Irish miles = 14 English miles, and since $11^2 = 121$ and $14^2 = 196$, it follows, that 121 Irish acres = 196 English acres, or 30¼ Irish = 49 English acres, or 3 a. 3 r. 5 p. Irish = 6 a. 0 r. 20 p. English. Irish measure is illegal except by special agreement.

XII. SURVEYORS' MEASURE.

100 links or 4 perches = 1 chain
10 square chains = 1 acre
1 mile = 80 chains
1 square mile = 640 acres

THE CIRCLE

Consists of 360 degrees, each degree = 60 minutes, and each minute = 60 seconds. Degrees, minutes, and seconds are marked thus, °, ′, ″. Every degree is = 60 geographical or $69\frac{1}{10}$ English miles.

XIII. COALS.

5 stones = 1 bushel
2 bushels = 1 bag
8 „ = 1 quarter
2 bags = 1 barrel
8 barrels = 1 ton

VARIOUS.

12 articles = 1 dozen
12 dozen = 1 gross
20 articles = 1 score
5 score = 1 hundred
6 score = 1 long hund.
24 shts. of paper = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
30 deals = 1 quarter of timber.
120 deals = 1 great hundred.

The Civil Service Commissioners usually examine Tidewaiters, Weighers, &c., on the Tables, thus:—

'Write out the tables of
Avoirdupois weight, Troy weight, Long measure, Square measure, Wine measure, Ale and Beer measure.'

The following have also been asked:—

'Write out—
Measure of Time.
Dry Measure.
Ale and Beer Measure.'

VI.

MISCELLANEOUS QUESTIONS.

1. Multiply ninety thousand seven hundred and eight by sixteen thousand and thirteen, and add 99 to the result.

2. From the product of thirteen thousand and nine and nine thousand and sixteen, take nine hundred and ninety-eight.

3. Divide the sum of 7896 and 8908 by 11.

4. Multiply the difference of 1000 and 53 by 79.

5. To the product of 1013 and 7009 add 73, and divide the sum by 2.

6. Take sixty thousand and fifty-four from seventy-eight thousand and ninety, and multiply the remainder by seven hundred and sixty.

7. From the product of one thousand and one hundred and three, take 99.

8. From the quotient of thirteen thousand and thirty-nine by thirteen, take 4.

9. Write down the sum, difference, product, and quotient of 9603 and 1067.

10. The product of two numbers, one of which is 365, is 389455; what is the other?

11. What is the continued product of the sum, difference, product, and quotient of 72 and 24?

12. The quotient of two numbers, the lesser of which is 49, is 347, and the remainder is 27; what is the greater?

13. How many days were in the first five months of the year 1860?

14. How many days from January 20, 1857, until August 14, 1863?

15. A regiment was supplied with 576 horses which cost £25 each; what was the entire cost?

16. How many pages in 30294 words, every page having 51 lines of 9 words each?

17. How many farthings are there in £497, there being 960 in a £?

18. How many pence should a person pay for 44 lbs. of beef at 8¾*d.* per lb.?

19. How many chickens at 10½*d.* each could a person buy for 115½*d.*?

20. Divide 25*s.* between two boys, giving one 3*s.* more than the other.

21. What is the number of lines in the page of a book which contains 1110 letters, every 37 letters forming a line?

22. What is the difference between the continued product of 17, 93, 46, 59, and that of 227, 247, and 58?

23. Add together 379876, 583974, 3978653, 98, 7894, 569, 3747, 7, 974869, 89767, 54869, 37897, 9879, 5678089, 24, and 397, and take 11796986 from the sum.

24. Add all the numbers ending in 9 from 1 to 300 together.

Proposed to Tidewaiters and Weighers.

25. Multiply four millions forty-nine thousand and three, by thirty-six thousand and twenty-nine.

VII.

MISCELLANEOUS QUESTIONS.

(a)

1. Explain the use of the *cypher* in our system of Notation.

2. Express in figures nine thousand and seventeen millions fifty-nine thousand and ninety-six; and write in words 7106597048.

3. At an election for two members of Parliament, one candidate had a majority of 189 out of 1135 votes; how many voted for each?

4. The population of Ireland in 1861 was 5798965, and in 1870 it was 5519641, if it decreased uniformly find its decrease every year.

5. There were in 1868, in England 20930779 sheep, in Wales 2668505, in Scotland 7112112, in Ireland 4648158, and in the Isle of Man and Channel Islands 73972; if each sheep be worth £1¾, give in words the value of the sheep of the United Kingdom.

6. If an elm produces 299580 seeds, and if one-half of these each produce a tree, find the number of trees that should be produced from 89 elms in 79 years.

(b)

1. Define the following terms: *Quantity*, *Number*, *Addition*, *Division*, &c.

2. How many years of 365¼ days each are in 18993 days?

3. In 1869 the capital of the 68 principal English railways was £418277679, of the 8 Scotch £56575607, and of the 17 Irish £27409601, find the excess in capital of the English railways above the Irish and Scotch together.

4. Write out the tables of troy and avoirdupois weight, and state for what they are respectively used?

5. Subtract two thousand and fifty-nine from forty-one thousand and thirty-three: then to the remainder add nineteen thousand and eighty, and express eleven times the result in words.

6. An estate has 7649 oak trees, 697 elms, 5073 beeches, 719 ash, and 838 poplar; if 24 be cut down every working day, in how many weeks will they be all cut down?

(c)

1. Add the following numbers and divide the sum by 17:—7986 8975, 3998, 976, 5899, 379, 5764, 7788, 9787, 377, 9789, 789, 6539, and 365.

2. Write in words 300060030; also the difference between one hundred and a million?

3. Divide 7890567 by 42, by long division; also by its factors 6 and 7, and prove the accuracy of the result by multiplication.

4. How many days will 9867 pounds of hay last 13 horses, if every horse eats 33 lbs. a day?

5. A boy has a bag containing 2960 nuts, he takes out 7 dozen for himself, and then divides the rest among 19 school-fellows, keeping those remaining over; how many had he and how many had each of the other boys? Had he at first divided equally among them all, how many nuts would each have got?

6. The distance of the earth from the sun is now ascertained to be about 92070000 miles; if the rays of his light reach the earth in 495 seconds, find their velocity per second.

(*d*)

1. From 5297849362972 take 942876043184 and divide the remainder by 13.

2. There are about twelve hundred and twenty-one millions of inhabitants on the earth: if this number die in 33 years, by how many do those dying every year exceed the population of the British Isles which is 30937487?

3. Divide 437 apples between two boys, giving one 29 more than the other.

4. If £1 be equal to 24¾ francs, find the number of francs equal to £999.

5. One of the London letter-carriers delivers letters 3 times a day, walking 9½ miles on each delivery. Suppose he continues for 29 years as a letter-carrier, find how far he will have walked neglecting Sundays and leap years.

6. If a franc be worth 9½d. British, find the number of francs equal to 2132113½d. Taking our sovereign equal to 24¾ francs, find how many sovereigns should be given for 2132113½ francs.

(*e*)

1. Multiply 949736018 by 4097080.

2. What number multiplied by 78079 will give as product 99 times the difference between 97889023 and 97654786?

3. A gentleman uses 14 dozen of sherry every month, worth £2 per dozen, 3 dozen of claret per week, worth £3 per dozen, and 3 dozen of port every fortnight, worth £2 10s. per dozen, how much has he remaining after paying for his wine out of a salary of £1000?

4. A city containing 758043 inhabitants contains five parishes, the population of the first is 78967, of the second 139753, of the third 145069, the other two parishes have each an equal number of inhabitants, find the population of each of these.

5. The pulse of an invalid, while he is awake, beats 87 times in a minute, and while asleep 69 times; how often will it beat in a week if he sleep 9 hours a day?

6. Give the rule for dividing by a composite number; divide by means of its factors 165 into 698966, and show how the true remainder can always be found.

(f)

1. Divide 21593754901082715 by 8697.

2. Explain the use of the "signs" of the first four rules, and find the value of the following:—(a) 96 + 17 − 39; (b) 75 + 96 ÷ 16; (c) 189 − 7 × 19; (d) 7 × 6 + 9 × 17; (e) 170 ÷ 17 + 365 ÷ 5 − 4; add the results and divide by 48½.

3. The gross rent of a farm at £3¼ per acre is £962¼; find the number of acres in the farm; and if the rent were increased to £4¾ per acre, what would the rent then be?

4. During the year 1867 there were delivered in the British isles 774831000 letters, and during 1868, 808117968 letters. If letters were delivered every day, Sunday and Christmas Day excepted, find the increase per diem during the year.

5. A postman delivers in a city twice as many letters on Monday and half as many on Sunday as any of the other days of the week; if his whole weekly delivery be 8505, how many does he deliver on Monday?

6. Two regiments went into battle of the same strength, 50 men of one were killed, and 450 of the other; after the battle just twice as many remained of the one as the number remaining of the other; how many men were in each before the battle?

REDUCTION.

43. Reduction is the bringing of a quantity, consisting of one or more denominations, to another quantity of the same kind, without altering its value.

Reduction Descending.

44. *To reduce a quantity to a lower denomination.*

Rule.—Multiply the highest denomination by the number which shows how many of the next lower denomination make 1 of it, adding in the number of that denomination (if any), and so on, until the required denomination is arrived at.

Ex. 1. Reduce £59 into pence.

Here we multiply £59 by 20, because there will be 20 *times* as many shillings as pounds. Again, we multiply the shillings, 1180, by 12, because there will be 12 *times* as many pence as shillings.

£59
20
1180 *shillings.*
12
14160 *pence. Ans.*

Ex. 2. Reduce 29 tons 17 cwts. 3 qrs. 27 lbs. 12 oz. into oz.

tons cwts. qrs. lbs. oz.
29 17 3 27 12
20
580 + 17 = 597 cwts.
4
2388 + 3 = 2391 qrs.
28
19128
4782
66948 + 27 =
66975 lbs.
16
1071600 + 12 = 1071612 oz.

Read the table of Avoirdupois Weight, and the reason of the annexed operation will be evident.

Ex. 3. Reduce £7869 19s. 11¾d. to farthings.

Here, in the operation of multiplying by 20 we add in the 19, and in the same manner the 11 in multiplying by 12.

£	s.	d.
7869	19	11¾
20		

157399 shillings.

12

1888799 pence.

4

7555199 farthings. *Ans.*

Ex. 4. Reduce 75 ells 3 qrs. 3 nls. to inches.

Having reduced to nails, we multiply by 2¼ for inches, as already explained.

ells.	qrs.	nls.
75	3	3
5		

378 ells.

4

1515 nails.

2¼

3030

¼ of 1515 = 378¾

3408¾ inches. *Ans.*

Ex. 5. Reduce 367 ac. 3 rds. 35 pls. 25 yards 8 ft. 74 in. to sq. in.

The learner must be careful in multiplying by 12 and 12, for 144, not to add the 74 inches until he is multiplying by the second 12.

ac.	rds.	pls.	yds.	ft.	in.
367	3	35	25	8	74
4					

1471 roods.

40

58875 poles.

30¼

1766275

14718¾

1780993¾ yards.

9

16028951¾ feet.

12

192347421

12

2308169126 inches. *Ans.*

Ex. 6. Reduce 18 miles 5 fur. 30 per. 2 yds. 2 ft. 10 in. into inches.

mls. f. p. yds. ft. in.
18 5 30 2 2 10
8

149 furlongs.
40

5990 perches.
5½

29952
2995

32947 yards.
3

98843 feet.
12

1186126 inches.

In this example the furlongs, perches, &c. are added in when multiplying, and not as in example 2.

VIII.

Post Office.

1. Reduce 15 guineas to farthings.
2. „ 15 miles 2 furlongs 36 perches 2 yards to feet.
3. „ 42 cwts. 3 qrs. 17 lbs. 6 oz. to oz.
4. „ 18 tons 17 cwts. 2 qrs. 1 lb. to lbs.

Admiralty (Dockyards).

5. Reduce 42 miles 5 fur. 13 poles to inches.

Candidates who are to know 'Elementary Arithmetic,' such as in Police Courts, Science and Art, Board of Trade, Fisheries, &c.

6. Reduce 3 tons 9 cwts. 2 qrs. 4 lbs. 6 oz. to oz.
7. „ 1 ton 4 cwts. 3 qrs. 2 lbs. 3 oz. to oz.
8. „ 16 tons 4 cwts. 3 qrs. 15 lbs. 13 oz. to oz.
9. „ 13 miles 5 fur. 9 poles 3 yds. to inches.
10. „ 42 miles 3 fur. 23 poles 3 yds. to inches.

Tidewaiters, &c.

11. Reduce 159 quarters of corn to quarts.
12. In 34 lbs. 10 oz. 12 dwts. 16 grs. troy how many grains?

India Board.

13. Reduce 3 acres 20 rods 12 sq. yards and 7 sq. feet to square feet.

War Office.

14. How many grains are there in 17 lbs. 3 oz. 7 dwts.?

Education and Constabulary Offices.

15. Reduce 15 lbs. 10 oz. 14 grs. troy to grains.

Candidates required to know 'Vulgar and Decimal Fractions.'

16. Reduce 2 miles 1 fur. 12 pls. 1 ft. 8 in. to inches.
17. „ 3 wks. 4 days 5 hrs. 54 mins. to seconds.
18. „ 13 yds. 7 ft. 19 inches (square measure) to inches.
19. „ 496 sq. yds. 2 sq. ft. to inches.
20. How many feet are there in 18 miles 6 fur. 12 pls. 2 yds.?
21. Reduce 17 lbs. 6 oz. 12 dwts. 7 grs. to grains.
22. „ 54 cwts. 3 qrs. 18 lbs. 14 oz. to drachms.

Various Departments.

23. Reduce £9 19*s.* 9¾*d.* to farthings.
24. „ 13 lbs. 7 oz. 16 grs. to grains.
25. „ 47 cwts. 1 qr. 16 lbs. 2 oz. to drachms.
26. „ £527 16*s.* 8½*d.* to farthings.
27. „ 375 cwts. 2 qrs. 1 st. 13 lbs. to ounces.
28. * In 11 miles 0 fur. 6 pls. 4 yds. how many feet?
29. Reduce 14 lbs. 11 oz. 11 grs. to grains.
30. „ 42 wks. 8¼ hrs. 15½ mins. 5 secs. to seconds.
31. How many seconds are there in 35 days 8 hrs. 48 mins. 29 secs.?

REDUCTION ASCENDING.

45. *To reduce a quantity to a higher denomination.*

RULE.—Divide the quantity by the number expressing how many of this lower denomination make *one* of the next higher denomination; and so on, step by step, until the required denomination is arrived at.

Remainders are always the same name as the dividends from which they are derived.

In REDUCTION ASCENDING we use division: in REDUCTION

---

* Proposed to Tidewaiters and Weighers.

DESCENDING we use multiplication. The *one case* proves the *other*.

Ex. 1. To reduce 7328 farthings to pounds.

4)7328 *farthings.*

12)1832 *pence.*

2,0)15,2 + 8

£7 12*s.* 8*d.* *Ans.*

Proof of Example 1.

Reduce £7 12*s.* 8*d.* to *farthings.*

```
 20
───
152
 12
───
1832
   4
────
7328 farthings.
```

We first divide by 4, because for every 4 farthings there will be *one* penny. We divide these pence, 1832, by 12; because for every 12 pence there must be *one* shilling; and lastly, we divide these shillings by 20 to bring them to pounds, using the *contracted* method.

Ex. 2. Reduce 73816 drachms to cwts.

```
    ⎧ 4)73816 drachms.
 16 ⎨ ─────
    ⎩ 4)18454
      ─────
    ⎧ 4)4613 oz. + 8
 16 ⎨ ─────
    ⎩ 4)1153 + 1
      ─────
    ⎧ 7)288 pounds + 5
 28 ⎨ ─────
    ⎩ 4)41 + 1
      ─────
      4)10 qrs. +  1 × 7 + 1
      ─────
```

2 cwts. 2 qrs. 8 lbs. 5oz. 8 drs. *Ans.*

Proof of Example 2.

Reduce 2 cwts. 2qrs. 8lbs. 5oz. 8drs. to drs

```
   4
  ──
  10 quarters.
  28
 ───
 288 pounds.
  16
 ───
4613 ounces.
  16
 ───
73816 drachms.
```

As 16 drachms make 1 ounce, we must first divide by 16: we use its factors 4 × 4. Having reduced to ounces, we divide by 16 again, to bring these to pounds; next, we divide by 28, since 28 lbs. = 1 qr.; for this we use the factors 7 and 4; and lastly, we divide by 4, to bring the quarters to cwts., and find 2 cwts. 2 qrs. 8 lbs. 5 oz. 8 drs. *Ans.*

Ex. 3. **In 3196800 cubic inches how many loads of hewn timber?**

```
1728)3196800
     1728
     ----- (1850 cubic feet.
      14688
      13824
      -----
        8640
        8640             Then 5,0)185,0
        ----                  ------
           0                  37 loads. Ans.
```

In this example we reduce the cubic inches to cubic feet, and find 1850, which divided by 50, the number of cubic feet in a load, gives 37, the answer.

IX.

Post Office.

1. In 6169 pence how many pounds?
2. In 15346907 oz. how many tons, cwts., &c.?

India Board.

3. In 523769 grains (Troy weight) how many lbs., oz., &c.?
4. In 569705 sq. ft. how many acres, roods, &c.?
5. In 29336935 seconds how many weeks, days, &c.?
6. How many lbs., oz., &c., are there in 111454 grains of gold?
7. How many miles, furlongs, &c., are there in 348164 inches?

Excise.

8. In 457241 lbs. (Avoirdupois) how many tons, cwts., &c.?
9. In 640825 seconds how many weeks, days, &c.?
10. In 2160000 cubic inches how many loads of hewn timber?
11. In 150000 eggs how many gross?

Education and Constabulary Offices.

12 Reduce 34677 square yards to acres, &c.

Where Candidates should know 'Vulgar and Decimal Fractions.'

13. In 23221 grs. of gold how many lbs., oz., &c.?
14. In 32391 oz. of sugar how many tons, cwts., &c.?
15. In 5792685 inches how many miles?

Various Departments.

16. How many miles, &c., are there in 247391 inches?
17. How many lbs., oz., &c., are there in 266450 grs. (Troy)?
18. Reduce 657918 cubic inches to yards, feet, and inches.

19. Reduce 138297 cubic inches to yards.
20. How many miles are there in 3126749 inches?
21. In 86754 oz. (Avoirdupois) how many tons, &c.?
22. Reduce 986877 cubic inches to yards, feet, and inches.

COMPOUND REDUCTION.

46. In several exercises, both of the cases of Reduction already considered have to be employed, as in the following questions:

Various Examinations.

Ex. 1. How many half-crowns are there in 83654 dollars, each worth 4s. 6d.?

dols.
83654
54d. = 4s. 6d.

334616
418270

a half-crown = 3,0d.)451731,6d.

150577 *h. crns.* 6d.

Post Office.

23. How many fourpenny pieces in 37689s.?
24. In £2470 how many crowns?
25. How many half-sovereigns in 7642 guineas?

Excise.

26. Reduce 564 English ells to nails.

Various Departments.

27. How many francs at 9½d. each are there in £1734 6s. 10½d.?

Not proposed at Examinations.

28. A person has 250 fourpenny pieces and 57 threepenny pieces, how much money has he?

29. How many labourers can a person employ with 35 guineas, each receiving half-a-crown?

COMPOUND ADDITION.

47. A Compound Number consists of quantities of the same kind, but of different denominations.

Rule.—1°. Set down the addends so that the same denominations may stand under each other.

2. Commencing at the right hand, or lowest denomination, add it up; find how many *ones* or *units* of the next higher denomination in the sum; set down the remainder (if any), and carry the *ones* to the next denomination, and so on.

Ex. 1. Add £67 16*s.* 8½*d.*; £100 13*s.* 7¾*d.*; £168 16*s.* 10½*d.* together.

We begin by adding the farthings, and find their sum 7*f.*=1 penny and ¾: we write the ¾ under the farthings, and *carry* the 1*d.* to the pence. We find the sum of the pence column 26, and 26*d.*=2*s.* 2*d.*, set down the 2*d.* and *carry* the 2*s.* In adding the shillings, we say, 2 and 6 are 8, 8 and 3 are 11, 11 and 6 are 17, set down 7 and carry 1. This 1 and the 3 *ones* in the tens' column make 4; we say half of 4 is two, and carry 2 to the pounds, because there will be half as many *pounds* as *tens* in the shillings. If the sum of the tens be odd we write down 1, and carry the half of the next smaller even number.

£	*s.*	*d.*
67	16	8½
100	13	7¾
168	16	10½
337	7	2¾

Ex. 2.—Add 148 cwts. 2 qrs. 16 lbs.; 96 cwts. 3 qrs. 22 lbs.; 149 cwts. 1 qr. 10 lbs.; 729 cwts. 3 qrs. 23 lbs. together.

In this example we add the column lbs. and find the sum 71. Now, as 28 lbs.=1 qr., we divide 71 by 28, which gives 2 qrs. 15 lbs. quotient; we set down the 15 lbs. and carry 2. We find the sum of the quarters =11; this divided by 4, gives 2 cwts. 3 qrs. We set down the 3 and *carry* 2, to the addition of the *cwts.*

cwts.	qrs.	lbs.
148	2	16
96	3	22
149	1	10
729	3	23
1124	3	15

The best way to add the *shillings' column* of an account is, to disregard the tens until the units have been added; then, having reached the top, add downwards, counting 10 for every 1 in the tens' column.

In the annexed example the sum of the units' column of shillings is found, 27; then proceeding downwards say, 27, 37, 47, 57, 67 shillings = £3 7*s.*

£	*s.*	*d.*	
48	16	10½	
56	19	7¾	
84	12	10	
78	18	6½	
£269	7	10¾	*Ans.*

We may proceed in the same manner in adding the column of lbs., adding 20 for every 2 in the tens' place, and 10 for every 1. Say in Ex. (2,) 3 + 2 + 6 = 11; then, adding downwards, say, 21, 41, 51, 71 lbs. = 2 qrs. 15 lbs. The perches' column may be *similarly* added.

X.

	£	*s.*	*d.*
1.	18	17	9
	16	11	4
	18	9	11
	82	14	6
	12	16	4
	4	3	7

	£	*s.*	*d.*
2.	18	17	9
	27	9	10
	9	17	4
	78	13	9
	93	15	7
	4	3	7

	£	*s.*	*d.*
3.	18	17	9
	27	9	10
	9	17	4
	64	15	8
	93	15	7
	4	3	7

	£	*s.*	*d.*
4.	37	10	6
	41	8	9
	73	13	3½
	19	9	4½
	23	15	2
	39	18	10

	£	*s.*	*d.*
5.	21	19	8½
	67	11	7½
	7	2	5
	89	15	7¾
	36	13	7½
	24	3	7½

	£	*s.*	*d.*
6.	21	19	8½
	67	11	7½
	7	2	5
	76	17	8½
	36	13	7½
	24	3	7½

	£	*s.*	*d.*
7.	366	13	4¼
	78	19	2½
	942	16	8¾
	1268	17	6½
	2876	15	2
	5534	2	0

	£	*s.*	*d.*
8.	21	19	8½
	16	13	6¾
	7	2	5
	43	17	8¾
	63	2	6¼
	24	3	7½

	£	*s.*	*d.*
9.	458	10	11½
	1087	2	3½
	942	16	7¼
	1234	5	6
	114	13	8
	29	19	11¾

	£	s.	d.
10.	5873	13	10½
	634	17	6½
	906	9	2½
	1347	9	11¾
	325	7	6
	923	13	11½
	949	17	7½
	138	16	8

	£	s.	d.
11.	3298	17	10
	634	17	6½
	906	9	2½
	1347	9	11¾
	325	7	6
	923	13	11½
	773	17	7½
	379	19	9

	£	s.	d.
12.	7089	14	5½
	508	3	9
	2530	6	10¼
	47	15	8¾
	3401	11	7
	9320	16	9¾
	477	19	4½
	63	16	7¼

	£	s.	d.
13.	7089	14	5½
	508	3	9
	8403	4	2½
	47	15	8¾
	3401	11	7
	65	5	5¼
	7019	12	10
	9320	16	9¾

	£	s.	d.
14.	2187	16	9
	634	17	6½
	8164	16	2½
	113	5	2½
	5016	15	7½
	923	13	11½
	732	9	6
	477	17	8¼

	£	s.	d.
15.	5873	13	10½
	634	17	6½
	906	9	2½
	1347	9	11¾
	325	7	6
	923	13	11½
	794	6	10
	39	17	10½

	£	s.	d.
16.	2917	10	2½
	27	1	2½
	533	7	6¼
	4456	8	9
	5001	17	8¾
	177	18	9
	309	16	8¼
	443	17	7½

	£	s.	d.
17.	912	17	8
	5	10	9¾
	79	13	10¼
	365	9	5½
	773	16	10¼
	47	9	9½
	751	10	10
	332	12	7½

	£	s.	d.
18.	998	18	7½
	233	7	4¼
	537	19	10
	498	16	11½
	96	12	10
	104	4	7
	137	7	8½
	689	16	6¼

	£	s.	d.
19.	7396	19	11¾
	8977	17	10½
	973	18	4¼
	36406	9	9
	6787	17	11½
	596	16	9¾
	498	17	11½
	9877	15	10¾

	£	s.	d.
20.	9745	16	10½
	7859	17	11¼
	897	19	9¼
	5788	18	6¾
	799	9	11½
	6849	18	10¼
	769	13	11¾
	9898	19	8½

	£	s.	d.
21.	9785	19	10¾
	7679	15	9½
	9886	9	6¾
	6749	18	11¼
	675	7	10½
	8979	19	11¾
	987	17	7½
	9608	8	10¼

	£	*s.*	*d.*
22.	79	19	9
	134	6	6½
	232	1	11¼
	67	17	9¾
	32	12	1
	416	9	8¾
	67	13	8¼
	239	2	5½
	409	12	8¼
	752	15	3¼
	63	9	5¾
	97	11	7¼
	257	8	11½

	£	*s.*	*d.*
23.	88	7	7
	204	15	2
	11	6	8¼
	743	13	3
	917	11	9¾
	376	15	10½
	15	9	5
	35	8	1½
	603	2	5¾
	498	13	10
	147	2	6½
	874	15	7¼
	925	4	11

	£	*s.*	*d.*
24.	98	8	10
	319	2	5¾
	911	6	7¼
	44	18	9
	76	0	9½
	63	9	11½
	732	17	1¼
	94	7	3
	69	19	10¾
	329	18	9
	41	2	7¾
	76	3	8¼
	28	3	6

	£	*s.*	*d.*
25.	276	5	9
	46	3	6½
	87	14	7
	917	10	8¾
	365	10	4¼
	369	15	5
	79	17	2¾
	9	9	10
	46	16	9½
	996	10	7¾
	22	5	10½
	49	19	10½
	444	4	4

	£	*s.*	*d.*
26.	631	7	7
	691	13	9½
	414	12	4¼
	16	18	2¾
	6	2	3¼
	56	4	10
	616	3	7¾
	89	2	3
	601	13	6½
	116	15	7¼
	917	18	8
	57	9	7¾
	555	5	5

	£	*s.*	*d.*
27.	397	7	10
	140	15	6
	563	4	10
	98	10	8
	42	1	6
	976	0	5
	49	3	3
	530	4	4
	811	5	5
	756	2	9
	76	11	10
	98	18	11¾
	666	6	6

	£	s.	d.		£	s.	d.		£	s.	d.
28.	431	7	9	29.	928	11	3	30.	603	3	7
	232	1	11¾		11	6	8½		210	6	7¾
	1967	17	9½		743	13	3		44	18	9
	32	12	1¼		917	11	9¾		376	0	9¼
	416	9	8		76	15	10		169	9	11½
	967	13	8¾		7215	9	5¼		732	17	1¼
	239	2	5		935	8	1		673	1	10¾
	397	8	7½		528	3	6¾		99	19	10½
	752	15	3¼		408	13	10½		8321	18	9
	35	12	8¾		58	15	1¼		978	11	11¾
	197	11	7		74	5	7½		9776	3	8½

	£	s.	d.		£	s.	d.		£	s.	d.
31.	8416	9	10½	32.	7319	8	4½	33.	3617	6	8½
	3276	5	9¼		631	7	7¼		1397	7	1¼
	46	3	6¾		691	13	9¾		1140	15	6
	1287	14	7½		414	12	4½		563	4	1¾
	4917	10	8¼		16	18	2¼		198	10	8½
	90365	10	4		436	2	3½		942	1	6¼
	366	5	5½		56	4	10¾		976	2	5
	1379	17	2¼		616	3	7		549	3	3
	9	9	10¾		189	2	8¾		530	4	4¼
	1340	16	9¼		401	3	6½		811	5	5
	906	10	7½		7146	15	7		756	2	2¼
	2222	5	1¾		917	18	8¾		76	11	10½

	£	s.	d.		£	s.	d.		£	s.	d.
34.	77	17	7	35.	88	18	8	36.	96	19	9
	39	18	11¼		4	19	7¾		96	6	6¾
	333	3	3		333	3	3		33	3	3
	647	9	9½		19	15	6¼		79	18	8
	778	18	10¼		77	6	3½		377	9	6¼
	56	16	7		717	7	10¾		98	5	5½
	998	13	4¾		9	9	9¼		647	17	7½
	77	17	7¼		627	7	7¾		39	0	3¾
	68	8	9¾		16	6	6½		726	15	11½
	797	7	7¾		729	3	3		10	10	10
	29	10	9		14	7	9¼		78	8	4½
	478	8	10		39	8	8		39	6	6½

	cwts.	qrs.	lbs.
37.	696	3	24
	523	1	16
	196	3	17
	567	3	18
	167	1	25

	cwts.	qrs.	lbs.	oz.
38.	69	3	14	10
	58	2	23	13
	68	3	10	12
	99	3	19	13

	lbs.	oz.	dwts.
39.	39	11	13
	56	10	9
	96	9	8
	59	10	10
	67	10	19

	lbs.	oz.	dwts.
40.	89	10	16
	56	9	19
	89	11	13
	56	10	17
	89	11	11

	acrs.	r.	pls.
41.	19	3	35
	17	2	27
	19	3	16
	15	2	29
	16	3	36

	acrs.	r.	pls.
42.	96	2	29
	10	3	36
	16	2	27
	19	3	30
	14	2	37

	acrs.	r.	p.	yds.
43.	64	2	30	20
	67	3	37	30
	69	2	36	27
	56	3	35	19
	19	3	25	25

	acrs.	r.	p.	yds.
44.	67	3	30	23
	19	2	17	19
	17	3	39	29
	18	2	36	20
	19	3	19	17

	degs.	m.	secs.
45.	106	30	35
	97	50	45
	109	20	25
	25	35	55
	317	30	30

	degs.	m.	secs.
46.	179	30	25
	806	26	36
	798	30	40
	779	36	50
	500	50	55

	gals.	qt.	nag.
47.	20	3	3
	17	2	2
	19	1	1
	23	3	2
	24	2	3

	gals.	qt.	pt.
48.	23	2	1
	27	2	1
	26	3	1
	25	2	1
	27	3	0

	tons	cwts.	qrs.	lbs.
49.	84	16	3	18
	1	19	2	14
	56	13	3	22
	19	18	2	16

	yds.	ft.	in.
50.	19	10	7
	3	7	9¾
	12	8	10½
	7	11	8¼

	hhds.	gals.	qrts.	pts.
51.	52	38	3	2
	19	58	3	1
	7	29	3	2
	85	48	2	1

	mls.	fur.	per.	yds.
52.	37	3	14	2
	28	14	16	2
	19	10	9	4
	10	5	6	3
	20	2	2	2
	30	1	1	4
	29	7	19	5

	hrs.	min.	sec.
53.	20	37	40
	17	20	35
	21	16	34
	16	27	46
	22	19	52
	19	22	16
	21	31	37

	yrs.	mnths.	wks.	dys.
54.	76	8	3	6
	57	11	2	3
	34	9	3	5
	57	6	1	2
	35	10	2	4
	56	9	3	3
	20	6	1	2

SEPARATE ADDITION.

	£	s.	d.		£	s.	d.		£	s.	d.
55.	632	5	11	56.	7125	13	4	57.	6147	1	8
	232	1	11		211	6	8		1211	6	7
	1067	17	9		7743	13	3		3144	18	9
	4032	12	1		6917	11	9		2376	9	9
	9416	9	8		14376	15	10		163	9	11
	1067	13	8		215	9	5		10732	17	1
	239	2	5		2935	8	1		6904	7	3
	6409	12	8		9603	2	5		109	19	10
	752	15	3		6408	13	10		4321	18	9
	56735	12	8		19058	15	1		25978	11	1
	6197	11	7		9374	5	7		9276	3	8
	40613	9	8		7312	2	9		6341	7	10
	3276	5	9		631	7	7		1397	7	1
	46	3	6		691	13	9		1140	15	6
	1287	14	7		1414	12	4		10563	4	1
	4917	10	8		5016	18	2		3198	10	8
	90365	10	4		436	2	3		78042	1	6
	360	5	5		56	4	10		25976	2	5
	1379	17	2		616	3	7		6549	3	3
	9	9	10		1189	2	3		7530	4	4
	1340	16	9		401	3	6		5811	5	5
	906	10	7		7116	15	7		756	2	2
	222	5	1		917	18	8		76	11	10

The Civil Service Commissioners allow fifty marks for "Separate Addition," such as the above, with which the learner is requested to familiarize himself by repeated trials. It appears from carefully reading the Civil Service Reports, that persons are examined in "Separate Addition," who are candidates for Clerkships of every kind, Surveyorships of Taxes, Inspectorships of Factories, and Cadetships of Constabulary. Candidates for the situation of Letter-carriers and other subordinate offices, and also those for Student Interpreterships, and for the Colonial Office, are not so tested.

COMPOUND SUBTRACTION.

48. Rule.—1°. Set down the less quantity under the greater, like two addends in the preceding rule.

2°. Commencing with the lowest denomination, take every term of the lower quantity from every term of the upper.

3°. If the lower term cannot be taken from the upper, borrow 1 from the next higher denomination of the upper line, then considering this so many *units* of the lower denomination, subtract, just as in *Simple Subtraction.*

Ex. 1. From £147 16*s.* 8½*d.* take £49 17*s.* 6¼*d.*

Here, we say, ¼ from ½, and ¼ remains, set down the ¼ under the farthings: again 6*d.* from 8*d.*, and 2*d.* remain, set down 2 under the pence. We cannot take 17*s.* from 16*s.*, therefore we borrow £1, calling it 20*s.*, this we add to 16*s.*, which makes 36*s.*, and say, 36—17 = 19*s.*, which we write under the shillings. Lastly, we carry 1 to the pounds and subtract as in Simple Subtraction.

£	*s.*	*d.*
147	16	8½
49	17	6¼
£97	19	2¼

Ex. 2. From 146 acrs. 2 r. 25 pls., take 19 acrs. 3 r. 30 pls.

Here, we take 30 from 40 + 25 = 65, as we must borrow one rood to enable us to subtract; or, we may say 30 from 40, and 10 remain; 10 and 25 are 35, which is set down. Carry 1 to 3 make 4, 4 from 3 we cannot, but 4 from 4 and 0 remains, 0 and 2 make 2, which is set down; carry 1 again, because we *borrowed* 1 acre, which we called 4 roods, and so on, as before.

acres	r.	pls.
146	2	25
19	3	30
126	2	35

XI.

	£	*s.*	*d.*
1.	109	7	4¼
	19	18	8½

	£	*s.*	*d.*
2.	169	16	4¼
	86	17	5½

	£	*s.*	*d.*
3.	1060	10	4½
	93	13	6¾

	£	*s.*	*d.*
4.	2019	16	4¼
	198	14	8½

	£	*s.*	*d.*
5.	107	10	4½
	19	16	8¼

	£	*s.*	*d.*
6.	123	13	11½
	19	17	10¾

	£	*s.*	*d.*
7.	2023	10	4¼
	127	12	9½

	£	*s.*	*d.*
8.	3020	19	6¼
	327	18	9½

	£	*s.*	*d.*
9.	1070	0	10
	57	18	0¾

	cwts.	qrs.	lbs.
10.	106	1	20
	19	2	18

	cwts.	qrs.	lbs.
11.	194	1	16
	14	2	19

	cwts.	qrs.	lbs.
12.	14	2	20
	9	3	24

	acrs.	r.	pls.
13.	134	2	30
	97	2	36

	acrs.	r.	pls.
14.	96	3	4
	19	2	30

	acrs.	r.	pls.
15.	306	2	16
	119	2	25

	acrs.	r.	pls.
16.	407	1	10
	296	2	35

	lbs.	oz.	drs.
17.	110	12	14
	96	13	15

	lbs.	oz.	drs.
18.	79	10	9
	17	11	13

	lbs.	oz.	drs.
19.	701	12	14
	19	12	15

	lbs.	oz.	drs.
20.	90	13	8
	17	10	11

	yrs.	m.	wks.
21.	90	10	3
	19	11	2

	yrs.	m.	wks.
22.	112	9	2
	96	10	3

	dys.	hrs.	m.
23.	16	12	40
	12	23	48

	dys.	hrs.	m.
24.	90	20	30
	47	22	36

	oz.	dwts.	grs.
25.	119	16	20
	39	18	23

	lbs.	oz.	dwts.
26.	76	10	13
	19	11	19

	lbs.	oz.	dwts.
27.	17	3	4
	9	10	9

	oz.	dwts.	grs.
28.	11	14	20
	9	17	23

	yds.	ft.	in.
29.	10	2	10
	7	2	11

	mls.	f.	pls.
30.	16	6	30
	9	6	36

	sq.yds.	sq.ft.	sq.in.
31.	13	7	70
	10	8	100

	sq.pls.	sq.yds.	sq.ft.	sq.in.
32.	30	20	6	100
	10	25	6	120

	yds.	qrs.	nls.
33.	117	3	2
	19	3	3

	ells	qrs.	nls.	in.
34.	67	1	2	1
	19	2	3	1¼

Post Office (Junior Appointments).

	£	s.	d.
35.	103	5	4
	49	18	10

	£	s.	d.
36.	92	6	2¼
	47	12	6¾

	£	s.	d.
37.	72	3	6
	58	18	9

	£	s.	d.
38.	201	7	5½
	53	16	7¾

	£	s.	d.
39.	61	7	3
	58	18	9

	£	s.	d.
40.	301	8	3¾
	53	16	7¾

Excise, Post Office, &c.

	£	s.	d.
41.	975	19	1¼
	9	16	10¾

		£	s.	d.
42.	From	453	6	2½
	Take	165	1	10¾

	£	s.	d.
43.	1687	1	7¾
	946	13	10½

	£	s.	d.
44.*	662303	11	3¼
	84606	16	4¾

Tidewaiter and Boatman.

	tons	cwts.	qrs.	lbs.
45.	359	17	2	19
	156	18	3	27

	yds.	ft.	in.
46.	252	1	1
	198	2	11

Examinations where 'Elementary Arithmetic' is required.

	£	s.	d.
47.	87341	10	9¼
	6442	19	11¾

	£	s.	d.
48.	641037	8	1¼
	83174	15	6¾

	£	s.	d.
49.	573156	11	2½
	92896	18	7¾

	£	s.	d.
50.	397612	4	5¼
	78360	16	9¾

	£	s	d.
51.	397612	4	5¼
	87406	18	6½

	mls.	fur.	pls.	yds.	ft.	in.
52.	38	7	13	1	1	2
	20	0	14	2	2	8

	mls.	fur.	pls.	yds.	ft.	in.
53.	79	6	12	1	1	6
	68	7	13	2	2	7

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* Admiralty (Dockyards).

COMPOUND MULTIPLICATION.

Case I.

49. *When the multiplier does not exceed* 12.

Rule.—Set down the multiplier under the lowest denomination of the multiplicand. Multiply the number of this denomination and every other by it, setting down and carrying as in addition.

Ex. 1. Multiply £118 12*s.* 6¼*d.* by 4.

In this example we say 4 × 1 (read times or multiplied by) = 4*f.* = 1*d.*: set down nothing, but carry 1. Then, 4 × 6 = 24 and 1 = 25*d.* = 2*s.* 1*d.*. set down 1. and carry 2*s.* 4 × 12 = 48, and 48 + 2 = 50*s.* = £2 10*s.* . set down 10*s.* and carry £2 to the pounds, and so on.

£	*s.*	*d.*
118	12	6¼
		4
474	10	1

Ex. 2. Multiply 16 yds. 2 qrs. 3 nls. by 6.

Here, we say 6 × 3 = 18, and 18 nls. = 4 qrs. 2 nls.: we write down 2 and carry 4. Again, 6 × 2 = 12, and 12 + 4 = 16 qrs. = 4 yds.: set down 0 and carry 4.

yds.	qrs.	nls.
16	2	3
		6
100	0	2

XII.

Post Office.

	£	*s.*	*d.*		£	*s.*	*d.*
1.	35	3	7 × 5.	2.	28	15	4¾ × 7.
3.	54	7	9 × 5.	4.	92	16	7¾ × 6.
5.	67	5	11 × 5.	6.	84	13	9¾ × 6.

50. *To multiply by a composite number.*

Rule.—Multiply successively by its factors.

Ex. 3. £736 18*s.* 10½*d.* × 16.

Here, as 16 is = 4 × 4, we first multiply by 4, and then multiply this product by 4, which must be the product by 16.

£	*s.*	*d.*	
736	18	10½	
		4	
2947	15	6	= *prod. by* 4.
		4	
11791	2	0	= *prod. by* 16.

XII.

	£	s.	d.			£	s.	d.	
7.	709	19	7½	× 24 and by 27.	8.	590	9	11¾	× 48 and by 50.
9.	719	19	9¼	× 64 and by 72.	10.	718	16	9½	× 90 and by 96.
11.	427	17	7¾	× 120 and by 121	12.	324	9	6½	× 132 and by 144.

CASE II.

51. ***When the multiplier is greater than* 12, *but not a composite number.***

RULE.—Multiply by the nearest ***composite*** number, and add or subtract the product found by multiplying the multiplicand by the difference between this composite number and the given multiplier.

Ex. 4. Multiply £12 10*s.* 6½*d.* by 19.

Here, we find the product by 18, as in *Case I.*; then we add the multiplicand, that is, once itself, which must give the product by 19. It will be useful to prove the accuracy of this result, by taking the factors of 20 instead of 18, and also using 12 + 7.

£	s.	d.	
12	10	6½	× 19
		6	
75	3	3	= *prod. by* 6
		3	
225	9	9	= *prod. by* 18
12	10	6½	= *prod. by* 1
238	0	3½	= *prod. by* 19

Ex. 5. Multiply £16 13*s.* 4¼*d.* by 327.

In this example we first find the product by 100; this product is multiplied by 3, which gives the product by 300. To find the product by 20, we multiply the product by 10 by 2; lastly, we multiply the multiplicand by 7, and then add.

	£	s.	d.	
(7)	16	13	4¼	× 327
			10	
(2)	166	13	6½	= *prod. by* 10
			10	
	1666	15	5	= *prod. by* 100
			3	
	5000	6	3	= *prod. by* 300
	333	7	1	= *prod. by* 20
	116	13	5¾	= *prod. by* 7
	5450	6	9¾	= *prod. by* 327

Since 327 = 320 + 7 = 4 × 8 × 10 + 7 we may proceed in the following manner:

£	s.	d.
16	13	4¼
		4
66	13	5
		8
533	7	4
		10
5333	13	4
116	13	5¾
5450	6	9¾

Ex. 6. Multiply 7 acrs. 2 rds. 20 pls. by 497.

Since 497 = 7 × 7 × 10 + 7 we proceed as follows:

acrs.	rds.	pls.
7	2	20
		7
53	1	20
		7
373	2	20
		10
3736	1	0
3789	2	20

Here we multiply 20 by 7 = 140 pls. which we divide by 40 and find 140 pls. = 3 rds. 20 pls., we write down 20 pls. and carry the 3 rds. Then 7 × 2 = 14, and 14 + 3 = 17 rds. = 4 acrs. 1 rd., we set down 1 rd. and carry 4 acrs., and say 7 × 7 = 49, and 49 + 4 = 53, which is written in full. In finding the product by 10, we say, since 20 pls. = one-half of a rood, 10 half-rds. = 5 rds. and carry the 5. In numerous examples we can proceed in a similar manner. Having obtained the product by 490 = 3736 acrs. 1 rd. 0 pls. we add to it the product by 7, already found, and thus arrive at the final result. Consequently we have an advantage in first using the factor 7.

£ s. d.
13. 568 9 10¼ × 69, 74.
14. 336 17 10¼ × 95, 98.
15. 360 19 7¼ × 137, 149.
16. 129 11 9¾ × 234, 378.

acrs. r. pls.
17. 19 3 30 × 147.
18. 23 1 16 × 523.

cwts. qrs. lbs.
19. 16 3 24 × 129.
20. 16 1 27 × 729.

yds. qrs. nls.
21. 20 3 2 × 709.
22. 27 3 3 × 999.

Tidewaiter and Weigher.

23. Multiply £227 10s. 5d. by 11.

24. Multiply 49 tons 13 cwts. 2 qrs. 26 lbs. 10 oz. 7 drs. by 9.

[Use factors for those marked thus.*]

Progressive Exercises.

25. £68 16*s*. 4¾*d*. × 11; £37 5*s*. 2½*d*. × 12.
*26. £13 19*s*. 9¼*d*. × 42; £45 7*s*. 10½*d*. × 63.
*27. £20 3*s*. 0¾*d*. × 28; £19 19*s*. 7¼*d*. × 88.
*28. £13 0*s*. 11½*d*. × 108; £29 13*s*. 2½*d*. × 121.
*29. £4693 11*s*. 10¾*d*. × 10; £73 16*s*. 6¾*d*. × 144.
30. £89 19*s*. 7¾*d*. × 46; £423 17*s*. 4½*d*. × 76.
31. £68 17*s*. 8½*d*. × 78; £79 16*s*. 10½*d*. × 109.
32. £401 11*s*. 9½*d*. × 43; £79 9*s*. 6½*d*. × 69.
33. £814 17*s*. 3½*d*. × 46; £87 7*s*. 7½*d*. × 127.
34. £401 11*s*. 2½*d*. × 94; £76 19*s*. 6¾*d*. × 243.
35. £680 15*s*. 8½*d*. × 89; £467 12*s*. 10¼*d*. × 346.
36. £904 5*s*. 11¼*d*. × 111; £49 19*s*. 11*d*. × 234.
37. £1647 8*s*. 2¼*d*. × 635; £407 8*s*. 10¼*d*. × 429.
*38. £19 9*s*. 7½*d*. × 224; £16 4*s*. 2¼*d*. × 392.
39. £54 17*s*. 6½*d*. × 365; £89 7*s*. 7½*d*. × 465.
*40. £54 7*s*. 7½*d*. × 693; £27 11*s*. 11¾*d*. × 637.
*41. £409 14*s*. 10½*d*. × 143; £48 8*s*. 7*d*. × 315.

COMPOUND DIVISION.

52. *This rule must be viewed in two aspects.*

I. To divide a compound quantity by an abstract number, which enables us to take the 3rd, 4th, or any part of the dividend.

II. To divide one compound quantity by another of the same kind, such as £1 5*s*. by 5*s*. This shows how often the dividend contains the divisor; and the quotient must always be an abstract number, expressing the number of times the dividend contains the divisor. (*See note page* 58.)

Under the first head we have cases corresponding to those in the preceding rule.

CASE I.

53. ***When the divisor is not greater than* 12.**

RULE.—1°. Set down the divisor to the left of the dividend, as in Short Division.

2°. Divide the left-hand term of the dividend by the divisor, and set down the quotient under that term. If there be any remainder reduce it to the next lower denomination, adding to it, thus reduced, the number of that denomination, and divide the sum by the divisor, and so on with the other terms.

Ex. 1. Divide £14 12*s*. 6*d*. by 6.

We say 14 contains 6 twice, set down 2: the remainder £2 = 40*s*., and 40 + 12 = 52*s*.; now, 52 ÷ 6 = 8 + 4 r*, set down 8: these 4*s*. must be reduced to pence, then 4*s*. = 48*d*., and 48 + 6 = 54, 54 ÷ 6 = 9, therefore, the quotient is £2 8*s*. 9*d*.

	£	*s*.	*d*.	
6)	14	12	6	
	2	8	9	*Ans.*

XIII.

Post Office.

1. £83 15*s*. ÷ 6.
2. £106 7*s*. 4½*d*. ÷ 9.
3. £88 9*s*. 6½*d*. ÷ 7.
4. £83 10*s*. 10½*d*. ÷ 6.
5. £81 3*s*. 1½*d*. ÷ by 7.
6. £77 11*s*. 4½*d*. ÷ by 6.

54. ***When the divisor is a composite number.***

RULE.—Divide *successively* by its factors.

Ex. 2. £28 12*s*. 3*d*. ÷ 18.

Here we first divide by 6, and then by 3, both divisors coming under CASE I.

	£	*s*.	*d*.	
6)	28	12	3	
3)	4	15	4½	
	£1	11	9½	*Ans.*

7. £5698 17*s*. 6*d*. ÷ 40, and by 49.
8. £8066 9*s*. 6*d*. ÷ 72, and by 81.
9. £7899 15*s*. 0*d*. ÷ 360, and by 432.

CASE II.

55. ***When the divisor is greater than* 12, *and not a composite number.***

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* Remainder.

Rule.—Proceed as in *Case* I., using *Long Division* instead of *Short.*

Ex. 3. Divide £167 16*s.* 1*d.* by 23.

The reason of the annexed example is obvious. We find 7, the first quotient figure; then, after reducing 6, the remainder, to shillings, we find 5, the next quotient figure. Again, after reducing to pence, we find 11, the last part of the quotient. The twenty-third part of £167 16*s.* 1*d.* is £7 5*s.* 11*d.* This may be proved by multiplying the quotient by the divisor.

```
      £   s.  d.
23)167   16   1
   161
   ——— (£7 5s. 11d. Ans.
     6
    20
   ———
23)136 (5
   115
   ———
    21
    12
   ———
23)253 (11
   253
   ———
```

Ex. 4. Divide 109 cwts. 3 qrs. 13 lbs. by 46.

The only difference between this example and the preceding one is in the reduction; after the first quotient figure we have 17 remainder, which we bring to quarters, adding the 3 qrs. in the multiplicand, and so on with the other denominations. After finding 15 lbs., we have a remainder. 23, which we reduce to oz., and find 8 oz.

Every other question coming under the rule is worked in a similar manner. The work may sometimes be contracted by at first dividing both divisor and dividend by some number that will divide them evenly. Let not the student be deterred from attempting this division because the dividend is a compound quantity.

```
   cwts. qrs. lbs.
46)109    3   13
    92
    —— (2 cwts. 1 qr. 15 lbs. 8 oz. Ans.
    17
     4
    ——
46)71 (1 qr.
   46
   ——
   25
   28
   ——
   203
   51
   ——
46)713 (15 lbs.
   46

   253
   230
   ———
    23
    16
   ———
46)368 (8 oz.
   368
   ———
```

XIII.

	£	s.	d.	
10.	794	16	5¼ ÷	78, and by 46.
11.	5760	15	4 ÷	165, „ 196.
12.	9648	17	8¾ ÷	427.

	cwts.	qrs.	lbs.	
13.	9006	3	20 ÷	120.
14.	8693	2	26 ÷	198.
15.	970	3	12 ÷	563.

	acrs.	r.	pls.	
16.	1768	3	20 ÷	530.
17.	4960	2	35 ÷	625.

Case III.

56. *To find how often one compound quantity contains another of the same kind.*[*]

Rule.—Reduce both to the same denomination, then divide as in Simple Division.

Ex. 5. Divide £1 11*s.* 6*d.* by 1*s.* 9*d.*

Here £1 11*s.* 6*d.* = 378*d.*; and 1*s.* 9*d.* = 21*d.*; then 378 ÷ 21 = 18. *Ans.*

Ex. 6. Divide 79 cwts. 2 qrs. 26 lbs. by 3 qrs. 10 lbs.

Here, 3 qrs. 10 lbs. = 94 lbs.
79 cwts. 2 qrs. 26 lbs. = 8930 lbs. } then 8930 ÷ 94 = 95. *Ans.*

	£	s.	d.	£	s.	d.
18.	1794	7	6 ÷	3	12	6.
19.	2777	10	0 ÷	2	10	6.
20.	2160	17	6 ÷	1	9	6.
21.	3186	18	4 ÷	2	13	4.

	cwts.	qrs.	lbs.	cwts.	qrs.	lbs.
22.	1468	3	7 ÷	1	1	0.
23.	1568	0	14 ÷	1	2	14.

Contractions.

57. *When we have to divide by* 100, 1000, *&c.*

Rule.—Point off so many places to the *right* as there are *cyphers* in the divisor.

---

[*] Although one compound quantity cannot be multiplied by another, for, to take 9*d.* 10*d.* times, or to take £1 a shilling times, is absurd; yet we can find how often one compound quantity contains another of the same kind—we can find how often £5 contains 10*s.* by considering them abstract numbers when reduced to the same denomination.

At first sight it might appear that pounds are multiplied by 20*s.*, to reduce them to shillings; this is not the case; there will be 20 *times* as many shillings as pounds, therefore we multiply 'the number denoting how many pounds' by 20—not by 20*s.* more than by 20 crowns.

Ex. 7. £3818 17s. 1d. ÷ 100.

Because 3818 ÷ 100 = 38 + 18r. we can point off the 18 at once, then multiply this 18 by 20, and adding 17, we get 377; we point off 77, and multiply by 12, to reduce to pence, and then point off 25, which we multiply by 4, and point off 00.

£ s. d.

1,00)38,18 17 1

20

3,77 (£38 3s. 9¼d. *Ans.*

12

9,25

4

1,00

£ s. d.

24. 7179 13 6½ ÷ 100.

£ s. d.

25. 2327 16 8 ÷ 1000.

To divide by 800, 1200, 3600, *&c.*

Rule.—First divide by 8, &c., then divide by the other factor, 100.

Ex. 8. Divide £28150 10s. by 900.

Since 900 ÷ 9 = 100, we can first divide by 9, then by 100, as in the annexed operation.

£ s. d.

9)28150 10 0

1,00)31,27 16 8

20

5,56

12

6,80

4

3,20; and 20 × 9 = 180*f.* = 3s. 9d.

*£*31 5*s.* 6¾*d.* + 3*s.* 9*d.* *Ans.*

£ s. d.

26. 14075 10 6 ÷ 800.

£ s. d.

27. 15816 15 0 ÷ 2400.

Post Office.

28. Divide £85 6s. by 72.

29. Divide £18 9s. 0¾d. by 78.

Education Office (London).

£ s. d.

30. 75919 11 7 ÷ 7.

£ s. d.

31. 611360 5 8 ÷ 622.

Admiralty.

£ s. d.

32. 648647 17 4 ÷ 32.

£ s. d.

33. 24727964 19 6 ÷ 236.

58. We can multiply or divide a compound quantity by a multiplier containing ½, ¼, or ¾, just as we have done with abstract numbers.

Ex. 9. Multiply £399 17s. 7d. by 9¾d.

	£	s.	d.	
	£399	17	7	
			9	
	3598	18	3	product by 9
½ of multiplicand =	199	18	9½	,, ,, ½
½ of this result =	99	19	4¾	,, ,, ¼
	£3898	16	5¼	*Ans.*

59. Ex. 10. Divide £3779 2 11¼ by 15¾.

15¾) £3779 2 11¼
4   4

63 { 7)15116 11 9
      { 9) 2159 10 3

239 18 11 *Ans.*

Miscellaneous Questions.

34. £7984 3 10½÷19½
35. £1616 10 9¾÷6¾
36. 2678492 15 6÷18
37. 3496852 19 5½÷94.
38. 1068091 12 11÷329, by 1316.
39. 276465420 7 8¾÷3469.
40. 15795 16 6¾÷399.
41. 158235 8 0¼÷3997.
42. 316470 16 0½÷£39 11 9¼.
43. 473874 16 10½÷395 17 8½.

44. If a tun of wine cost £60 7s. 6d., find the rate per gallon.

45. Find the weight of 1 hhd. if 27 hhds. weigh 475 cwts. 1 qr. 16 lbs.

46. Multiply £32576 2s. 1d. by 16½, and divide the result by 65.

47. Divide £8230 18s. 5¼d. by 13½, multiply the quotient by 11, and from the product take £499 19s. 9¾d.

48. Reduce 40 lbs, 8 oz. 5 drs. 1 sc. 7 grs. to grains (apothecaries' weight), and then express the result in troy weight.

49. In a tract of country 12 miles square, how many square miles? how many acres?

50. Multiply £28 14s. 7d. by 103, and by 1130; and say what number would give a product equal to both results together.

51. Find the number of half-crowns and also the number of half-guineas in 12600 fourpenny pieces.

52. If 328 dollars = £74 2s. 10d., how much is a dol. more than 4s.

53. Find the number of pints in 1008 quarters 1 bus. 1 gal. of corn.

54. If £12287 14s. 8d. be divided among 2816 persons, how much does each receive?

55. Find the produce of 529 acres at 5 qrs. 1 bus. 3 pks. per acre.

Ex. 11. Reduce 5702751 inches to miles, &c.

12)5702751 inches.

3)475229 3 inches.

158409 2 feet.
2

1 pl. = 11 h. yds.)316818 half-yards.

$$\left.\begin{array}{ll} 4{,}0)2880{,}1 & \frac{7}{2} = 3 \text{ yds. } 1 \text{ ft. } 6 \text{ in.} \\ 8)720 & 1 \text{ pl. } \quad 2 \text{ ft. } 3 \text{ in.} \end{array}\right\}$$

90 mls. 1 pl. 4 yds. 9 in.

Ex. 12. Reduce 100196196 square inches to acres, &c.

$$144\left\{\begin{array}{l} 12)100196196 \text{ sq. in.} \\ 12)8349683 \end{array}\right.$$

9)695806 11 × 12 = 132 in.

77311 7 ft.
4

$$30\tfrac{1}{4}\left\{\begin{array}{ll} 11)309244 & \text{quarter-yds.} \\ 11)28113 & 1 \end{array}\right.$$

$$\left.\begin{array}{llll} 4{,}0)255{,}5 & \frac{89}{4} = 22 \text{ yds.} & & 2 \text{ ft. } 36 \text{ in.} \\ 4)63 & & 35 \text{ pls.} & 7 \text{ ft. } 132 \text{ in.} \end{array}\right\}$$

15 ac. 3 rds. 35 pls. 23 yds. 1 ft. 24 in.

Miscellaneous Questions.

56. Find number of inches in 1 mile 1 furlong 3 poles.
57. Reduce 15 lbs. 13½ oz. 12 dwts. to grs.
58. Reduce 1 ac. 2 roods 11 pls. 2 yards to square inches.
59. In 758,372 grains how many lbs., &c.?
60. Reduce 150 tons 75 lbs. to ounces.
61. In 3,947,652 ounces how many tons, cwts., &c.?
62. In 327,461 pints how many quarters, bushels, &c.?
63. Divide £26 9s. 7d. by 82, and multiply the quotient by 67.
64. Reduce 2 miles 3 fur. 10 pls. 2 feet to feet.
65. Reduce 8,868,097 square feet to acres, &c.
66. Reduce 973,376 grs. to lbs. ozs., &c. (troy weight).
67. In 32,478,621 inches how many miles, &c.?

68. Reduce 10 acres 3 pls. 10¼ square yards to square inches.

69. In 9,503,707 pints how many quarters, &c.?

70. In 120,505 square feet how many acres, &c.?

71. In 3,060,752 seconds how many weeks, days, &c.?

72. If a yard of silk cost 4s. 7½d., find the cost of 417 yards.

73. What weight of iron can be bought for £2058 1s. 3d., if one cwt. cost 3s. 3d.?

74. Find the value of 888 cricket bats at 17s. 6d. a dozen.

75. Multiply 3 mls. 2 fur. 11 pls. 1 yard 6 inches by 62.

76. Divide 12 tons 9 cwt. 1 qr. 12 lbs. 5 ozs. by 11.

77. In 371,976,825 square inches how many acres, &c.?

78. In 6 qrs. 3 bush. 2 pks. 1 pint how many pints?

79. Find the price of 1 yard of cloth, if 17 cost £6 12s. 5½d.; find also the value of 110½ yards at same rate.

80. A person sold 8 tons 4 lbs. of steel for £466 15s. 5d., find the rate per lb.

81. Bring 142,114½ square feet to acres, &c.

82. How often can a vessel which holds 2⅓ pints be filled out of 56 gallons?

83. Find the cost of half an acre of building ground at 15s. 9d. per square yard.

84. Reduce 53,280,352 drs. to tons, &c.

85. How many pieces 6¼ in. long can be cut off a plank 67 feet in length, and how much will remain over?

86. The distance between two telegraph posts is 18 miles 1 fur. 44 yds. How many posts will be required between them, the posts being 8 rods apart?

87. A person laid out £65 3s. 4d. in the purchase of articles at 5s. 8d. each, how much does he gain by retailing them at 70s. a dozen?

88. Find the price of 29 articles at £3 16s. 6d. a dozen.

89. What is the remainder left when £15 10s. is divided by 71⅓?

90. A gentleman pays 23 guineas income tax at 4d. in the pound, find his income.

91. Reduce 7,865,375 cubic inches to cubic yards, &c.

92. Bring 77,356,935 drs. to tons, &c.

93. Reduce 7 yrs. 37 days 15 hrs. 17 m. 57 secs. to secs.

94. By an increase of the income tax from 3d. to 7d. in the pound a person would lose £84 7s. 6d., find his income.

95. If the income tax on £155 15s. be £2 11s. 11d., find how much it is in the pound.

96. If a kilometer be 1093 yards, reduce 7640 kilometers to English measure.

97. Reduce 5 quarters 2 bushels 2 pecks 1 quart to pints.

98. A person bought 318 yards of velvet at £1 7s. 9½d. each, how much will he gain by selling it at £1 13s. 11d. if he lose 5½ yards by shrinkage?

99. Find the cost of 9,367 inland telegraph messages, and 761 continental messages, each of the former being 1s., and each of the latter 5s.

100. A bicycle goes 2½ miles in 8 minutes, find its rate per hour.

QUESTIONS WORKED OUT.

Ex. 1. A person bought 560 yards of cloth at 11s. 3d. a yard, and retailed it at 14s. 1d. a yard; find his total profit.

On one yard his profit is 14s. 1d. — 11s. 3d. = 2s. 10d.

Therefore his profit on 560 yards is 560 times 2s. 10d., which we find as below :—

$$\begin{array}{rrrl}
 & \text{s.} & \text{d.} & \\
 & 2 & 10 & \\
 & & 8 & \\
\hline
1 & 2 & 8 & \text{profit on 8 yards.} \\
 & & 7 & \\
\hline
7 & 18 & 8 & \text{profit on 56 yards.} \\
 & & 10 & \\
\hline
£79 & 6 & 8 & \text{profit on 560 yards.}
\end{array}$$

Ex. 2. A merchant bought 279 cwts. of sugar for £785 17s.; find what it cost him per cwt., and how much per cwt. he will gain by selling the whole for £789 6s. 9d.

Cost of 279 cwts. = £785 17 0

$$\text{Therefore cost of 1 cwt.} = \frac{£785\ 17\ 0}{279} = £2\ 16\ 4$$

Again, gain on 279 cwts. = £789 6 9 — £785 17 = £3 9 9

$$\text{Therefore gain on 1 cwt.} = \frac{£3\ 9\ 9}{279} = \frac{837\text{d.}}{279} = 3\text{d.}$$

Ex. 3. If 27 tons 4½ cwts. of coal be divided among 44 poor persons, how much should each receive, and find the value of the whole at 25s. per ton?

Each person evidently received one-forty-fourth part of 27 tons 4½ cwts.

Since ½ cwt. = 2 qrs., we have :—

$$\begin{array}{r|rrr}
 & \text{tons} & \text{cwts.} & \text{qrs.} \\
44\left\{\begin{array}{r} 11) \\ \\ 4) \end{array}\right. & \begin{array}{r}27 \\ \hline 2\end{array} & \begin{array}{r}4 \\ \hline 9\end{array} & \begin{array}{r}2 \\ \hline 2\end{array} \\
\hline
 & \multicolumn{3}{r}{12\text{ cwts. } 1\text{ qr. } 14\text{lbs.}}
\end{array}$$

$$\text{Again, since 25s. per ton} = \frac{25\text{s.}}{20} = 1\text{s. } 3\text{d. per cwt.}$$

	s.	d.
Cost of 1 cwt. =	1	3
		4
Cost of 4 cwt. =	5	0
½ of 1s. 3d. =	0	7½
Cost of 4½ cwts.	5	7½

£1	5	
	9	
11	5	cost of 9 tons.
	3	
33	15	,, ,, 27 tons
	5 7½	cost of 4½ cwts.
£34	0 7½	*Ans.*

Ex. 4. A franc being value for 10d., and a dollar 4s. 3¾d., find the number of half crowns there are in 48 francs and 40 dollars.

	£	s.	d.
Here 48 francs = 10d. × 48 =	2	0	0
And 40 dols. = 4s. 3¾d. × 40 =	8	15	6
	10	12	6

Now it is evident there will be as many half-crowns as 2s. 6d. is contained in £10 12s. 6d.; and 2s. 6d. is contained 8 times in £1, it is contained 10 times 8 or 80 times in £10, and 5 times in 12s. 6d. ∴ *Ans.* = 85.

Ex. 5. "A wine merchant mixes 2 casks of wine worth 12s. 6d. a gallon, with one cask worth 18s. per gallon, and sells the mixture at 15s. per gallon; what profit does he make on every gallon?"

	s.	d.		£	s.	d.
Cost of 2 gallons of first kind =	12	6	× 2 =	1	5	0
,, 1 ,, second ,, =	18	0	× 1 =	0	18	0
,, 3 ,, of the mixture				2	3	0

Therefore gain on 3 gallons = 15s. × 3 — £2 3s. = 2s.
∴ *Ans.* = 2s. ÷ 3. = 8d. per gallon.

Ex. 6. "A grocer buys 300 lbs. of tea at 2s. 4½d. a pound, but 60 lbs. of it being damaged, the price for this is reduced to 1s. 9d. a pound; what is the cost of the whole, and the average cost per lb."

Taking the 60 lbs. damaged from the whole, we have 300 — 60 = 240 lbs. left.

Now the whole cost = 2s. 4½d. × 240 + 60 × 1s. 9d. = £28 10s. + £5 5s. = £33 15s.

Now 300 lbs. cost £33 15s., therefore 1 lb. will cost £33 15s. ÷ 300 = 2s. 3d., the average cost per lb.

Ex. 7. "If the telegraph posts be 8 poles 1 yard apart, how many of them will there be in a mile?"

There will be just as many as the number of times 8 poles 1 yard is contained in a mile: That is $\frac{1 \text{ mile}}{8 \text{ poles } 1 \text{ yd.}} = \frac{1760 \text{ yards}}{45 \text{ yards.}} = 39$, with a small remainder which in this case must, from the nature of the question, be overlooked.

Ex. 8. Two travellers set out at the same time, one from London, who walks 4 miles an hour, and the other from Brighton, who walks 3 miles an hour; how far from London will they meet, and how long after starting, the whole distance being 42 miles?

The two travellers approach each other at the rate of $4 + 3 = 7$ miles an hour, consequently they will meet after $42 \div 7 = 6$ hours, and $6 \times 4 = 24$, the number of miles from London.

Ex. 9. Divide £100 between A and B, so that A may have £5 12s. 6d. more than B.

From £100 0 0 the whole sum
Take 5 12 6 what A has more than B.

2) 94 7 6

47 3 9 B.'s share; then £47 3s. 9d. + £5 12s. 6d. = £52 16s. 3d. A.'s share.

Ex. 10. "Divide £20 among A, B, and C, so that B may have 2 guineas more than A, and that C may have 2s. less than B."

From £20 take £2 2s. and £2, and divide the remainder by 3 for the smallest share.

Therefore $\frac{£20 - £4\ 2\text{s.}}{3} = \frac{£15\ 18\text{s.}}{3} = £5\ 6\text{s.}$ A.'s share, and £5 6s. + £2 2 = £7 8s. B.'s share, and £7 6s. = C.'s share.

Ex. 11. "What is the amount of income-tax on an annuity of 500 guineas at 7d. in the £1?"

500 guins. $= 500 \times 21 = 10500$s., and 10500s. $= \frac{10500}{20} = £525$.

Now the tax being 7d. in every £1, the whole tax $= 525 \times 7\text{d.} = 3675\text{d.}$

Then 3675d. = £15 6s. 3d. *Ans.*

Ex. 12. Divide £45 between 3 persons, such that as often as the first gets £3, the second gets £5, and the third £7.

It is evident that if we take the fifteenth part, that is, the part designated by the sum of 3 + 5 + 7, and give the first 3 parts, second 5, and the third 7 parts, we shall have done what was required.

$$\frac{£45}{3+5+7} = £3, \text{ the fifteenth part.}$$

$$\begin{array}{lll}
\text{Then } £3 \times 3 = £9, & \text{the share of the first.} \\
\phantom{\text{Then }} £3 \times 5 = £15, & \quad ,, \quad ,, \quad \text{second.} \\
\phantom{\text{Then }} £3 \times 7 = £21, & \quad ,, \quad ,, \quad \text{third.}
\end{array}$$

Ex. 13. "If £433 7s. be paid in crowns, half-crowns, shillings, and sixpences, of each an equal number, how many coins of each kind will there be?"

Here 1 crown + 1 half-crown + 1 shilling + 1 sixpence = 10 + 5 + 2 + 1 = 18 sixpences, and £433 7s. = 8667s. = 17334 sixpences. There will be as many coins of each sort as 18 sixpences are contained in 17334 sixpences.

$$\therefore \textit{Ans.} = 17334 \div 18 = 963.$$

Ex. 14. From Dublin to Belfast is 101 miles; how often will the wheel of a railway carriage, which is 2¾ yards in circumference, turn in this distance?

It is evident it will turn as often as 2¾ yards are contained in 101 miles.

Since 101 miles = 101 × 1760 = 177760 yards, we have

$$\begin{array}{r|l}
2\frac{3}{4}) & 177760 \\
4 & 4 \\
\hline
11 &)711040 \\
\hline
 & 64640 \; \textit{Ans.}
\end{array}$$

MISCELLANEOUS EXERCISES IN THE PREVIOUS RULES.

XIV.

1. How much does a collector receive in 2 years, his weekly collection being £7 4s. 3d., and find his commission at 1s. 8d. in the £?

2. What sum will a labourer earn in 313 days, his wages being 11¾d. per day, and how much will he earn in 15 weeks?

3. Divide the difference between a guinea and a crown by 128.

4. From one thousand pounds take £982 14s. 0¼d., multiply the remainder by 57, and divide the product by 19.

5. Divide £100 between two persons, giving one £4 13s. 9d. more than the other. Find one-twelfth of the difference of their shares.

6. Multiply £16 10s. 9d. by 4½, divide the product by 13½.

7. Reduce 6 cwts. 5 sts. 7 lbs. to lbs. and divide the result by 14.

8. How long will £48 19*s.* 1½*d.* pay a labourer who earns 10½*d.* per day ?

9. How much was gained by selling a quantity of oats for £170 7*s.* 4¼*d.* which was purchased for £113 10*s.* 10½*d.* ?

10. What ground, at 4*s.* 8½*d.* per square perch, can be rented for £523 11*s.* 4*d.* ?

11. What quantity of tallow, at 7½*d.* per lb., could be purchased for £27 18*s.* 9*d.* ?

12. Bought 59 horses for £1040 19*s.* 7½*d.* ; what was the cost of each ?

13. How many dollars are there in £30 2*s.* 3*d.*, each dollar being worth 4*s.* 1½*d.* ?

14. From 347 tons 13 cwts. 2 qrs. 23 lbs. take 29 tons 15 cwts. 3 qrs. 25 lbs., and divide the remainder by 56.

15. What annual income would enable a person to expend 4*s.* 6*d.* per day, and save £2 6*s.* 10½*d.* per month ?

16. What was the salary of a person who paid £17 10*s.* income-tax when this tax was 14*d.* in the pound ?

17. How many shirts can be made from 60 ells of linen, each shirt requiring 3 yds. 3 qrs. ?

18. How much loaf sugar, at 8*d.* per lb., is equivalent in value to 30 sts. of moist sugar at 7*s.* 10*d.* per stone ?

19. How many dollars each equivalent to 4*s.* 2½*d.* should be received for £105 4*s.* 2*d.* ?

20. The price of a quantity of broadcloth at 10*s.* 8*d.* per yd. is £26 12*s.*, how much does it contain ?

21. What will be the cost of seeding 3 acres of potatoes at 6½*d.* per st., when 20 sts. are required for 30 perches ?

22. Divide £300 18*s.* 10½*d.* between two individuals, giving one £50 12*s.* 6*d.* more than the other.

Tidewaiters, &c.

23. Add £491 16*s.* 9*d.* ; £272 15*s.* 6¼*d.* ; £889 17*s.* 10½*d.* ; £647 19*s.* 2¾*d.* ; £398 16*s.* 7*d.* ; £563 16*s.* 10¼*d.* ; £770 0*s.* 5¾*d.* ; £945 17*s.* 7*d.* ; £240 13*s.* 9¼*d.* ; and £150 10*s.* together.

24. In 50 acres, how many sq. inches ?

25. In 3 leagues, how many inches ?

War Department.

26. Reduce £43 11*s.* 9¾*d.* to farthings.

27. Find the number of ounces in 18 tons 9 cwts. 3 qrs. 15 lbs.

28. If 1 yard of cloth cost 15*s.* 6½*d.*, what will be the price of 32½ yds. at the same rate?

Post Office.

29. Find the value of 120 doz. of candles, at 9*s.* 9½*d.* per doz.

30. Find the price of 137 articles, at £1 17*s.* 6¼*d.* each.

31. In 42 mls. 4 fur. 3 pls. 4 yds., how many feet?

Excise.

32. Subtract 169 lbs. 6 oz. 14 dwts. 17 grs. from 187 lbs. 9 oz. 12 dwts. 20 grs.

33. Divide 467 tuns 3 hhds. 26 gals. 2 qrts. 1 pt. by 5.

Proposed to those who are to know 'Elementary Arithmetic,' except Letter-Carriers and Rural Messengers.

34. A sum of £10465 was divided between two persons, so that one had £547 10*s.* more than the other; what did each receive?

35. If a man rows at the rate of 7 miles an hour with the stream, the rate of which is 2½ miles, how fast will he row against it?

36. A person mixed 8 lbs. of tea at 3*s.* 6*d.*, 10 at 3*s.* 8*d.*, 12 at 4*s.* 2*d.*, and 10 at 4*s.* 6*d.*, and sold the mixture at 4*s.* 8*d.* per lb.; what did he gain out of the whole quantity?

37. If 120 dollars be given for 111 crowns, find the value of a dollar?

38. A man borrowed a plough, for which he was to pay 7½d. for every working day he kept it, on Saturday, the 13th March, 1869; he returned it on 14th May following. How much had he to pay?

39. How often will a wheel, 3¼ yds. in circumference, turn between London and York, the distance being 198 miles?

40. How many solid yards, feet, and inches, are there in 175983 solid inches?

Police Courts, Prisons, &c.

41. Reduce 4 ac. 20 pls. to sq. feet.

42. How many miles, furlongs, &c., are there in 174082 inches?

43. How many lbs. oz., &c., are there in 228908 grs. of gold?

44. Reduce 4 miles 3 fur. 4 pls. 5 yds. to inches.

45. In 327471 oz., how many tons, cwts., &c.

Committee of Council on Education.

46. There are two mountains such that if 126 ft. are added to 4 times the height of the lower one, the sum is half the difference between their heights. Given that the lower one is 441 ft. high, find the height of the other.

47. What is the length of the longest day where the sun rises on that day at 3 hrs. 49 mins. 51 sec.?

Surveyorship of Taxes.

48. Reduce 14 tons 7 cwts. 1 qr. to ounces.

49. In 3784593 inches, how many miles?

Constabulary

50. In 6 tons 7 cwts. 3 qrs. 15 lbs., how many ounces?

51. If a man rows 10 miles in 2½ hrs. against the stream, the rate of which is 3 miles an hour; how long would he be in rowing 5 miles with the stream?

52. In 1721793 inches, how many miles, furlongs, &c.?

53. Find the circumference of a carriage wheel which revolves 1848 times in a journey of 3½ miles.

Where Candidates are to know 'Vulgar and Decimal Fractions.'

54. Reduce 6 miles 5 fur. 7 pls. 2 yds. to feet.

55. In 767914 sq. inches, how many yards, feet, and inches?

56. Reduce 36 miles 3 fur. 36 pls. 5 yds. to feet.

57. Reduce 63 cwts. 2 qrs. 5 oz. to drachms.

58. Reduce 14 lbs. 11 oz. 11 grs. to grains.

59. Reduce 787312 drachms to cwts., qrs., &c.

60. Reduce 13 yds. 5 ft. 19 in. (cubic measure) to inches.

Various Departments.

61. How many feet are there in 9 miles 3 fur. 6 pls. 1 yd.?

62. How many lbs. of silver are there in 270 spoons, each of which weighs 1 oz. 13 dwts. 8 grs.?

63. In 1847638 inches, how many miles, &c.?

64. How many francs, at 9½*d.* each, are there in £693 14*s.* 9*d.*?

65. Reduce 46 lbs. 3 oz. 2 dwts. 2 grs. to grains.

66. Reduce 17 tons 13 cwts. 1 lb. to ounces avoirdupois.

67. In 537086 inches, how many miles, furlongs, &c.?

Admiralty.

68. In 36845371 oz. avoirdupois, how many tons, cwts., &c.?

Various departments.

69. In 259566 lbs. avoirdupois weight, how many tons, cwts., and qrs.?

70. In 1749134 seconds, how many weeks, days, &c.?

71. In 9503707 pints, how many quarters, bushels, &c. ?

72. In 120505 square feet, how many acres, &c. ?

73. Reduce 3 square miles, 3 roods, 3 poles, to square feet.

74. Find the value of 47 sq. yards of carpet, at 9d. per sq. foot.

75. A farm of 329 acres produces 11 bushels of corn per acre; find the value of the crop at 6s. 6d. per quarter.

76. By an increase of the income tax from 3d. to 7d. in the pound, a person would lose £84 7s. 6d.; find his income.

77. A bar of gold, at £4 4s. 6d. per ounce, is worth £340 12 9¾d.; find its weight.

78. Find the number of yards in 42 pieces of cloth, each containing 40 yards 30 inches; then find the value of the whole at 5s. 6½d. per ell.

79. Bring 1392 tons 18 cwts. 2 qrs. 25 lbs. 7 oz. 15 drs. to drachms, and find the value of the result at 3d. for every 2 drachms.

80. How many half-crowns, shillings, and farthings, of each an equal number, are there in £338 ?

81. A grocer buys 200 lbs. of sugar at 5¼d. per lb.; what does it cost him ? At what price per lb. must he sell it to clear £1 13s. 4d. ?

82. Express 4040789 grains in pounds, ounces, pennyweights, &c.

83. At the International Exhibition, £2857 15s. was received on a day on which the cost of admission was 1s.; how many must be admitted on a half-crown day, so that the receipts may be doubled ?

84. A person buys 758 articles at 3s. 6d. each, and 534 more at 3s. 7d. each; he sells the former at 4s. 10½d., and the latter at 4s. 4¼d. each; find his entire gain.

85. Reduce 5 ac. 2 r. 11 pls. 7 yds. 5 ft. 29 in. to inches.

86. State the rule for finding the true remainder in dividing by a composite number: and divide, by means of factors, 7865327 by 105.

87. A bankrupt owes £9075 10s.; he pays 3s. 4d. in the pound; find how much his creditors lose.

88. The bounty money for 1408 recruits is £13 per man, but from this is deducted £6143 17s. 4d. for clothing, how much does each man receive ?

89. A person buys 22 cwt. 2 qrs. of sugar for £48 19s. 0½d., how much does he gain or lose by selling it at 5d. per lb. ?

90. A person bought 97 tons of iron for £840 17s. 4½d., find its cost per ton ? If it rise 11½d. per cwt., find his gain in selling the whole ?

91. Divide £1520 among A, B, and C, so that B may have £100 more than A, and C £270 more than B.

92. If 451 tons of copper ore are exchanged for 8003 tons 17 cwts. 1 qr. 7 lbs. 6 oz. of iron ore, how much iron ore is given for every ton of copper ore ?

93. Find the income tax on £7980 10s. at 7d. in the pound?

94. By what number should £27 17s. 10½d. be multiplied to make it £13500 11s. 6d.?

95. Find the value of 3546 pieces of cloth at £1 6s. 7¾d. each.

96. If the solar year contains 365 days 5 hours 48 minutes and 57 seconds, how many seconds in 25 years 25 days?

97. Reduce 17 qrs. 7 bus. 3 gals. 3 qrts. 3 pints to pints, and find the value of the result at 7¾d. per pint.

98. A purse containing £78 7s. 6d. made up of a certain number of sovereigns, twice as many half-sovereigns, and 3 times as many half-crowns, find the number of half-crowns?

99. Incomes under £200 a year pay income tax after a deduction of £60; find the largest tax such incomes can pay on an exact number of pounds at 5d. in the pound.

100. A man can buy a white hat which lasts 4 months for 4s. 6d.; he can buy a black one which lasts 9 months for 10s. 6d., which should he wear? What is saved in 3 years by wearing the cheaper?

101. If the moon moves forward in her orbit 57° 13′ 18″ in 13 days, find the distance she moves in one day?

102. If the pendulum of a certain clock vibrate 47 times every minute, how often will it vibrate in 196 days 49 minutes?

103. A British merchant owes 2020 francs to a Paris merchant, and 3035 dollars to a New York merchant, how much will discharge both debts, the franc being worth 9½d. and the dollar 4s. 6d.?

104. Reduce 41601764336 sq. inches to acres, &c.; subtract 29 acres 25 yards 5¼ feet from the result, and find one-thirty-second part of the remainder?

105. How often should a coach wheel 7 feet 9½ inches in circumference turn in passing over a distance of 42½ miles, and in what time will it perform the journey, the rate of driving being 8 miles an hour?

106. If a merchant give 795 yards of cloth worth 7s. 9½d. per yard for silk worth 18s. 9¼d., how many yards of silk should he receive, and what is its value?

107. A person travels 378 miles 1120 yards in 8½ days, find how far he travels each day supposing his rate uniform?

108. A person gave 240 gallons of wine, worth 2 guineas per gallon, in exchange for 224 barrels of ale, what should the ale be worth per barrel?

109. Find the rent on 8 farms of land, 4 containing 10 ac. 1 rd. 20 pls. each, and the others containing 8 ac. 3 rds. 20 pls. each, at £1 7s. 9d. per acre?

110. The latitude of St. Paul's in London is 51° 30′ 49″, and that of St. Peter's in Rome 41° 53′ 54″, what is one-fifth of the difference, and what one-fourth the sum of their latitudes?

111. Divide £55 6s. 10½d. equally among 35 persons, and find how much 3 of them will receive.

112. Add together ¾ of a square mile, ¾ of an acre, and 98 square yards, giving the answer in square feet.

113. Bring 875679 chains 25 links to miles, furlongs, &c.

114. Bring 7658 square chains 625 links to acres, roods, &c.

115. A spirit merchant mixes 4 gallons of water with every 17 gallons of spirits which cost him 3s. 9d. per gallon; at what price per gallon must he sell the mixture, in order to gain 1s, 6d. per gal.?

116. Divide £328 3s. 6½d. between A, B, C, and D, giving A £3 as often as B £5, as often as C £8, and D £9.

117. How much time, in the course of 33 years, will a person, who rises at 5 o'clock in the morning, gain over another, who rises at 8 o'clock, supposing both to go to bed at same hour, and a year to be 365¼ days?

118. Find the cost of 256 lbs. of tea at 3s. 3¾d. per lb. If 16 lbs. of it be spoiled, how much is gained by selling the remainder at 4s. 1½d. per lb.?

119. The pound troy contains 5760 grains, and a pound avoirdupois, 7000 grains; find the number of lbs. troy equal to 78 lbs. 4 oz. avoirdupois.

120. A bankrupt owes £2225, and pays a dividend of 6s. 8½d. in the £; after 6 months he pays an additional dividend of 4s. 3¾d.; how much is still due to his creditors?

121. How much water must be added to a cask containing 80 gals. of spirits at 14s. a gal., so as to reduce the price to 10s. 6d. per gal.?

122. An equal number of men, women, and boys, earned £24 15s. in 6 days, the daily wages of a man being 1s. 4d., a woman, 10d., and a boy, 4d.; how many were there of each?

123. A purse and its contents are worth £1 0s. 3d.; the contents are 8 times the value of the purse; what is the value of the purse?

124. A cask of wine containing 38 gallons was bought for £25; by leakage, 8 gallons were lost; at what price per gallon must the remainder be sold to gain £2 10s. on the whole?

125. In a school of 600 children, there are 18 more boys than girls, and the number of boys and girls is together double the number of infants; find how many boys there are?

126. If oranges be bought at 20 for a shilling, how much will be gained by selling 3000 of them at 10d. per dozen?

127. If 87 quarters of corn be divided as equally as possible, among 139 horses, no smaller measure than a pint being used; what quantity will remain after the distribution?

128. Two shepherds owning a flock of sheep agree to divide them equally between them; one takes 72 sheep, and the other 92, and pays the former £35; find the value of a sheep?

129. If the telegraph posts be 88 yards apart, and one is observed to pass the window of a railway carriage every 3 seconds, find the rate of the train per hour?

130. The lb. avoirdupois being equal to 7000 grains troy, convert 35 lbs. 8 oz. 7 dwts. 17 grs. troy into avoirdupois weight?

131. A person expends £1000 in the purchase of railway shares, at £12 17s. 6d. each; how many shares does he buy, the expenses of purchase being £8 12s. 6d?

132. A person holds 350 acres of land, for which he pays a tax of 3s. 6d. per acre, and a corn rent of 100 quarters of wheat, and the same quantity of barley, oats, and beans; find the cost of his land per acre when wheat is 38s. 9d., barley, 27s. 4d., oats, 17s. 4d., and beans, 33s. 10d. per quarter?

133. If the mean solar year be 365 days, 5 hrs., 48 m., 57 secs., show how much 400 such years differ from calendar years, viz.; 303 of 365 days each, and 97 leap years of 366 days each?

134. The Irish sq. pole is equal to 49 sq. yards, the English 30¼; find the quantity of land in Irish measure = 367 ac. 2 rds. English.

135. A truck with its load of 128 equal packages, weighs 2 tons 2 qrs. 4 lbs.: if the truck weighs 1 ton 1 cwt. 1 qr., find the weight of each package?

136. Divide £16 2s. 6d. between three persons, such that the second may have three times as much as the first, and the first twice as much as the third.

137. If a machine cuts 120 shingle-nails in a minute, find how many gross it will cut in 47 days 7 hrs., if it be at work 10 hrs. a day.

138. If the value of a sheep be £1 17s. 6d. find the price of an ox, so that by giving 80 sheep in exchange for 12 oxen I may gain £13 3s. 9d.?

139. If 27 bullocks worth £13 14s. each, and 140 sheep worth 36s. each, serve 4146 soldiers with meat rations for 9 days; find the daily cost of meat rations for each man?

140. A person has 4988 francs worth 9¾d. each, the same number of dollars worth 4s. 2½d., and half as many rupees worth 2s. 1½d., and one-fourth as many Spanish reals worth 2½d. each. If he receives £1500 for all of them, how much does he gain or lose?

VULGAR FRACTIONS.

61. A FRACTION is an expression for a part or parts of anything considered as a whole.

A *fraction* may be either greater or less than a *unit*; if it be *one-half* of *ten* it is equal to *five units*.

A *fraction* is expressed by means of two numbers placed one over the other with a line between them.

Thus, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{8}{5}$, read, *one-half*, *two-thirds*, *eight-fifths*.

The number below the line is called the *Denominator*, because it shows the number of parts into which the whole or unit is divided. It gives the name or denomination to the fraction, just as *cwt.* gives the name *hundredweight* to any number after which it is written. The number above the line is called the *Numerator*. The numerator shows or enumerates the number of parts expressed by the fraction. If we divide anything into four equal parts, we express three of these parts by the fraction $\frac{3}{4}$. The numerator and denominator are called the *Terms* of the fraction.

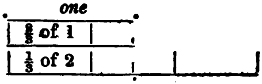
62. A fraction corresponds to an example in division before the process is performed, the *numerator* corresponding to the *dividend* and the *denominator* to the *divisor*. Therefore the TRUE OR REAL VALUE of any fraction is the *quotient* obtained by dividing its numerator by its denominator. Hence, the greater the numerator in comparison with the denominator, the greater the value of the fraction. For, the more parts we take of anything, the greater must be the particular fraction used to express these parts.

Increasing the numerator, the denominator remaining the same, increases the value of the fraction: it increases the number of parts; or, which amounts to the same thing, it increases the dividend, the divisor remaining the same. Increasing the denominator decreases the value of the fraction; as it makes a greater number of parts, each part must be smaller. It increases the divisor, the dividend remaining the same.

The *terms* of a fraction may be both multiplied or both divided by the same number, without affecting the value of the fraction. The quotient will remain the same, therefore the value of the fraction will not be changed. For, $\frac{6}{3}=\frac{24}{12}=\frac{48}{24}$, &c.

Two-thirds of one is equal to *one-third* of two, as may be understood from the illustration.

For, $\frac{2}{3}$ of a shilling $= 8d.$, and $8d. =$ one-third of $24d.$, or 2 shillings.



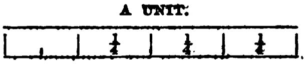
In the same manner it may be shown that $\frac{1}{2}$ of $1=\frac{1}{4}$ of 2, and $\frac{3}{4}$ of $1=\frac{1}{4}$ of 3, &c.

63. A PROPER FRACTION has its numerator less than its denominator:—$\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, are proper fractions.

An IMPROPER FRACTION has its numerator equal to or greater than its denominator, and is equal to or greater than a unit:—$\frac{4}{4}$, $\frac{5}{4}$, $\frac{7}{3}$, are improper fractions.

A SIMPLE FRACTION represents a part or parts of one or more units; as we have seen, $\frac{2}{3}$ may be considered $\frac{2}{3}$ of 1, or $\frac{1}{3}$ of 2.

A COMPOUND FRACTION represents a part or parts of a fraction, and is known by having 'of' interposed; as, $\frac{1}{2}$ of $\frac{1}{4}$, $\frac{1}{3}$ of $\frac{1}{2}$, &c. It bears a similar relation to a fraction that a simple fraction bears to a whole number.



Observe that the $\frac{1}{4}$ to the left is divided into two parts, each of which represents the fraction $\frac{1}{2}$ of $\frac{1}{4}$. Again, since $\frac{1}{4}$ of a shilling $= 3d.$, and one-half of $3d. = 1\frac{1}{2}d.$, and $1\frac{1}{2}d. = \frac{1}{8}$ of a shilling, therefore $\frac{1}{2}$ of $\frac{1}{4}=\frac{1}{8}$.

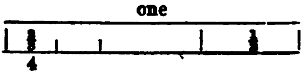
A MIXED NUMBER consists of a whole number and a fraction; as, $3\frac{1}{2}$, $2\frac{3}{4}$, $6\frac{1}{3}$.

A COMPLEX FRACTION represents a part or parts of a fraction, of a mixed number, or a fractional part of a whole number; as,

$$\frac{\frac{2}{3}}{4} \quad \frac{4}{\frac{2}{3}} \quad \frac{3}{4\frac{1}{2}} \quad \frac{\frac{1}{2}\text{ of }3}{2+\frac{1}{3}}$$

The first means that one-fourth of $\frac{2}{3}$ is to be taken, and not $\frac{1}{4}$ of 1. The second expresses $4 \div \frac{2}{3}$, and not $\frac{2}{3}$ of 4. They will be easily understood from the following explanation.

Since $\frac{2}{3}$ of a shilling $= 8d.$,
$\frac{\frac{2}{3}}{4} = 8d. \div 4 = 2d. = \frac{1}{6}$ of a shilling,



therefore $\frac{\frac{2}{3}}{4} = \frac{1}{6}$. Also, $\frac{2}{3}$ of a pound $= 13s.\ 4d.$, and $13s.\ 4d. \div 4 = 3s.\ 4d.$ $= \frac{1}{6}$ as before.

REDUCTION.

64. *To reduce an improper fraction to its equivalent whole or mixed number.*

RULE.—Divide the numerator by the denominator, the quotient is the whole number; under the remainder (if any) write the denominator, in the form of a fraction.

Ex. 1. Reduce $\frac{15}{4}$ and $\frac{79}{14}$ to whole numbers.

Here, $15 \div 4 = 3 + 3$ r. $= 3\frac{3}{4}$ *Ans.*

And, 14)79 ($5\frac{9}{14}$ *Ans.*
70
—
9
—

REASON.—The denominator shows the number of parts to be taken to make up a unit; therefore, as many parts must be put together to make 1, as there are units in the denominator; this is effected by the rule. Since the operation of division does not alter the nature of the quantity divided, the remainder is a part of the given fraction, and is expressed as such by writing the denominator under it.

XV.

Reduce to whole or mixed numbers:—

1. $\frac{23}{3}$, $\frac{29}{7}$, $\frac{83}{17}$.
2. $\frac{1862}{23}$, $\frac{7625}{[illegible]}$, $\frac{901}{11}$.
3. $\frac{2045}{144}$, $\frac{1969}{19}$, $\frac{2847}{31}$.
4. $\frac{19}{2}$, $\frac{365}{17}$, $\frac{649}{27}$.

65. *To reduce a mixed number to its equivalent improper fraction.*

RULE.—Multiply the whole number by the denominator of the fraction, and to the product add its numerator; under this sum write the denominator of the fraction.

Ex. 1. Reduce $14\frac{11}{30}$ to an improper fraction.

$$\begin{array}{r} 14\frac{11}{30} \\ 30 \\ \hline 431, \end{array} \textit{ then } \frac{431}{30} \textit{ Ans.}$$

This rule is the reverse of the preceding. The *reason* is the same in principle as that for the reverse operation.

Reduce to improper fractions:—

5. $7\frac{1}{9}$, $7\frac{3}{5}$, $18\frac{3}{7}$.
6. $129\frac{11}{40}$, $16\frac{12}{23}$, $5\frac{7}{9}$.
7. $18\frac{10}{17}$, $70\frac{1}{5}$, $69\frac{5}{11}$.
8. $32\frac{1}{9}$, $16\frac{7}{12}$, $113\frac{117}{118}$.

66. Any number may be written in a fractional form by putting 1 under it for a denominator.

We may give a whole number *any* denominator, if we first multiply it by that denominator.

For, $8 = 8 \times \frac{6}{6} = \frac{48}{6}$, and $10 = \frac{50}{5}$; also, $20 = 20 \times \frac{5}{5} = \frac{100}{5}$.

9. Reduce 19, 10, 30, to fractions whose denominators shall be 11, 17, 29, respectively.

10. Reduce 27, 39, 165, to fractions whose denominators shall be 39, 46, 57, respectively.

We may also reduce a fraction to another fraction having *any* denominator, if the denominator of such a fraction be a *factor* of the number to which it is to be reduced.

Thus, $\frac{1}{4}$ can be brought to a fraction whose denominator is 16; since $16 = 4 \times 4$, we have merely to multiply the terms of the fraction by 4, and we have $\frac{1 \times 4}{4 \times 4} = \frac{4}{16}$.

We cannot, however, reduce $\frac{1}{4}$ to a simple fraction whose denominator shall be 18, since 4 is not one of the factors of 18.

We may reduce $\frac{1}{2}$ and $\frac{7}{8}$ to equivalent fractions, having 48 for denominator. For, $\frac{1}{2}=\frac{24}{48}$ (multiplying the terms by 24), and $\frac{7}{8}=\frac{42}{48}$ (multiplying the terms by 6).

67. If one number divides another without a remainder, it is said to be a MEASURE of that other.

A *Measure* is another name for *Factor*.

If any number be a measure or factor of two or more others, it is called a *common measure* of them; that is, a measure common to them. The numbers 12 and 18 have each the following measures:—12 has 12, 6, 4, 3, 2; 18 has 18, 9, 6, 3, 2. Therefore 12 and 18 have 6, 3, 2, common measures. 6 is their *greatest common measure*, that is, the greatest measure common to them.

68. *To find the greatest common measure of any two numbers.*

RULE.—1°. Divide the greater number by the less; divide the less by the remainder; continue the process, always dividing the preceding divisor by the remainder, until there is no remainder.

2°. The last divisor will be the greatest common measure.*

Ex. 1. Find the greatest common measure of 12 and 18.

We divide by 12, and get 6 for remainder; we then divide 12, the first divisor, by 6, and find no remainder: therefore 6 is the greatest common measure of 12 and 18.

```
12)18(1
   12
   --
   6)12(2
     12
     --
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* Before we prove the reason of the rule it is necessary to show:—

I. 'If a number measure two other numbers it will measure their sum and difference. Thus, 4 is a measure of 12 and of 20, it is a measure of 32=12+20, and also a measure of 8=20—12.

II.—' If one number *measure* another, it will measure any multiple of that other.' (A multiple of a number is its product when multiplied by any number.) Thus, 4 is a measure of 12; it is also a measure of 24=12×2, and of 36=12×3.

III. The *greatest common measure* that two or more numbers can have is the least of those numbers.

To find the G. C. M. of 72 and 156. The G. C. M. these two numbers can have is 72 (III.); hence, if 72 measures 156, it is their G. C. M. We find by trial that 72 does not measure 156; therefore their G. C. M. is less than 72 (otherwise it would not be a common measure). In trying if 72 measures 156, that is, in dividing by 72, we have divided 156 into

```
72)156(2
   144
   ---
   12)72(6
      72
      --
```

XV.

Find the greatest common measure of :—

11. 66 and 154 ; and of 195 and 819.
12. 378 and 1467 ; and of 18996 and 29984.
13. 365 and 7345 ; and of 1964 and 78688 ; and of $\frac{15}{16}$, $2\frac{1}{2}$, 4, and $5\frac{1}{3}$.

For more than two numbers.

RULE.—Find for *two*, then of this measure and a *third*, and so on.

Find the greatest common measure of :—

14. 12, 42, and 66 ; and of 19, 57, and 380.
15. 324, 612, and 1032 ; and of 96, 216, 88, and 16.
16. 14, 70, and 1484 ; and of 8128, 14816, 75288, and 8472.

69. *To reduce a fraction to its lowest terms.*

RULE I.—Divide the terms of the fraction by any common factors ; the new terms in the same manner, and so on, until terms are obtained which have no common factors.

RULE II.—Divide the terms of the fraction by their greatest common measure.

Ex. 1. Reduce $\frac{48}{112}$ to its lowest terms.

Here, $\frac{48}{112}=\frac{24}{56}=\frac{12}{28}=\frac{6}{14}=\frac{3}{7}$ *Ans.* This result is found by dividing by 2 continually. But we might have divided by 4, or by any other common factor.

The greatest common measure of 48 and 112, we find is 16 ; then dividing the terms by 16, we get $\frac{3}{7}$, as before.

The following observations will be found useful in finding common factors :—

1. All even numbers are divisible by 2 at least.
2. All even numbers are divisible by 4, when the number expressed by their last two figures is divisible by 4.

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two parts; one is a multiple of 72, the other (12) is the difference between that multiple and 156. Whatever measures 144 and 12 will measure 156 (I.); but the G. C. M. of 144 and 12 may not measure 72 ; therefore, instead of trying 144 and 12 we try 72 and 12, as we know the number that measures 72 will measure 144 (II.). Now, the *greatest* measure that 72 and 12 can have is 12 (III.), which we find will answer. It is obvious that 12 is the G. C. M. of 72 and 156, since the greatest possible measure was tried in every case.

In dividing the greater number by the less, that *multiple* of any divisor may be taken which comes nearest the dividend, whether greater or less than it. We may divide any divisor and its corresponding dividend by common factors, at any step during the process. See APPENDIX, page 238.

3. A number is divisible by 8, if its *units' period* be divisible by 8.

4. An equal number of cyphers cancel when numbers end in one or more cyphers.

5. Numbers ending in 0 or 5, are divisible by 5, and no others.

6. If the *sum* of all the digits of a number be divisible by 3 or 9, the number is divisible by 3 or 9.

7. A number is divisible by 11, if the *sums* of the alternate digits be equal, beginning at either right or left; or, if one sum exceed the other by 11, or 22, 33, &c.*

XV.

Reduce to their lowest terms:—

17. $\frac{72}{168}$, $\frac{48}{160}$, $\frac{272}{1496}$.
18. $\frac{1638}{24102}$, $\frac{1572}{1965}$, $\frac{7956}{9724}$.
19. $\frac{5544}{8008}$, $\frac{48016}{108036}$, $\frac{25025}{35035}$.
20. $\frac{49}{294}$, $\frac{1235}{2223}$, $\frac{13068}{15444}$.

Proposed in various Departments.

21. Reduce $\frac{7040}{7392}$, to its lowest terms.
22. " $\frac{6398}{7812}$, to its lowest terms.
23. " $\frac{69615}{92872}$, to its lowest terms.

THE LEAST COMMON MULTIPLE.

70. When a number is a *factor* of another number, it has been called a *measure* of that other.

A number is called a MULTIPLE of any of its measures or factors. Thus 12 is a *multiple* of 6, and of 4.

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* REASON.—1. This is self-evident.

2. Since 100 is divisible by 4, any number of hundreds is divisible by 4. But any number may be considered as a certain number of hundreds + the number expressed by the figures in its units' and tens' places. Therefore if these last be divisible by 4, the entire number must be divisible by 4. Thus, $1728=1700+28$; $736=700+36$.

3. The reason of this is similar to that of the preceding, only say 1000 instead of 100, and 8 instead of 4.

4. This is obvious.

5. Go over the MULTIPLICATION TABLE by 5.

6. Every number is equal to two other numbers, one of which is a multiple of 9 and the other the sum of its digits; thus, $324=3\times100+2\times10+4$, and since $10=9+1$, $100=99+1$ &c., we have, $324=3(99+1)+2(9+1)+4=297+18+3+2+4=315$ (a multiple of 9)+9 (the sum of the digits).

When a number is a multiple of two or more others, as 12 of 6, 4, and 3, it is called their *Common Multiple.* The numbers 6, 4, and 3, have also 24, 36, 48, &c., common multiples. A prime number is one which is divisible only by itself and unity, as 7, 11, 13, &c.

71. *To find the Least Common Multiple of numbers.*

RULE.—1°. Place the numbers in a line; strike out those which are measures of any of the others, then divide by a measure or divisor which will divide as many as possible of the given numbers.

2°. Set down the quotients and undivided numbers.

3°. Proceed in the same manner with the quotients and undivided numbers, until no number can be found to measure any two of them.

4°. Multiply together the undivided numbers and the divisors used: the product will be the least common multiple.

When numbers are all *prime* to one another, their product is their least common multiple.*

Ex. 1. Find the least common multiple of 6, 12, 9, 18, 24, and 30.

$$\begin{array}{r} 6\underline{)\ \cancel{6}\ \cancel{12}\ \cancel{9}\ 18\ 24\ 30} \\ 3\times4\times5\times6= \end{array}$$

360 the least common multiple.

In this example 6 is a measure of 18, 12 of 24, and 9 of 18, consequently they are struck out. Now, 6 will measure the three numbers 18, 24, and 30, hence we divide by 6, and write the quotients 3, 4, and 5, no two of which can be divided by any number. Then we multiply 3, 4, 5, and the divisor 6 together for the L. C. M.

Ex. 2. Find the L. C. M. of 24, 10, 33, 22, 30, 18, and 45.

$$\begin{array}{r} 3\underline{)\ 24\ \cancel{10}\ 33\ 22\ 30\ 18\ 45} \\ 2\underline{)\ 8\ \ \ \cancel{11}\ 22\ 10\ \ 6\ 15} \\ 4\ \times\ 11\times\cancel{5}\ \ \cancel{3}\ 15\times2\times3= \end{array}$$

3960 the least common multiple.

In this example we strike out 10 because it is a factor of 30: we divide by the common factor 3, then strike out 11 because it is a factor of 22. Having divided by 2, the second common factor, we strike out 5 and 3 because they are factors of 15. Lastly, we multiply 4, 11, 15 (no two of which have a common factor), and the divisors used, 3 and 2, and their product 3960 is the L. C. M.

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* The divisors used must be *prime* to the numbers which they do not measure, for if not, the result obtained will not be the L. C. M. For instance, if one of the numbers were 12, we could not use 9 as a divisor, since 12=3×4, and 9=3×3, the factor 3 is common to both, and would enter twice into the final product, and consequently cause it to be three times the L. C. M.

REASON.—This process is merely using *one* of the factors common to several numbers instead of *all* the factors the same as itself: therefore, as only *prime* and *common* factors are used, the product is the *least possible*; and since no factors have been *omitted*, the product must be a *multiple of all* the numbers.

72. *To reduce fractions to equivalent ones having a Common Denominator.*

RULE.—Find the least common multiple of all the denominators, which will be the required common denominator; then proceed as in **66.**

This is merely an *extension* of **66**, and we proceed on the same principle.

REASON.—We have seen that all the denominators must measure that denominator to which they can be reduced: hence the reason of the rule.

Ex. 1. Reduce $\frac{1}{2}$ and $\frac{1}{3}$ to equivalent fractions having the same denominator.

Here, $\frac{1}{2}=\frac{3}{6}$, and $\frac{1}{3}=\frac{2}{6}$; therefore $\frac{3}{6}$ and $\frac{2}{6}$ *Ans.*

In this operation we multiply the terms of each fraction by the same number; therefore we do not alter the value of any of the fractions used. The multiplier for the terms of any fraction will always be the *quotient* obtained by dividing the least common multiple by the denominator of that fraction. In the fractions $\frac{1}{2}$ and $\frac{1}{3}$, 6 is the least common multiple of 2 and 3; then $6 \div 2 = 3$, the multiplier for $\frac{1}{2}$; and $6 \div 3 = 2$, the multiplier for $\frac{1}{3}$. Then $\frac{1 \times 3}{2 \times 3}=\frac{3}{6}$, and $\frac{1 \times 2}{3 \times 2}=\frac{2}{6}$, as before.

Ex. 2. Reduce $\frac{1}{2}$, $\frac{1}{4}$, $\frac{5}{8}$, $\frac{11}{16}$, $\frac{5}{18}$, to others having a common denominator.

$$\begin{array}{l|l} 2 & \cancel{2} \quad \cancel{4} \quad \cancel{8} \quad 16 \quad 18 \\ \hline & \qquad\qquad\quad\ \ 8 \quad\ \ 9 \end{array}$$

$\frac{1}{2}=\frac{72}{144}$
$\frac{1}{4}=\frac{36}{144}$
$\frac{5}{8}=\frac{90}{144}$
$\frac{11}{16}=\frac{99}{144}$
$\frac{5}{18}=\frac{40}{144}$

Then $2 \times 8 \times 9 = 144$, the *least common multiple.*

We write the denominators in a line, and find their least common multiple 144, which is the denominator of their equivalent fractions. We find the multipliers thus:—$144 \div 2$, the denominator of the first, $= 72$, the numerator of the equivalent to $\frac{1}{2}$; again, $144 \div 4 = 36$. For the third, we say $144 \div 8 = 18$, and $18 \times 5 = 90$, the numerator of the equivalent to $\frac{5}{8}$; and so on.

Reduce the following to their *equivalents* having the same denominator: *—

XV.

24. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$.
25. $\frac{3}{4}$, $\frac{7}{12}$, $\frac{5}{16}$, $\frac{7}{8}$.
26. $\frac{3}{11}$, $\frac{17}{22}$, $\frac{4}{5}$, $\frac{19}{33}$.
27. $\frac{7}{9}$, $\frac{8}{11}$, $\frac{13}{18}$, $\frac{9}{17}$.
28. $\frac{4}{7}$, $\frac{5}{6}$, $\frac{11}{18}$, $\frac{13}{15}$.

73. ***To express one quantity as the fraction of another of the same kind.***

RULE.—1°. Write the former as numerator, the latter as denominator.

2°. If they are of different denominations, they must be reduced to the same denomination, and for simplification the fraction reduced to its lowest terms.

Ex. 1. What fraction of 18*s.* is 6*s.*? Here $\frac{6}{18}=\frac{1}{3}$ *Ans.*

Ex. 2. What fraction of 2*s.* is 2¼*d.*?

$$\text{Here, } \frac{2\frac{1}{4}d.}{2s.}=\frac{2\frac{1}{4}d.}{24d.}=\frac{9f.}{96f.}=\frac{3}{32}\ \textit{Ans.}$$

Reduce to the fraction of a *pound*:—

29. 17*s.* 6*d.*; 15*s.* 9*d.*; 1*s.* 10½*d.*
30. 2*s.* 9*d.*; 1*s.* 7½*d.*; 9*s.* 3*d.*

Reduce to the fraction of a *shilling*:—

31. 9*d.*, 7½*d.*, 4½*d.*

Reduce to the fraction of a *cwt.*:—

32. 2 qrs. 14 lbs.; 3 qrs. 14 lbs.; 1 qr. 7 lbs.

Reduce to the fraction of an *acre*:—

33. 2 r. 20 p.; 3 r. 10 p.; 1 r. 30 p.; 3 r. 25 p.

34. Express 2*s.* 0¾*d.* as a fraction of 4*s.* 9¾*d.*
35. Express £2 6*s.* 8*d.* as a fraction of £3 13*s.* 4*d.*
36. What fraction of £27 17*s.* 1¼*d.* is £23 11*s.* 4¾*d.*?
37. Find the L. C. M. of $\frac{1}{15}$, $2\frac{1}{2}$, 5, $6\frac{1}{3}$, and $\frac{1}{11}$.

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[*] It is necessary that we should be able to bring fractions to the *same* denomination or kind without altering their value; for fractions, like any other quantities, can neither be added nor subtracted when they are of different kinds. Making them have a common denominator is reducing them to others *equal in value*, but all of the same kind. By looking at $\frac{5}{7}$ and $\frac{8}{11}$ we cannot say which is the greater, but since $\frac{5}{7}=\frac{55}{77}$, and $\frac{8}{11}=\frac{56}{77}$, we see at once that $\frac{8}{11}$ is the greater.

ADDITION.

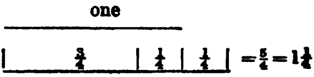
CASE I.

74. *When the fractions have a common denominator.*

RULE.—Add the numerators together and under their sum write the common denominator.

Ex. 1. Add $\frac{3}{4}$ and $\frac{2}{4}$. Here we say $2+3=5$, and write 4 the common denominator under it; $\frac{5}{4}=1\frac{1}{4}$ *Ans.*

REASON.—Since $\frac{3}{4}s.=9d.$, and $\frac{2}{4}s.=6d.$, we have $\frac{3}{4}+\frac{2}{4}=9d.+6d.$ $=15d.=1\frac{1}{4}s.$ Or, because $\frac{3}{4}=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$, and $\frac{2}{4}=\frac{1}{4}+\frac{1}{4}$; we have $\frac{3}{4}+\frac{2}{4}=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{5}{4}=1\frac{1}{4}$; compare this with the illustration. The fractions to be added should always be written as in **75**, *Ex.* 2.



XVI.

Add the following:—

1. $\frac{1}{4}+\frac{3}{4}+\frac{1}{4}$; $\frac{3}{7}+\frac{6}{7}+\frac{5}{7}+\frac{4}{7}$.
2. $\frac{7}{11}+\frac{6}{11}+\frac{9}{11}$; $\frac{11}{13}+\frac{12}{13}+\frac{6}{13}$.

CASE II.

75. *When the fractions to be added have not a common denominator.*

RULE—1°. Reduce them to others equal in value, and having a common denominator.

2°. Add these others as in CASE I.

Customs and Excise.

Ex. 1. Add $\frac{3}{4}$, $\frac{5}{8}$, $\frac{2}{3}$, together.

As we cannot add 4ths to 8ths, nor to 6ths, we must find a *common denominator*, 24, and reduce the fractions into equivalent ones having a common denominator, namely to 24ths. Adding these we get $\frac{49}{24}=2\frac{1}{24}$ *Ans.*

$\cancel{4}$, $8\times3=24$ *common denominator*

$$\begin{aligned}\tfrac{3}{4}&=\tfrac{18}{24}\\ \tfrac{5}{8}&=\tfrac{15}{24}\\ \tfrac{2}{3}&=\tfrac{16}{24}\\ \hline \tfrac{49}{24}&=2\tfrac{1}{24}\end{aligned}$$

Various Departments.

Ex. 2. Add together $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{12}$.

The explanation is similar to that given in Ex. 1, and the work will be as the annexed.

$\cancel{3}\ \cancel{4}\ 5 \times 12 = 60$ *common denominator.*

$$\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{7}{12}$$

$$=\frac{40+45+48+35}{60}$$

$$=\frac{\overset{14}{\cancel{168}}}{\underset{5}{\cancel{60}}}=2\tfrac{4}{5}.\quad \textit{Ans.}$$

In actual practice the denominators of the equivalent fractions are not usually written.

Various Departments.

3. Add together $\frac{1}{4}$, $\frac{5}{12}$, $\frac{7}{15}$, $\frac{8}{20}$.

Post Office.

4. Add the following $\frac{3}{4}$, $\frac{2}{7}$, and $\frac{5}{6}$.

Excise.

5. Add $\frac{3}{4}$, $\frac{2}{9}$, and $\frac{5}{6}$ together.
6. „ $\frac{3}{4}$ and $\frac{6}{7}$ together.

War Office.

7. Add together $\frac{7}{18}$, $\frac{7}{15}$, and $\frac{11}{18}$.

Customs and Excise.

8. Add $\frac{5}{9}$, $\frac{3}{4}$, and $\frac{2}{3}$ together.
9. „ $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{9}$ together.

Audit Office.

10. Add $\frac{3}{8}$ and $\frac{4}{7}$ together.

Inland Revenue.

11. Add together $\frac{9}{16}$, $\frac{5}{7}$, and $\frac{1}{3}$.

76. *To add mixed numbers.*

Rule.—Add the fractional parts by the rules given above, set down the fraction of the sum, if any, and carry the ***whole number***, if any, to the sum of the whole numbers.

Ex. 3. Add together $7\frac{1}{3}$, $\frac{3}{14}$, $\frac{10}{21}$, and $2\frac{7}{9}$.

In this example the fractions when added make $1\frac{101}{126}$; we set down the fraction $\frac{101}{126}$ and carry the 1 to the whole numbers, and find the answer $10\frac{101}{126}$.

$$7)\underline{\cancel{3} \quad 14 \quad 21 \quad 9}$$
$$2 \times \cancel{3} \quad 9 \times 7 = 126$$
common denominator.

$$7\tfrac{1}{3} + \tfrac{3}{14} + \tfrac{10}{21} + 2$$
$$= 9 + \frac{42 + 27 + 60 + 9}{126}$$
$$= 9 + \frac{227}{126} = 10\frac{101}{126}.$$ *An*

12. $\frac{1}{5}$, $4\frac{1}{3}$, and $\frac{2}{5}$; also $3\frac{1}{11}+2\frac{13}{22}+\frac{17}{44}$.
13. $1\frac{3}{8}+\frac{1}{6}+\frac{5}{18}+2\frac{1}{12}$; also $5\frac{3}{8}+1\frac{13}{16}+2\frac{1}{2}$.
14. $\frac{3}{4}+\frac{5}{7}+1\frac{5}{6}+2\frac{1}{8}$; also $17\frac{4}{7}+3\frac{11}{14}+1\frac{19}{20}+\frac{27}{28}$.
15. $\frac{4}{5}+\frac{5}{6}+4\frac{5}{9}+1\frac{7}{10}$; also $3\frac{2}{3}+\frac{114}{27}+\frac{17}{18}+4\frac{5}{6}$.
16. $\frac{2}{5}+\frac{3}{7}+\frac{4}{11}$, and $\frac{2}{13}$ of $7\frac{4}{5}$; also $\frac{11}{13}$ of $3\frac{5}{7}+\frac{6}{9}+\frac{10}{17}$.
17. $3\frac{1}{9}+5\frac{3}{5}+1\frac{4}{15}+\frac{2}{3}$; also $3\frac{1}{3}+7\frac{1}{9}+5\frac{11}{12}+\frac{7}{16}+1\frac{7}{18}$.
18. $5\frac{4}{9}+\frac{3}{8}+\frac{7}{12}+2\frac{5}{24}$; also $3\frac{11}{17}+\frac{82}{85}+\frac{13}{17}+7\frac{11}{34}$.
19. $7\frac{1}{8}+\frac{2}{3}+3\frac{4}{5}+1\frac{11}{12}$; also $13\frac{6}{19}+\frac{36}{57}+23\frac{1}{2}+7\frac{24}{95}$.
20. $\frac{4}{5}+\frac{7}{8}+\frac{11}{12}+3\frac{3}{20}$; also $3\frac{17}{23}+14\frac{5}{8}+27\frac{7}{46}+\frac{3}{4}$.
21. $\frac{2}{5}+2\frac{5}{6}+2\frac{1}{8}+\frac{5}{12}$; also $17\frac{3}{5}$ of $2\frac{1}{11}+\frac{5}{9}$ of $7\frac{1}{5}$.
22. $3\frac{5}{12}+3\frac{1}{6}$; also $67\frac{3}{11}+30\frac{5}{8}+3\frac{5}{6}+1\frac{19}{22}$.
23. $4\frac{2}{7}+\frac{13}{14}+2\frac{19}{21}$; also $7\frac{7}{10}+4\frac{4}{5}+7\frac{19}{20}+23\frac{14}{15}$.
24. $9\frac{1}{7}+1\frac{3}{4}+1\frac{2}{21}+\frac{13}{42}$; also $16\frac{11}{13}+1\frac{19}{39}+\frac{2}{3}+\frac{7}{26}$.

25.	£16	13	$10\frac{1}{2}$	26.	£76	16	$10\frac{2}{3}$	27.	£79	19	$9\frac{3}{4}$
	17	19	$11\frac{2}{5}$		77	19	$7\frac{1}{2}$		64	4	$4\frac{1}{12}$
	69	13	$9\frac{9}{10}$		56	3	$4\frac{5}{6}$		19	17	$7\frac{11}{24}$
	16	14	$10\frac{3}{4}$		19	17	$11\frac{3}{4}$		58	8	10
	19	9	$7\frac{7}{10}$		27	8	$7\frac{1}{4}$		94	16	$6\frac{3}{4}$

28. $45\frac{11}{16}+1\frac{17}{18}+3\frac{3}{4}+9\frac{9}{32}$; also $4\frac{4}{29}+1\frac{2}{3}+3\frac{84}{87}$.
29. $59\frac{3}{4}+10\frac{11}{12}+\frac{3}{8}+14\frac{17}{18}$; also $\frac{7}{19}+3\frac{35}{38}+3\frac{14}{19}+\frac{11}{38}$.
30. $45\frac{3}{5}+49\frac{13}{15}+\frac{13}{20}+\frac{4}{9}$ of $1\frac{3}{5}+3\frac{11}{30}+4\frac{4}{5}+\frac{7}{8}$.
31. $7\frac{7}{8}+5\frac{5}{16}+13\frac{13}{14}+1\frac{5}{6}+4\frac{4}{9}+5\frac{5}{6}+1\frac{11}{14}+\frac{7}{16}$.
32. $4\frac{3}{13}+6\frac{1}{17}+5\frac{2}{19}+4\frac{188}{221}+15\frac{6}{17}+\frac{318}{323}$.

SUBTRACTION.

CASE I.

77. *When the fractions have a common denominator.*

RULE.—Take the numerator of the less fraction from the numerator of the greater, and under the remainder write the common denominator.

EX. 1. From $\frac{5}{6}$ take $\frac{1}{6}$.

Take 1 from 5 and 4 remain, under which we write 6; then $\frac{4}{6}=\frac{2}{3}$ *Ans.*

$$\begin{array}{r} \frac{5}{6} \\ \frac{1}{6} \\ \hline \frac{4}{6}=\frac{2}{3}\ \textit{Ans.} \end{array}$$

REASON.—$\frac{1}{6}$s.$=2d.$, $\frac{5}{6}=10d.$; hence $\frac{5}{6}-\frac{1}{6}=10d.-2d.=8d.=\frac{8}{12}=\frac{2}{3}$ as before.

XVII.

1. From $\frac{5}{8}$ take $\frac{3}{8}$; also $\frac{11}{12}-\frac{7}{12}$; $\frac{4}{7}-\frac{3}{7}$.
2. „ $\frac{13}{14}$ „ $\frac{11}{14}$; „ $\frac{8}{19}-\frac{5}{19}$; $\frac{13}{16}-\frac{7}{16}$.

CASE II.

78. *When the fractions have not a common denominator.*

RULE.—Bring the fractions to equivalent ones having a common denominator, then proceed as in CASE I.

Commissariat.

EX. 2. Subtract $\frac{5}{12}$ from $\frac{7}{8}$.

In this question we find 24 the common denominator, and the given fractions are (72) equivalent to $\frac{21}{24}$ and $\frac{10}{24}$; then $21-10=11$, under which we write the common denominator. Or, since $\frac{1}{8}$ of a shilling$=1\frac{1}{2}d.$, $\frac{7}{8}=10\frac{1}{2}d.$; and since $\frac{1}{12}$ s.$=1d.$, $\frac{5}{12}=5d.$ then $10\frac{1}{2}d.-5d.=5\frac{1}{2}d.=\frac{11}{24}$s. as before.

$$\begin{array}{r} \frac{7}{8}=\frac{21}{24} \\ \frac{5}{12}=\frac{10}{24} \\ \hline \frac{11}{24}\ \textit{Ans.} \end{array}$$

Inland Revenue.

3. Subtract $\frac{3}{10}$ from $\frac{8}{11}$.

Customs.

4. Subtract $\frac{7}{8}$ from $\frac{9}{10}$.

79. *To subtract mixed numbers.*

RULE.—1°. Subtract just as in pence and farthings.

2°. Take care to *borrow* one when the fraction of the smaller number is greater than the other fraction: this 1 always must be considered as divided into the number of units expressed by the common denominator.

Proposed where Fractions are to be known.

Ex. 3. Subtract $5\frac{7}{10}$ from $8\frac{1}{2}$.

$$\begin{array}{rl} \text{From } 8\frac{1}{2} & = 8\frac{5}{10} \\ \text{Take } 5\frac{7}{10} & = 5\frac{7}{10} \\ \hline & 2\frac{8}{10} = 2\frac{4}{5}\ \textit{Ans} \end{array}$$

As we cannot take $\frac{7}{10}$ from $\frac{5}{10}$ we borrow 1 from the 8 and call it $\frac{10}{10}$ its equivalent. We then say $\frac{7}{10}$ from $\frac{15}{10}=\frac{10}{10}+\frac{5}{10}$ and $\frac{8}{10}$ remain $=\frac{4}{5}$. We proceed in a similar manner in every example having a fraction in the subtrahend greater than that in the minuend. If there be no fraction in the minuend we subtract the numerator of the fraction in subtrahend from the common denominator.

(5.) $7\frac{2}{5}-5\frac{2}{7}$; also $19\frac{3}{28}-12\frac{7}{8}$; also $37\frac{2}{11}-3\frac{8}{11}$.

(6.) $16\frac{2}{15}-11\frac{7}{8}$; also $5\frac{1}{9}-3\frac{1}{10}$; also $7\frac{1}{10}-4\frac{1}{8}$.

(7.) $1\frac{4}{25}-\frac{3}{4}$; also $37\frac{1}{5}-19\frac{4}{15}$; also $13\frac{11}{14}-1\frac{6}{7}$.

(8.) $6\frac{8}{15}-4\frac{1}{3}$; also $5\frac{3}{8}-3\frac{2}{3}$; also $19\frac{1}{5}-10\frac{7}{15}$.

(9.) $7\frac{4}{5}-5\frac{6}{35}$; also $8\frac{11}{12}-7\frac{13}{16}$; also $7\frac{7}{8}-3\frac{11}{12}$.

(10.) $33\frac{11}{13}-\frac{19}{26}$; also $3\frac{5}{8}-2\frac{7}{9}$; also $16\frac{7}{17}-\frac{33}{34}$.

(11.) $18\frac{2}{3}-8\frac{3}{7}$; also $\frac{3}{8}-\frac{11}{36}$; also $5\frac{1}{17}-3\frac{1}{18}$.

(12.) $9\frac{3}{4}-6\frac{5}{12}$; also $37\frac{1}{13}-19\frac{4}{39}$; also $30\frac{7}{19}-3\frac{3}{4}$.

(13.) $32\frac{3}{4}-19\frac{7}{8}$; also $4\frac{1}{2}-\frac{2}{5}$; also $12\frac{13}{14}-9\frac{9}{10}$.

(14.) $5\frac{3}{4}-1\frac{7}{9}$; also $41\frac{11}{23}-19\frac{5}{6}$; also $33\frac{1}{7}-17\frac{1}{8}$.

(15.) $61\frac{2}{11}-18\frac{3}{13}$; also $100\frac{1}{4}-53\frac{2}{9}$; also $102\frac{3}{17}-2\frac{16}{17}$.

(16.) $11\frac{3}{20}-7\frac{7}{45}$; also $114\frac{3}{10}-96\frac{17}{30}$; also $1\frac{3}{14}-\frac{6}{7}$.

(17.) $5\frac{11}{12}-3\frac{3}{5}$; also $31\frac{2}{7}-18\frac{5}{9}$; also $4\frac{1}{4}-3\frac{11}{12}$.

(18.) $27\frac{19}{20}-6\frac{9}{10}$; also $(\frac{1}{2}+\frac{9}{10}+\frac{3}{16})-\frac{19}{24}$; also $(3\frac{2}{13}+5\frac{4}{11})$ $-6\frac{6}{7}$.

(19.) $3\frac{1}{7}-2\frac{13}{14}$; also $(\frac{1}{9}+\frac{1}{3}+\frac{1}{5})-\frac{2}{7}$; also $(\frac{1}{5}+\frac{2}{7}+\frac{3}{8})-\frac{5}{14}$.

MULTIPLICATION.

80. *To multiply fractions.*

RULE.—Multiply the numerators together for the numerator of the product, and the denominators for its denominator.

Customs.

Ex. 1. Multiply $\frac{3}{4}$ by $\frac{7}{8}$.

Here, $\frac{3}{4}\times\frac{7}{8}=\frac{3\times 7}{4\times 8}=\frac{21}{32}$ *Ans.*

Compound fractions are reduced to *Simple* fractions in the same manner.

Thus $\frac{2}{3}$ of $\frac{5}{6}=\frac{2}{3}\times\frac{5}{6}=\frac{10}{18}=\frac{5}{9}$; and $\frac{1}{2}$ of $\frac{4}{5}=\frac{1}{2}\times\frac{4}{5}=\frac{4}{10}=\frac{2}{5}$.

The operation may be contracted by dividing any numerator and any denominator by common factors. Thus $\frac{15}{16}\times\frac{8}{15}=\frac{1}{2}\times\frac{1}{1}=\frac{1}{2}$, found by dividing the numerator of the first and the denominator of the second by 15, and denominator of first and numerator of second by 8.

This rule includes the multiplication of whole nnmbers, for $2\times 3=\frac{2}{1}\times\frac{3}{1}=\frac{6}{1}=6$. It also includes the multiplication of a whole number by a fraction, or a fraction by a whole number; in either case, we write the whole number in a fractional form and multiply. Thus, $\frac{5}{6}\times 3=\frac{5}{6}\times\frac{3}{1}=\frac{15}{6}=2\frac{3}{6}=2\frac{1}{2}$, and $3\times\frac{5}{6}=\frac{3}{1}\times\frac{5}{6}$, as before $=2\frac{1}{2}$.

It follows from the method of contraction explained above that when the whole number by which we are to multiply is a *measure* of the *denominator* of the fraction, we have merely to divide that denominator by the whole number for the product; thus,

$$\frac{5}{6}\times 3=\frac{5}{6\div 3}=\frac{5}{2}=2\frac{1}{2}.$$

81. In the same manner the value of a fraction of any quantity can be found, by considering that quantity as numerator of a fraction, and then multiplying the quantity by the numerator of the given fraction, and dividing the product by its denominator.

Thus, to find $\frac{4}{5}$ of £3; we say £$\frac{3}{1} \times \frac{4}{5} =$ £$\frac{12}{5} =$ £2 8*s.*

By first principles, $\frac{4}{5}$ of £3; here $\frac{1}{5}$ of £3 = £3 ÷ 5 = 12*s.*: therefore $\frac{4}{5} = 12s. \times 4 =$ £2 8*s.*

REASON.—Take the example $\frac{1}{2} \times \frac{3}{4}$. Now, $\frac{1}{2} \times 3 = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$ by *addition*; but we have seen that $\frac{3}{4}$ is $= \frac{1}{4}$ of 3; therefore we have multiplied by a quantity *four times* too great, and hence our product $\frac{3}{2}$ is *four times* too great. Now, $\frac{3}{2}$ is the quotient of 3 by 2; this quotient we still require to divide by 4; but any number divided by 2 and then by 4, is the same as if divided by 8; therefore $\frac{3}{8}$ is the answer, which is a verification of the rule.

Mixed numbers should be reduced to improper fractions, and whole numbers written in fractional forms, before multiplying.

XVIII.

Various Competitions.

1. $\frac{3}{10} \times \frac{8}{11}$; also $8\frac{5}{6} \times \frac{3}{5} \times 17\frac{19}{20} \times \frac{3}{4}$; also $\frac{4}{11} \times 1\frac{4}{7}$.
2. $\frac{3}{8} \times \frac{3}{8}$; also $\frac{9}{14} \times \frac{5}{27} \times 9\frac{4}{5} \times 2\frac{2}{7}$; also $13 \times 2\frac{6}{11} \times \frac{1}{7} \times \frac{2}{39} \times \frac{5}{32}$.
3. $\frac{5}{9}$ of $\frac{8}{13} \times \frac{6}{7}$; also $2\frac{1}{3} \times \frac{1}{4} \times \frac{5}{14} \times 4\frac{1}{2}$; also $5\frac{5}{6} \times 3\frac{3}{11}$.
4. $2\frac{3}{4} \times 5\frac{20}{21}$; also $\frac{3}{8} \times 6\frac{1}{2} \times \frac{13}{14} \times \frac{7}{26}$; also $\frac{7}{11} \times 2\frac{1}{2} \times \frac{3}{7} \times \frac{21}{2}$.
5. $\frac{1}{4}$ of $\frac{3}{8} \times \frac{5}{7}$; also $3\frac{3}{4} \times 2\frac{1}{2} \times \frac{7}{13} \times 5\frac{1}{5}$; also $\frac{2}{3}$ of $1\frac{1}{7} \times 2\frac{2}{5} \times 4\frac{3}{8}$ of $1\frac{1}{9}$.
6. $\frac{3}{4}$ of $9 \times \frac{7}{8}$; also $3 \times \frac{5}{17} \times 6\frac{1}{3} \times \frac{17}{30} \times 5\frac{2}{5}$; also $12\frac{3}{8} \times 1\frac{5}{11}$.
7. $\frac{2}{7} \times \frac{17}{45}$; also $4\frac{1}{8} \times \frac{1}{2} \times 1\frac{3}{11} \times \frac{16}{21}$; also $\frac{7}{11} \times 2\frac{1}{2} \times \frac{3}{7} \times 10\frac{1}{2}$.
8. $22\frac{2}{11} \times 4\frac{2}{5}$; also $\frac{4}{7} \times \frac{3}{8} \times 4\frac{9}{10} \times \frac{5}{6} \times 1\frac{2}{5}$; also $\frac{3}{8} \times \frac{5}{7} \times 1\frac{3}{4} \times \frac{1}{8}$.

Admiralty.

9. Multiply $161\frac{1}{5}$ by $2\frac{3}{4}$.

Constabulary.

10. Multiply $10\frac{2}{11}$ by $8\frac{1}{4}$.

Surveyor of Taxes.

11. Multiply $18\frac{2}{5}$ by $5\frac{7}{16}$.
12. „ $13\frac{2}{7}$ by $19\frac{1}{12}$.

Various Departments.

13. Multiply $9\frac{2}{7}$ by $3\frac{2}{11}$.
14. „ $10\frac{3}{7}$ by $3\frac{11}{12}$.
15. „ $11\frac{2}{5}$ by $8\frac{3}{7}$
16. „ $11\frac{5}{9}$ by $7\frac{7}{8}$.
17. „ $42\frac{3}{5}$ by $\frac{10}{27}$.
18. „ $9\frac{4}{5}$ by $5\frac{5}{7}$.
19. What is the value of $\frac{5}{36}$ of a guinea?
20. Find the value of $\frac{2}{3}$ of £1 + $\frac{3}{7}$ of £2 10*s.* 2*d.* + $2\frac{2}{3}$ of 16*s.*
21. Add together $\frac{4}{15}$ of a pound, $3\frac{3}{13}$ of a shilling, and $\frac{3}{13}$ of a penny.
22. Multiply $19\frac{5}{6}$ by $3\frac{3}{17}$.

DIVISION.

82. *To divide one fraction by another.*

RULE.—Invert the divisor and multiply.

Thus, $\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3} = \frac{10}{9} = 1\frac{1}{9}$: $\frac{7}{12} \div \frac{5}{8} = \frac{7}{12} \times \frac{8}{5} = \frac{56}{60} = \frac{14}{15}$.

Mixed numbers must be reduced to improper fractions and compound fractions to simple ones, before applying the rule. A *Complex* fraction is a short mode of expressing the division of one fractional quantity by another, a fractional quantity by a whole number, or a whole number by a fractional quantity. *Complex* fractions are reduced to *Simple* ones by this rule.

Thus, $\dfrac{\frac{2}{3}}{\frac{4}{5}} = \frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$.

Ex. 2. Divide $\frac{9}{10}$ by 3.

Here $\frac{9}{10} \div 3 = \frac{9}{10} \div \frac{3}{1} = \frac{9}{10} \times \frac{1}{3} = \frac{9}{30} = \frac{3}{10}$ *Ans.*

From the above examples the following Rules will be easily derived.

83. *To divide a fraction by a whole number or the contrary.*

Rule.—Either multiply its denominator or divide its numerator by that number.

84. *To divide any quantity by a fraction.*

Rule.—Multiply the quantity by the denominator of the fraction, and divide the product by the numerator.

For, $3 \div \frac{4}{5} = \frac{3}{1} \times \frac{5}{4} = \frac{15}{4} = 3\frac{3}{4}$; also, 3*s.* 6*d.* $\div \frac{4}{5} = \frac{3s.\ 6d.}{1} \times \frac{5}{4} = \frac{17s.\ 6d.}{4} =$ 4*s.* 4$\frac{1}{2}$*d.*

Reason.—It is evident that fractions having a common denominator may be divided like any other quantities, by omitting the denominators, and dividing the numerators only. Thus, $\frac{2}{3} \div \frac{3}{5} = \frac{10}{15} \div \frac{9}{15} = 10 \div 9 = \frac{10}{9} = 1\frac{1}{9}$; but $\frac{10}{9} = \frac{2}{3} \times \frac{5}{3}$, which is the rule.*

XIX.

Find the quotient of:—

1. $\frac{2}{9} \div \frac{7}{8}$, and of $\frac{1}{4} \div \frac{1}{8}$.
2. $1\frac{1}{9} \div 2\frac{5}{6}$, and of $10 \div \frac{8}{11}$.
3. $16\frac{3}{4} \div 9$, and of $\frac{12}{23} \div \frac{4}{9}$.

---

* If we divide $\frac{2}{3}$ by 3; the fraction $\frac{2}{3}$ implies the quotient of 2 by 3, which quotient is still to be divided by 3; but dividing twice by 3 is the same as dividing by 9; hence $\frac{2}{3} \div 3 = \frac{2}{9}$. Now if the divisor be $\frac{3}{5}$ instead of 3, it is evident that we have divided by a quantity 5 times too great; consequently, the quotient is 5 times too small; hence we must multiply $\frac{2}{9}$ by 5, $= \frac{10}{9}$; then $\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3} = \frac{10}{9}$, which is the rule.

When one proper fraction is *divided* by another, the quotient is greater than the dividend. Division, like Multiplication of Fractions, implies *multiplication* by one number, and *division* by another; and in proper fractions the multiplier is greater than the divisor.

In dividing a mixed number by a whole number, proceed as in the annexed example. After the whole number, 8, is found, the remainder is $3\frac{3}{4}$; then, *mentally*, say, $3\frac{3}{4} = \frac{15}{4}$, write 15 for numerator, and say, $8 \times 4 = 32$, denominator.

$$\begin{array}{r} 8)67\frac{3}{4} \\ \hline 8\frac{15}{32} \end{array} \textit{ Ans.}$$

We should be prepared to avail ourselves of such examples as $24\frac{1}{2} \div 7 = 3\frac{1}{2}$. Here, $24\frac{1}{2}$ contains 7, 3 times, with $3\frac{1}{2}$ remainder; then, since $3\frac{1}{2} = \frac{1}{2}$ of 7, we can write $\frac{1}{2}$ at once. Again, $62\frac{1}{2} \div 10 = 6\frac{1}{4}$; as the remainder, $2\frac{1}{2}$, is $\frac{1}{4}$ of 10, we write $\frac{1}{4}$ at once.

When the divisor is a fraction, the quotient will express *what part or parts the dividend is of the divisor.*'

Find the quotient of:

4. £7 12*s.* 6*d.* $\div\frac{11}{12}$. 6. 16 cwts. 2 qrs. 14 lbs. $\div\frac{5}{7}$.
5. £16 13*s.* 4*d.* $\div\frac{7}{8}$. 7. 4 acrs. 2 rds. 15 pls. $\div\frac{5}{8}$.

Reduce to simple fractions:—

8. $\dfrac{\frac{3}{4}+\frac{1}{2}}{\frac{3}{4}}$; $\dfrac{\frac{3}{5}}{4}\div\frac{6}{7}$; $\dfrac{\frac{1}{2}+\frac{1}{4}}{\frac{1}{3}\times\frac{1}{4}}\div\frac{3}{5}$.

9. $\dfrac{\frac{7}{8}}{\frac{4}{5}}\div 7$; $\dfrac{4\frac{1}{2}}{3\frac{1}{4}}\div\dfrac{\frac{1}{2}+\frac{1}{4}}{\frac{3}{8}}$; $\frac{7}{8}\div\dfrac{\frac{1}{2}\text{ of }\frac{1}{3}}{7}$

85. *To reduce a fraction of one quantity to the fraction of another of the same kind.*

RULE.—If from a lower denomination to a higher, divide: if from a higher to a lower, multiply the fraction by as many of the lower as make one of the higher denomination.

Ex. 1. Reduce $\frac{2}{3}$*d.* to the fraction of a shilling.
Here, $\frac{2}{3}\div 12$, the number of pence in a shilling, $=\frac{2}{36}=\frac{1}{18}$ *Ans.*
Ex. 2. Reduce $\frac{2}{3}$ of a crown to a fraction of a shilling.
Here, $\frac{2}{3}\times 5=\frac{10}{3}=3\frac{1}{3}$*s.* *Ans.*

10. Reduce £$\frac{7}{8}$ to the fraction of £2 10*s.*
11. Reduce $\frac{3}{4}$*s.* to the fraction of 10*d.*
12. Bring $\frac{4}{7}$ lb. to the fraction of 2 qrs.

Commissariat.

13. Divide $\frac{6}{7}$ by $\frac{2}{3}$.

Customs.

14. Divide $\frac{3}{8}$ by $\frac{5}{6}$.

Expectants of Excise.

15. Divide $\frac{3}{4}$ by $\frac{8}{10}$.
16. ,, $\frac{3}{20}$ by $\frac{4}{5}$.
17. ,, $\frac{11}{13}$ by $\frac{12}{11}$.
18. ,, $\frac{9}{16}$ by $4\frac{1}{2}$.

Admiralty.

19. Divide $15\frac{5}{12}$ by $7\frac{3}{8}$.

Miscellaneous.

20. Divide $4\frac{1}{3}$ by $7\frac{11}{13}$; $9\frac{1}{11}$ by $2\frac{3}{11}$; $4\frac{1}{5}$ by $6\frac{3}{7}$.
21. „ $7\frac{5}{9}$ by $4\frac{2}{19}$; $6\frac{2}{3}$ by $2\frac{1}{9}$; $\frac{9}{11}$ by $4\frac{2}{7}$.
22. „ $2\frac{4}{15}$ by $4\frac{1}{3}$; $7\frac{8}{9}$ by $\frac{13}{24}$; $48\frac{1}{6}$ by 17.
23. „ $4\frac{5}{6}$ by $6\frac{7}{18}$; $2\frac{7}{19}$ by 15; $73\frac{17}{18}$ by $\frac{11}{12}$.
24. „ $7\frac{5}{7}$ by $4\frac{2}{21}$; $1\frac{1}{4}$ by $2\frac{1}{9}$; ·76 by $\frac{19}{23}$.
25. „ $18\frac{2}{7}$ by $5\frac{1}{7}$; $4\frac{3}{8}$ by $1\frac{13}{16}$; 85 by $6\frac{3}{8}$.
26. „ $5\frac{1}{6}$ by $3\frac{2}{3}$; $2\frac{19}{26}$ by $16\frac{5}{13}$; $35\frac{2}{11}$ by $4\frac{7}{9}$.
27. What fraction of half-a-crown is $\frac{2}{9}$ of 6*s.* 8*d.*?
28. Reduce £7 9*s.* 6*d.* to the fraction of £13 4*s.* 6*d.*
29. Reduce 6 acres 1 rood and 4 poles to the fraction of $2\frac{1}{2}$ roods.
30. What fraction of a pound is 9*s.* 10*d.*?

Customs, Post Office, &c.

31. Add $\frac{3}{4}$ of a pound to $\frac{5}{8}$ of a shilling.
32. From $\frac{1}{6}$ of a yard take $\frac{2}{3}$ of an inch.

DECIMALS.

86. We now come to consider another class of fractions, called Decimal Fractions, or more briefly Decimals. Any fraction whose denominator is 10 or any power of 10, or, more simply, which has 1 followed by any number of cyphers for its denominator, is called a decimal fraction.

The denominators of decimals are generally *suppressed*; hence it becomes necessary to have a method of distinguishing their numerators from whole numbers. If we omit the denominator of $\frac{7}{10}$ it becomes 7; to prevent this 7 from being read 7 units, we place a point before it; thus, ·7.

87. A more extended *Numeration Table* must now be given, but almost *identical* with that of whole numbers (**9.**)

Thousands.	Units.		Thousandths.	Millionths.
0 0 0	0 0 0	•	0 0 0	0 0 0
⋯ ‥ ·	⋯ ‥ ·		⋯ ‥ ·	⋯ ‥ ·

This Table, like that just referred to, has an *indefinite* number of periods; each period has *three* places—units, tens, hundreds. Therefore decimals may be *written* and *read* like whole numbers. This Numeration Table also increases towards the left. In fact it is merely a continuation of the former table to the right; and, like it, increases in a tenfold ratio to the left. In the above table, the period immediately to the left of the *units' period* is the *thousands'*, and that immediately to the right the *thousandths';* because every 1 in this period is the one-thousandth part of a unit; just as 1 unit is the one-thousandth part of 1000, and 1000 the one-thousandth part of a million. In the same manner 12 in this period are twelve-thousandths, and so on. The next period to the right is called the *millionths'* period; we may continue to form periods to the right as far as we please.

If it is required to write 165 thousandths, we must place it in its *proper* period; for this purpose prefix the *point*; thus, ·165, read, *one hundred and sixty-five thousandths.* To set down 3 thousandths, prefix 00, to have 3, that is, 3 *units of thousandths* in its proper place; thus, ·003. We read 3·63, three units and six hundred and thirty thousandths, or sixty-three hundredths.

88. If the point be removed *one* place to the *right*, the number will be increased *tenfold.* For 6 = ten times ·6, and ·6 = ten times ·06. This is evident from the nature of notation. On the contrary, if the point be removed *one* place to the *left*, the number will be divided by 10; if two places, by 100; and so on. When the point is removed one place to the right the units become tens, &c.

Any number of cyphers to the right of a decimal has no effect, as they do not alter the position of the point. Thus, ·6 = ·60 = ·600. But a cypher placed between the point and a decimal diminishes the decimal tenfold; for ·6 = ten times ·06: two cyphers, placed in this position, decrease the decimal one hundred-fold: for ·6=one hundred times ·006, since $\frac{6}{10} = \frac{6}{1000} \times 100$.

With a little practice the learner will be able to write decimals with the same facility as whole numbers.

XX.

Write down in figures :—

1. Ninety-five thousandths.
2. Thirty-nine thousandths.
3. Twelve units and sixty-eight thousandths.
4. Six hundred and nine thousandths.
5. Fifty units and sixty thousandths.
6. Twenty-nine units and nine thousandths.
7. Forty-seven units and four hundred thousandths.
8. Sixty-nine units six hundred and seven thousandths and sixty-four millionths.
9. One hundred and twenty units fourteen thousandths and nineteen millionths.
10. Fifty-five units six hundred and forty-nine thousandths and four hundred millionths.
11. Sixty units sixty thousandths and sixty millionths.
12. Seventeen units seventeen thousandths and seventeen millionths.

ADDITION.

89. RULE.—1°. Set down the addends with their points in the same vertical line.

2°. Then add as in *Simple Addition*, and place the point of the sum under the other points.

REASON.—It has been shown that the *notation* of decimals is exactly the same as that of whole numbers; therefore we may say, '*Add and Subtract as in whole Numbers.*

Various Departments.

Ex. 1. Add together 3·406, ·0212, 47·9 and ·6.

This is added just as an example in whole numbers; care should be taken in writing the decimal points directly under each other.

```
 3·406
  ·0212
47·9
  ·6
-------
51·9272 Ans.
```

XXI.

Various Departments.

1. Add together 52·38, 367·4, ·172, and 6·0053.
2. „ 672·5, 4·923, 80, and ·076.

Customs.

3. Add together 2·8146, ·0938, 8, ·875, 31·2788, 4·0087.
4. „ 1·3046, 15·031, ·0082, and 3·29.

Various Departments.

5. Add 74·36, 5·473, ·9872, 340·03, and 5984.
6. Add 72·5, 2·1574, 371·4, and 2·75.

India Board.

7. Add together 42·79, ·2105, ·047, and 140.

Various Departments.

8. Add together 27·03, 452·0091, ·37, 1·873592, and 83.
9. „ 70·1046, 701, ·6, ·16, and 7·304.
10. „ 501·1306, ·96, 6·401302, and 72.
11. „ 35·2176, 201·00541, 3·1482, ·054, 216·945, 7543·4.

Science and Art, Prisons, &c.

12. Add together 62·7, 2·30073, 450·923.

Constabulary.

13. Add together ·813, 420·91, ·0093, 7·043, 12560.

SUBTRACTION.

90. RULE.—Write the numbers as two addends are written: subtract as in whole numbers: and place the *point* in the remainder under the other points.

Various Departments.

Ex. 1. Subtract 19·7564 from 26.

In this example we may suppose cyphers annexed to the minuend, as such cyphers have no effect (**88**) on its value. In subtracting we begin just as in whole numbers and place the point in its proper place as directed by the rule

26·0000
19·7564
6·2436 *Ans.*

XXII.

India Board.

1. Subtract 42·946 from 161·06.

Expectants of Excise.

2. From 10154271· take 9178965·8742.
3. From 991742865·010101 take 881745522·201002.

Inland Revenue.

4. Subtract 17·2463 from 29.
5. „ 337·55 from 350·111.

Constabulary.

6. From 506·222 take 59·87.

Science and Art, War Department, Prisons.

7. Subtract 5·74 from 121·0239.

Admiralty Dockyards.

8. Subtract 3·901 from 6·01.

Various Departments.

9. From 3017·215 take 5·7124.
10. Subtract 83·450392 from 1210·3.
11. Subtract 901·53629 from 30640·48.
12. Add together 5·3, ·00015, 643·8797, 62·09, 1035; and subtract 109·000365 from the result.
13. Subtract 236·932 from 270·00086.

MULTIPLICATION.

91. RULE.—1°. Multiply as in whole numbers disregarding the points.

2°. Point off so many places of decimals in the product as are in *both* multiplicand and multiplier, prefixing cyphers if necessary.

Ex. 1. 7·64 × 8·9.

In this example there are two places of decimals in multiplicand and one in multiplier; therefore the product must have three places of decimals.

$$\begin{array}{r} 7{\cdot}64 \\ \underline{8{\cdot}9} \\ 6876 \\ \underline{6112} \\ 67{\cdot}996 \text{ product.} \end{array}$$

Ex. 2. Multiply 476 by ·0137.

In this example after multiplying we prefix 00 to make up 7 places of decimals, the number in both multiplicand and multiplier.

$$\begin{array}{r} ·476 \\ ·0137 \\ \hline 3332 \\ 1428 \\ 476 \\ \hline ·0065212 \text{ product} \end{array}$$

Reason.—The nature of this rule, as well as its reason, will be best understood from the following explanation.

If we have to multiply 7·9 by 6, that is, to add 7·9, 6 times, we say 6 × ·9 = 5·4: this can be shown by *Addition*. Set down ·4, and carry 5; therefore 7·9 × 6 = 47·4, and has only one decimal place. Now, if we make the multiplier ten times less, we have 7·9 × ·6, which must give a product ten times less than 47·4, that is 4·74 having two places of decimals. Also, 7·9 × ·06 = ·474; and 7·9 × ·006 = ·0474, having four decimal places, which establishes the rule.

XXIII.

Customs and Excise.

1. Multiply 75·6 by 6·75.
2. „ 19·378 by 19·38.
3. „ 33·21 by 4·41.
4. „ 5·874 by 58·139.
5. „ 1·021 by ·0037.

Post Office, Police Courts, &c.

6. Multiply 3·46 by ·489.
7. „ 79·004 by ·00473.

India Board.

8. Multiply 65·43 by ·00376.

Admiralty Dockyards.

9. Multiply 6·4073 by ·42.

Constabulary Office.

10. Multiply 757·04 by 15·8.

Various Departments.

11. Multiply 9·436 by 67·49.
12. „ 4·81 by ·0074.

13. Multiply 3·05 by ·25 and ·32 by ·231.
14. „ 1·342 by ·2057.
15. „ 5412·384 by 1·0023.
16. „ ·1356 by ·1458.
17. „ 5·0103 by 6·503.
18. „ 80·46 by ·00392.

DIVISION.

92. RULE.—I.—1°. If the dividend and divisor do not contain an equal number of decimal places, make the number of places equal by annexing cyphers.

2°. Then disregarding the points, divide as in whole numbers; the quotient thus obtained will be a whole number; after which the point is placed.

3°. Any other quotient figures, found by adding cyphers, will be decimals.

Ex. 1. Divide 478·9875 by 7·5.

There are four places of decimals in the dividend, we annex 000 to the divisor to have an equal number in it. Having brought down 5, the last figure of the dividend, and having found its corresponding quotient figure, we write the point, and add cyphers for the remaining figures of the quotient.

```
7·5000)478·9875
       450000
       ------ (63·865
        289875
        225000
        ------
         648750
         600000
         ------
          487500
          450000
          ------
           375000
           375000
           ------
```

93. ***When the dividend contains more places of decimals than the divisor.***

RULE.—1°. Set a mark after an equal number of decimal places in the dividend.

2°. Then divide as in *Rule I.*, remembering to bring down the other figures of the dividend in the process.

Ex. 2. 478·9875÷7·5.

This is the same example as given under *Rule I.* Compare the two methods and the superiority of this one when applicable will be evident.

```
7·5)478·9,875
    450       (63·865 Ans
    ───
     289
     225
     ───
      648
      600
      ───
       487
       450
       ───
        375
        375
        ───
```

XXIV.

Customs and Excise.

1. Divide 3·24 by ·25.
2. „ 2·5 by ·32.
3. „ 9065 by ·049.

Science and Art, War Department, &c.

4. Divide 307·8 by 23·8 to 3 places of decimals.
5. „ 8·747 by 413·4 to 3 places of decimals.

Post Office and Excise, &c.

6. Divide 476·532 tons by 60 and bring it to its proper value.*
7. „ £880·7643 by 5·48 and bring it to its proper value.

India.

8. Divide 154·28 by ·0064.

Admiralty.

9. Divide 240·13 by 73·4 to 3 places of decimals.
10. „ ·0045 by ·03.

Surveyor of Taxes.

11. Divide 327·6 by 4·89 to 3 places of decimals.
12. „ 55·8 by ·08125 to 3 places of decimals.
13. „ 6·045 by 62·5.

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* The learner should omit Nos. 6, 7, 13, 15, 17, 20, 21, 32, 33, 34, 37, 38, and 48 until he has read Circulators.

Various Departments.

14. Divide 274·6 by 3·672 to 3 places of decimals.
15. „ 89·2 by ·0074 to 3 places of decimals.
16. „ 16 by ·0004.
17. „ 21546·872 by ·054.
18. „ 34·952 by 53·07 giving the first 4 figures of the quotient.
19. „ ·1 by ·001 and the quotient by 2.
20. „ 721·42 by 21·9 to 4 places of decimals.
21. „ ·169 by 13, by 130, and by ·0013.
22. „ 91·6 by 8931·61 to 4 places of decimals.
23. „ 43·2 by ·0351 to 3 places of decimals.

94. *To reduce a decimal to its equivalent vulgar fraction.*

Rule.—Omit the point, and write the figures of the decimal as numerator, and 1 with as many cyphers as there are decimal places for denominator.

Thus, $\cdot755 = \frac{755}{1000} = \frac{151}{200}$; also, $\cdot8 = \frac{8}{10} = \frac{4}{5}$; also $\cdot025 = \frac{25}{1000} = \frac{5}{200} = \frac{1}{40}$.

This is merely supplying the denominator which is suppressed in every decimal.

Various Departments.

24. Express ·0125 as a vulgar fraction in its lowest terms.
25. Express ·00175 as a vulgar fraction in its lowest terms.
26. Reduce 2·0445 and ·000625 to vulgar fractions.
27. Reduce 23·038125 and ·0006875 to vulgar fractions.

95. *To reduce a vulgar fraction to a decimal.*

Rule.—Divide the numerator by the denominator according to the rule for the division of decimals. The quotient will be the required decimal.

Thus, $\frac{3}{4} = 3 \div 4 = \cdot75$; and $\frac{3}{8} = 3 \div 8 = \cdot375$.

Reason.—We require to bring $\frac{3}{4}$ to a decimal—one whose denominator is 10, or some power of 10; a method of doing which was given (**66**). By the rule referred to, we find, as 4 measures 100, the multiplier is 25, for any fraction whose denominator is 4; therefore $\frac{3}{4} \times \frac{25}{25} = \frac{75}{100} = \cdot75$, the same result as found above. But multiplying by 25 is the same as adding 00 and dividing by 4; hence the rule.

A fraction to produce an *exact* decimal must have for denominator a measure of 10, 100, 1000, &c.

Reduce to decimals :—

28. $\frac{1}{2}$; $\frac{1}{5}$; $\frac{7}{8}$; $\frac{4}{5}$; $\frac{19}{25}$. 29. $\frac{7}{16}$; $\frac{11}{25}$; $\frac{13}{32}$; $\frac{47}{64}$; $\frac{97}{625}$

96. *To reduce one quantity to the Fraction of another.*

Ex. 1. Reduce 12*s.* 6¾*d.* to the decimal of a £.

Here we begin at the lowest denomination, ¾*d.*, which we express as a decimal of next higher, and write along with the higher; then dividing by 12, we express 6·75*d.* as the decimal of a shilling, writing the number of shillings before it, and so on.

4) 3f.
12) 6·75*d.*
2,0)1,2·5625*s.*
·628125£
Or, *Ans.*

Since 12*s.* 6¾*d.*=603 farthings, we have $\frac{603}{960}=\frac{201}{320}=$ ·628125.

Reduce to the decimal of a pound :—

30. 15*s.* 9¾*d.*; 17*s.* 9*d.*; 7½*d.*; 18*s.* 10½*d.*

31. 8*s.* 1½*d.*; 9*s.* 4¾*d.*; 4*s.* 2½*d.*; 1½*d.*

Reduce to the decimal of a guinea :—

32. 1*s.* 9*d.*; 5*s.* 3*d.*; 10*s.* 11¼*d.*; 2*s.* 7½*d.*

33. $\frac{5}{8}$ of a crown ;* 1·85 of 3*s.* 4*d.*

Reduce to the decimal of a ton: —

34. 12 cwt. 1 qr. 7 lbs.; 7 cwt. 3 qrs. 21 lbs.; 14 lbs.

Reduce to the decimal of an acre :—

35. 2 rds. 18½ pls.; 2 rds. 10 pls.; 35 pls.; 1 rd. 7½ pls.

36. Bring $\frac{5}{7}$ of 5*s.* 3*d.* to the decimal of £2.

37. Reduce $\frac{4}{5}$ of 7*s.* 6*d.* to the decimal of £2 16*s.* 8*d.*

38. Reduce $\frac{5}{64}$ and $\frac{3}{14}$ to decimals.

39. Bring $\frac{21}{250}$ to a decimal; and 5½ in. to decimal of 1 *m.*

97. *To reduce any number of shillings, pence and farthings, accurately to the decimal of a pound, at once.*

RULE.—1°. Conceive an 0 to be annexed to the shillings, and take one-half that number for the first part of the decimal.

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* The chapter on Circulators may now be explained.

2°. Falling back one place to the right, write the number of farthings in the pence and farthings.

3°. Falling back another place to the right, set down one-sixth of the pence and farthings, considering the farthings expressed as the decimal of a penny.

4°. Then add these three lines of figures for the decimal required.*

Ex. 1. Reduce 13*s.* 9¾*d.* to the decimal of a pound.

Here we take one-half of 130 = 65 ; then 9¾*d.* = 39*f.*, the next part ; and 9¾*d.* ÷ 6 = 9·75*d.* ÷ 6 = 1·625, the last part.

$$\begin{array}{l} 13s.\ 9\frac{3}{4}d. \\ \hline \cdot65 \\ \quad 39 \\ \qquad 1625 \\ \hline \cdot690625£.\ \textit{Ans.} \end{array}$$

Ex. 2. Reduce 19*s.* 4¾*d.* to the decimal of a pound.

Here we take one-half of 190 (a 0 annexed to the shillings) = ·95 ; next, we write the number of farthings in 4¾*d.* = 19 ; and lastly, we write one-sixth of 4·75 (4¾*d.*) = ·0791$\dot{6}$.

$$\begin{array}{l} 19s.\ 4\frac{3}{4}d. \\ \hline \cdot95 \\ \quad 19 \\ \qquad 0791\dot{6} \\ \hline \cdot969791\dot{6}£.\ \textit{Ans.} \end{array}$$

Reduce to the decimal of a pound:—

	s.	*d.*		*s.*	*d.*
40.	17	6,	and	13	7½
41.	16	9¾,	and	19	6
42.	17	9¾,	and	7	11¼
43.	10	10½,	and	4	4½

98. ***To find the value of the decimal of a given quantity.***

Rule.—1°. Multiply the decimal, as in reduction, by the number of units of the next lower denomination which makes one of the given denomination.

---

* Since 1*s.* = £$\frac{1}{20}$, 1*s.* = £·05 ; and 2*s.* = ·1 : hence the reason of taking half the shillings. Adding the 0 is unnecessary when the number of shillings is even ; for *one* shilling write ·0 before dividing by 2. Again, £1 = 960*f.*, which wants 40 of being 1000, the number of thousandths in a unit (in a £) ; 40 wanted in 960 is $\frac{40}{960}=\frac{1}{24}$, that is, 1 in 24 ; in other words, there will be more thousandths than farthings by 1 in every 24*f.*, or 1 in every 6*d.*; hence the reason of adding ⅙ of the pence. Conversely, there will be *less* farthings than thousandths by 1 in 25, for there are 40 *less* in 1000, and $\frac{40}{1000}=\frac{1}{25}$; hence the decimals may be considered farthings, after rejecting 1 in every 25.

2°. The whole number (if any) thus obtained will be in the lower denomination.

3°. Multiply the decimal part in the same manner, for the next lower denomination, and so on.

Ex. 1. Find the value of ·675 of a *cwt.*

Here we multiply by 4, and find 2 a whole number, which is 2 qrs.; again, we multiply by 28 and find 19·6 lbs., and so on.

$$\begin{array}{rl} & \text{cwt.} \\ \cdot 675 & \\ 4 & \\ \hline 2\cdot 700 & \text{qrs.} \\ 28 & \\ \hline 19\cdot 600 & \text{lbs.} \\ 16 & \\ \hline 9\cdot 6 & \text{oz.} \\ 16 & \\ \hline 9\cdot 6 & \text{drs.} \end{array}$$

2 qrs. 19 lbs. 9 oz. 9·6 drs. *Ans.*

Ex. 2. Find the value of ·526 of £1 10s.

Here, ·526 × 30 = 15·78*s.*; and ·78 × 12 = 9·36*d.*; then ·36 × 4 = 1·44: therefore 15*s.* 9¼*d.* *Ans.*

Find the value of: —

44. ·7689£., ·465*s.*

45. ·976 of a day.

46. ·634 of 6*s.* 8*d.*

47. ·7958 of an acre.

99. *To find the value of any decimal of a pound at once.*

Rule.—1°. Take the first three figures of the decimal; increase the third by 1, if the fourth decimal figure be greater than 5.

2°. Then, take one-fifth of the number expressed by the first two figures for shillings.

3°. Prefix the remainder (if any) to the third, and after rejecting one twenty-fifth, consider the result as farthings.

Ex. 1. Find the value of ·7967£.

As the fourth figure is 7, we add 1 to the third; then we have ·797; now, 79 ÷ 5 = 15. + 4r., and 47 − 2 = 45*f.* = 11¼*d.* We subtract 2 because 47 is nearly 50; hence 15*s.* 11¼*d.* *Ans.*

REASON.—Considering the first and second figures as a whole number is the same as multiplying by 100; and we divide this by 5. Since 100÷5=20, multiplying by 100, and dividing by 5, is equivalent to multiplying by 20, the number of shillings in a pound. (*See* **97** *and note.*)

Find the value of:—

48. ·6583£., and of ·97832£.

Customs and Excise.

49. Add the following and find the value of it in £. *s. d.*

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1·521
 ·675
 ·145
3·184
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50. Add together ·0204£., 1·326£., and ·003£., and find the value of the sum.

Admiralty.

51. Find the value of ·75 of a £.
52. What is the value in money of ·25 of a £.?
53. What is the value in money of ·73125 of a £.?
54. What is the value in money of ·025 of a £.?

Various Departments.

55. What is the value of ·4375 of a shilling?

CIRCULATING DECIMALS.

100. One fraction can be reduced to another equal in value and having *any denominator*, when the denominator of the given fraction is a *factor* of that to which it is to be reduced (**66.**) Now, as the denominator of every decimal is 10 or some power of 10, we cannot reduce those vulgar fractions to equivalent decimals which have denominators that are *not* factors of 10, or some power of 10. In the reduction of such fractions to decimals, the division does not terminate, and the same figures will be repeated over again, continually. If we try to reduce $\frac{2}{3}$ to a decimal, it becomes

$\frac{2}{3} = \cdot6666$ &c.; also, $\frac{5}{9} = \cdot5555$ &c. Decimals of this kind, in which the same figures are continually repeated, are called *Repeating*, *Recurring*, or *Circulating* decimals. The part repeated is called the *Period* or *Repetend*, and is termed a *Simple Repetend* when it consists of one figure; when it consists of more than one figure it is called a *Compound Repetend.* Decimals, arising from vulgar fractions, whose denominators give *exact* quotients, are called *Terminate.* The period of a circulator is marked by means of dots or accents, placed over its first and last figures. When only one figure is repeated, one dot is sufficient. Thus, $\frac{2}{3} = \cdot\dot{6}$, and $\frac{1}{9} = \cdot\dot{1}$.

When the period begins *immediately* after the point, it is called a *Pure Circulator;* as, $\frac{1}{3} = \cdot\dot{3}$.

A *Mixed Circulator* has a finite and an infinite, or circulating part. Thus, $\frac{1}{6} = \cdot1\dot{6}$. The finite part is $\cdot1$, and the recurring part is $\cdot\dot{6}$.

We find by using the nine figures as denominators and 1 as numerator, that 2, 4, 5, and 8, give *terminate;* 3, 7, 9, *pure circulators;* and 6, a *mixed circulator*, as in the annexed equations.

$\frac{1}{2} = \cdot5$
$\frac{1}{3} = \cdot\dot{3}$
$\frac{1}{4} = \cdot25$
$\frac{1}{5} = \cdot2$
$\frac{1}{6} = \cdot1\dot{6}$
$\frac{1}{7} = \cdot\dot{1}4285\dot{7}$
$\frac{1}{8} = \cdot125$
$\frac{1}{9} = \cdot\dot{1}$

101. Any fraction in its lowest terms whose denominator contains only powers of 2 and 5 as factors, can be reduced to a terminate decimal.

Reason.—A fraction brought to its *lowest terms* can have no factor common to both numerator and denominator: therefore it is in consequence of the *addition* of cyphers to the numerator, that is, the multiplication by 10 or some power of 10, that the denominator measures the numerator; in other words, it is the number by which the numerator is multiplied that the denominator measures. But 10 contains only the factors 2 and 5; any power of 10 will only contain powers of 2 and 5. Therefore the denominator, to measure any power of 10, must be made up of 2 and 5, or powers of these factors. If the denominator be so made up, the decimal will terminate; but if the denominator be made up of any other factors, as 3, 7, 9, &c., it will produce a *circulator.*

102. The number of figures in a period must always be less than the number of units in the denominator.

Because different figures must arise from different remainders, and the number of remainders must be less than the divisor or denominator. Thus, $\frac{1}{7}=\dot{1}4285\dot{7}$. When dividing by 7, we can only have 1, 2, 3, 4, 5, 6, as remainders. In dividing by any number, the greatest remainder must be less than the divisor.

103. *To reduce a pure circulator to a vulgar fraction.*

RULE. Write the period for numerator, and so many nines as there are figures in it, for denominator. Thus, $\cdot\dot{3}=\frac{3}{9}=\frac{1}{3}$; and $\cdot\dot{2}\dot{7}=\frac{27}{99}=\frac{9}{33}=\frac{3}{11}$.*

REASON.—Since $\frac{1}{3}=\cdot3333$, &c., multiply by 10, and we have ten-thirds $=3\cdot333$, &c.; then, subtracting $\cdot333$, &c., from $3\cdot333$, &c., we get 3, which must be equal to nine-thirds, as we took the value of $\frac{1}{3}$ from ten-thirds. Now, since $\frac{9}{3}=3$, one-third must equal $\frac{3}{9}$, therefore $\cdot333$, &c., $=\frac{3}{9}$.

XXV.

Reduce to equivalent vulgar fractions:—

1. $\cdot\dot{7}$, and $\cdot\dot{6}$.
2. $\cdot\dot{4}\dot{5}$, and $\cdot\dot{1}3\dot{5}$.

104. *To reduce a mixed circulator to a vulgar fraction.*

RULE.—1°. From the finite part and period, considered as one number, subtract the finite part; the difference will be the numerator.

2°. For the denominator, write as many nines as there are figures in the period, followed by as many cyphers as there are figures in the finite part. Thus, $\cdot1\dot{6}=16-1=15$, and $\frac{15}{90}=\frac{3}{18}=\frac{1}{6}$.†

PROOF.—$\frac{1}{6}=\cdot1\dot{6}$.

---

* $\frac{1}{9}=\cdot1111$, &c.; $\frac{2}{9}=\cdot2222$, &c.; $\frac{5}{9}=\cdot555$ &c. Also, $\frac{1}{99}=\cdot010101$, &c.; and $\frac{15}{99}=\cdot1515$, &c.: therefore any figure divided by 9 gives that same figure as period, and any two figures divided by 99 will give those *two figures* as period; hence the reason of the rule.

† Take $\frac{7}{30}$, which is $=\cdot2\dot{3}$; multiply this mixed circulator by 10, and we get $2\cdot\dot{3}$, which is $2\frac{3}{9}$ (**103**). Now, $2\frac{3}{9}$ is 10 times too great, as we multiplied by 10; then dividing by 10 we get $\frac{2}{10}+\frac{3}{90}$ (dividing both separately). By adding, we get $\frac{2\times90}{900}+\frac{30}{900}=\frac{2\times9}{90}+\frac{3}{90}$ and since $9=10-1$, we have $\frac{2\times(10-1)}{90}+\frac{3}{90}=\frac{2\times10-2}{90}+\frac{3}{90}$; adding numerators, we have $\frac{20+3-2}{90}=\frac{23-2}{90}=\frac{21}{90}=\frac{7}{30}$ which is the rule.

Reason.—Multiply $\cdot1\dot{6}$ by 10, and we have ... ... $1\cdot\dot{6}$ *Take*
Again, multiply $\cdot1\dot{6}$ by 100, and we have ... ... $16\cdot\dot{6}$ *From*
Subtracting the former product from the latter we get 90 times $\cdot1\dot{6} = 15$; therefore $\cdot1\dot{6} = \frac{15}{90}$, which is the rule. 15

Reduce to vulgar fractions :—

3. $\cdot\dot{2}\dot{7}$, $\cdot5\dot{9}\dot{0}$.
5. $\cdot7\dot{3}$ and $\cdot8\dot{3}$.
4. $\cdot58\dot{3}$, $\cdot\dot{6}9230\dot{7}$.
6. $\cdot\dot{1}\dot{8}$ and $\cdot7916$.
7. $\cdot4\dot{1}8\dot{9}$, $\cdot07\dot{8}5714\dot{2}$.
8. $\cdot\dot{8}1\dot{9}$, $\cdot05\dot{7}6923\dot{0}$.
9. Multiply $\cdot0\dot{0}2\dot{1}$ by $48\cdot02\dot{6}$, and $\cdot0\dot{2}9\dot{7}$ by $\cdot6\dot{7}\dot{2}$.

105. Since (**103.**) $\cdot\dot{3} = \frac{3}{9}$, it follows that $\cdot\dot{9} = \frac{9}{9} = 1$. It is evident that we cannot reverse this operation and bring 1 to $\cdot99$, &c.; but if we repeat the period we shall find that it approaches so near 1, that the difference will be absolutely nothing; for $\cdot999 + \frac{1}{1000} = 1$, and $\cdot9999 + \frac{1}{10000} = 1$, &c. Therefore when we meet $\cdot\dot{9}$ in any decimal we may add 1 to the preceding figure, and say, $\cdot69\dot{9}$, &c., $= \cdot7$; $\cdot4\dot{9} = \cdot5$; and $\cdot7\dot{9} = \cdot8$.

When *great* accuracy is required, circulators should be reduced to vulgar fractions; but, in general, it will be sufficient for practical purposes to repeat the period to *three* or *four* places.

In reducing vulgar fractions to decimals, having found half the circulating period, take the difference between each digit, and 9 for remaining figures: thus, $\frac{5}{7} = \cdot714$, &c. Now, taking these figures from 999, we have $\frac{5}{7} = \cdot\dot{7}1428\dot{5}$.

Again, $\frac{7}{19} = \cdot368421052$; now, taking these from 9 respectively, we have $\frac{7}{19} = \cdot\dot{3}6842105263157894\dot{7}$. [This, however, does not invariably hold good.]

Again, since $\frac{7}{19} = \cdot36842\frac{2}{19}$; now dividing by 7 we have $\frac{1}{19} = \cdot05263\frac{2}{19}$; therefore $\frac{2}{19} = \cdot10526\frac{6}{19}$; consequently by substituting $\frac{7}{19} = \cdot3684210526\frac{6}{19}$. Now substituting the value of $\frac{6}{19}$, we get more figures, and so on until we arrive at the actual value. The following will be found useful to the learner :—

$\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$; now, $\frac{1}{3} = \cdot 333$, &c., and $\frac{1}{6} = \cdot 166$, &c.; then $\cdot 333 \times \cdot 166 = \cdot 055278$, the product of $\frac{1}{3}$ and $\frac{1}{4}$ when reduced to *circulators* of *three* figures; but $\frac{1}{18} = \cdot 0555$, &c.; the two products only coincide in three figures. The number of decimal places to be used will depend very much on the nature of the case. If we have three true places in any result, it is sufficient in commercial matters. For, since £1 = ·960*f.*, 1*f.* is greater than $\frac{1}{1000} = \cdot 001$. If the fourth place were 9, it would be much less than a farthing.*

106. From the preceding observations it is evident that the kind of decimal any vulgar fraction will give, can be determined from its denominator in its lowest terms; if a mixed circulator, the number of figures in its finite part; and if a finite decimal, the number of decimal places. Any *pure circulator* must have so many figures in its period as will be equal to the least number of *nines* which will be measured by its denominator. The denominator must be a factor of a certain number of nines, (**66.**)

A *mixed circulator* arises from a denominator made up of either *twos* or *fives* and some other factors. Thus, 6 is $= 3 \times 2$; here 2 will give a *finite* part, and 3 an *infinite* part.

Again, take $\frac{13}{54}$; now $54 = 2 \times 3 \times 3 \times 3$; hence any vulgar fraction having 54 as denominator, will give a decimal having one figure in its finite part, corresponding to the factor 2, and three figures in its period corresponding to $3 \times 3 \times 3$. We find $\frac{13}{54} = \cdot 2\dot{4}0\dot{7}$. This is true irrespective of the numerator; for $\frac{1}{54} = \cdot 0\dot{1}8\dot{5}$, and $\frac{53}{54} = \cdot 9\dot{8}1\dot{4}$.

A *Finite* decimal will have as many decimal places as are equal to the greatest number of *twos* or *fives* in the factors of its denominator in its lowest terms. If different denominators be compared with the decimals they produce, the reason of this will be evident; and such a comparison will afford an excellent exercise for the student.

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* When we have *two* decimal places in any result, the *error* by omitting any other decimal figures, must be less than $\frac{1}{100}$, for ·009 is less than ·01. If we have *three* places of decimals, the *error* must be less than $\frac{1}{1000}$, for ·001 is greater than ·0009; this is called *the Limit of the Error*; and since £$\frac{1}{1000}$ is less than 1*f.*, three true decimal places in any result will be quite sufficient for mercantile calculations.

107. We can frequently dispense with the rule given in 104. The circulator $\frac{1}{7} = \cdot\dot{1}4285\dot{7}$; we find that any other numerator with 7 as denominator will produce the same figures, but in a different order: thus, $\frac{3}{7}$ = a circulator commencing with the figure 4, that is $\frac{3}{7} = \cdot\dot{4}2857\dot{1}$, and so on with $\frac{5}{7}$, $\frac{6}{7}$, &c. Now the *sum* of the figures of the above period equals 27, a multiple of 9; and, indeed, the sum of very many of the figures which form periods is either $=9$ or some multiple of 9. In finding a vulgar fraction equal to $\cdot3585714\dot{2}$, we see at once by inspection $\cdot\dot{8}5714\dot{2} = \frac{6}{7}$: hence $\cdot35\dot{8}5714\dot{2} = \cdot35\frac{6}{7}$, which is obviously $= \cdot\frac{251}{7}$, that is, $= \frac{251}{700}$, the fraction required. Passing over the circulators arising from the denominators 3 and its multiples 6 and 9, we have the next number in succession, 11, giving 2 figures in its period, the sum of which $=9$: thus, $\frac{3}{11} = \dot{2}\dot{7}$, and $\frac{10}{11} = \cdot\dot{9}\dot{0}$. The circulators arising from the denominator 13, though a little more complicated than those arising from 7, are also easily known by inspection; these also circulate on 6 figures, but there are, in regular rotation, two sets of figures that form the period. We find $\frac{1}{13} = \cdot\dot{0}7692\dot{3}$, the sum of the figures being $=27$, but $\frac{2}{13} = \cdot\dot{1}5384\dot{6}$, a new set of figures. On examining the 12 proper fractions which can have 13 for denominator, we find 6 of them, those with, 1, 3, 4, 9, 10, and 12, as numerators, circulating on the first set of figures given above, and the others taking the second set for their circulating period.

We can now easily reduce $\cdot75\dot{3}8461\dot{5}$ and $\cdot46\dot{7}6923\dot{0}$ to vulgar fractions; for they are $= \cdot75\frac{5}{13}$ and $\cdot46\frac{10}{13}$ respectively, which are $= \frac{980}{1300}$ and $\frac{608}{1300} = \frac{49}{65}$ and $\frac{152}{325}$. Although 17, 19, and many other prime numbers, as denominators will only circulate on a number of figures $=$ one less than the units in the denominator, yet 37 circulates on 3 figures: thus $\frac{1}{37} = \cdot\dot{0}2\dot{7}$; again, 73 produces a period of 8 figures.

COMPLEX FRACTIONS.

108. Few questions give more trouble to learners than those which contain Complex Fractions.

Brackets have the effect of binding two or more quantities so closely as to cause them to be considered collectively, and subjecting them to the same laws as if they were all expressed in one quantity. For example $(4+6)\times10 = 10\times10 = 100$, the 4 and 6 being added before their relation to the number 10 is considered. Had no bracket existed we should have $4 + 6 \times 10 = 4 + 60 = 64$.

As a general rule it may be laid down that the signs × and ÷ bind quantities more closely than signs + and —. The last two signs, though uniting the quantities, imply distinct operations.

Ex. 1. Simplify:— $\dfrac{\frac{3}{4}+\frac{6}{7}-\frac{5}{8}}{\frac{3}{14}+\frac{2}{7}+\frac{3}{16}}+\frac{11}{14}\times 2\frac{1}{3}$.

We have here two parts, separated by the sign +, which should be kept distinct until the end of the operation,

$$=\frac{1\frac{17}{28}-\frac{5}{8}}{\frac{24+32+21}{112}}+\frac{11}{14}\times\frac{7}{3}=\frac{\overset{5}{\cancel{55}}}{\cancel{56}}\times\frac{\overset{2}{\cancel{112}}}{\underset{7}{\cancel{77}}}+\frac{11}{\underset{2}{\cancel{14}}}\times\frac{\cancel{7}}{3}=1\tfrac{3}{7}+1\tfrac{5}{6}=3\tfrac{11}{42},$$

Ex. 2. Simplify:— $\dfrac{7\frac{1}{2}}{6\frac{1}{2}}+\dfrac{11\frac{1}{2}-2\frac{2}{5}}{11\frac{1}{2}+2\frac{2}{5}}\times 10\tfrac{9}{13}-7\tfrac{1}{8}$

Here we have three distinct parts,

$$=\frac{15}{2}\times\frac{2}{13}+\frac{9\frac{1}{10}}{13\frac{9}{10}}\times\frac{139}{13}-7\tfrac{1}{8}.$$

$$=\frac{15}{\cancel{2}}\times\frac{\cancel{2}}{13}+\frac{\overset{7}{\cancel{91}}}{\cancel{10}}\times\frac{\cancel{10}}{\cancel{139}}\times\frac{\cancel{139}}{\cancel{13}}-7\tfrac{1}{8}=1\tfrac{2}{13}+7-7\tfrac{1}{8}$$

$$=1\tfrac{2}{13}-\tfrac{1}{8}=1\tfrac{3}{104}$$

Ex. 3. Simplify:— $\frac{3}{4}+\cfrac{\frac{5}{8}}{1+\cfrac{\frac{15}{4}}{\frac{2}{7}+\cfrac{\frac{3}{2}}{4}}}$

In this example we commence at the end, and we have,

$$\frac{2}{3} \div 4 = \frac{1}{6}, \quad \text{and} \quad \frac{15}{4} \div \left\{\frac{3}{7} + \frac{1}{6}\right\} = \frac{15}{4} \div \frac{25}{42} = \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{21}{\cancel{42}}}{\underset{5}{\cancel{25}}} = \frac{63}{10}$$

$$\text{Again,} \quad \frac{5}{9} \div \left\{1 + \frac{63}{10}\right\} + \frac{3}{4} = \frac{5}{9} \times \frac{10}{73} + \frac{3}{4} = \frac{2171}{2628} \quad \textit{Ans.}$$

XXV.(*a*)

1. Define "Multiplication," and deduce the rule for the multiplication of fractions.

Simplify $(\frac{2}{3} + \frac{3}{4}) \times (2\frac{1}{2} - \frac{2}{5}) \div \left\{\frac{1}{2} + \frac{1}{4\frac{4}{5}}\right\}$

2. "Express twenty and a hundredth, and by it ÷ ·650727201."

3. What fraction of a sovereign is $4\frac{14}{21} - 10\frac{44}{56} + 9\frac{3}{45} - \frac{53}{117}$ of a penny?

4. Add $9{\cdot}14\dot{3} \times 11{\cdot}\dot{0}\dot{6}$ of £3 15s. 7½d. to $7{\cdot}\dot{3}$ of 3s. 4½d. ÷ $\cdot\dot{0}0433767\dot{1}$; and if a French métre is = 39·371 inches, what decimal of our mile is a kilometer (1000 métres).

5. Simplify $(16\frac{3}{4} + 17\frac{5}{12}) \div (10\frac{8}{19} \times 3\frac{9}{55}) + 5\frac{3}{13} \div \frac{17}{36} - 1\frac{9}{10}$

6. Simplify $(184{\cdot}3\dot{2}\dot{1} \times 4{\cdot}532 - 825{\cdot}\dot{5}587\dot{0}\dot{3}) \div 322{\cdot}906$

7. Simplify $\left\{2\frac{3}{4} + \frac{5}{2} \text{ of } \frac{7}{3\frac{4}{5}} - \frac{1\frac{2}{3}}{2\frac{1}{2}}\right\} \div 1\frac{77}{228}$

8. Simplify $\dfrac{3\frac{1}{3} - 2\frac{1}{6}}{\frac{1}{4}\text{ of}(\frac{1}{5} + \frac{1}{7})} \div 15\frac{5}{9}$; also $\dfrac{2\frac{3}{4} - 1\frac{2}{9}}{\frac{19}{36} + \frac{3}{2}\text{ of}\frac{1}{4}} \div 1\frac{9}{13}$

9. Reduce $\dfrac{1 + \dfrac{2}{3\frac{4}{5}}}{1 - \dfrac{2}{3\frac{4}{5}}} \div \left\{1 + \dfrac{4}{9 + \dfrac{3}{1 - \frac{3}{5}}}\right\}$ to a decimal.

10. Simplify $\dfrac{3{\cdot}5 \text{ of } 1{\cdot}\dot{3}}{2{\cdot}1\dot{6} \times 3{\cdot}06} + \dfrac{2\frac{1}{7} \times \frac{9}{7}}{3{\cdot}18\dot{3}\dot{7}} \text{ of } \dfrac{7{\cdot}\dot{1} \times 2\frac{1}{14}}{\cdot\dot{4}3\dot{2} - \cdot\dot{1}7\dot{1}}$

11. Show what kind of vulgar fractions will produce terminate decimals, and simplify

$$\frac{\cdot\dot{6}9230\dot{7}+\cdot\dot{7}1428\dot{5}}{\cdot3\dot{5}7142\dot{8}+\cdot3\dot{4}6153\dot{8}}\times\cdot03\dot{9}1\dot{8}+\cdot\dot{8}219178\dot{0}+\cdot1\dot{4}794520\dot{5}.$$

12. Simplify $\frac{2\frac{1}{2}-\frac{5}{6}}{2\frac{1}{2}+\frac{5}{6}}+\frac{7}{12}$ of $\frac{9\times10}{14\times3}-\frac{22\frac{1}{2}}{30}$; and $6\frac{1}{3}+\frac{5}{8}+(4\frac{7}{8}\div6\frac{1}{2})$

13. Simplify $\frac{2\frac{1}{2}+1\cdot\dot{3}}{\frac{5}{12}+\frac{7}{8}+\cdot375}\times\frac{\frac{4}{49}-\frac{1}{81}}{\frac{4}{27}-\frac{16}{121}}\div\frac{3\frac{2}{7}}{\frac{13}{1331}}$

14. Find value of £$\frac{2}{5}$ $(6\frac{2}{3}+2\frac{1}{2})+\frac{2\frac{1}{4}-\frac{2}{3}\text{ of }1\frac{5}{6}}{\frac{1}{5}\text{ of }3\frac{1}{3}+\frac{13}{36}}\times\cdot95$ of 5s.

$+\frac{\text{16s. 8d.}}{\cdot024}$

15. Simplify $\frac{284\cdot3\dot{2}\dot{1}}{2+\cdot\dot{6}\times\cdot\dot{5}\dot{4}}\times16\cdot58\dot{3}+\frac{572\cdot\dot{0}2\dot{7}\times6\frac{1}{6}}{\cdot\dot{2}8571\dot{4}\times24\frac{55}{49}}+\frac{11}{17}.$

16. Simplify $\left\{\frac{1}{2}\text{ of }3\frac{4}{17}\text{ of }6\frac{2}{11}-\frac{3\frac{5}{9}}{1\frac{2}{3}}\right\}+\frac{6\frac{3}{8}+5\frac{2}{3}}{9\frac{3}{4}-5\frac{5}{6}}$

17. Find the difference between

$$\frac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}+\frac{1}{4}}+\frac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{4}+\frac{1}{5}}\text{ and }\frac{\frac{1}{6}-\frac{1}{8}}{\frac{1}{6}+\frac{1}{8}}-\frac{\frac{1}{8}-\frac{1}{10}}{\frac{1}{8}+\frac{1}{10}}$$

18. Find the value of

$\frac{\frac{7}{10}-\frac{2}{3}}{\frac{9}{12}+\frac{1}{5}}\div\frac{\frac{1}{3}}{9\frac{1}{2}}$; and of $\frac{1}{2}+\frac{2}{9}-\frac{4}{15}+\frac{2}{15}$;

add both results and give the fraction the sum is of $93\frac{2}{3}$.

19. Simplify $\frac{3}{11}+\frac{\frac{4}{5}\times2\frac{1}{2}+\frac{5}{9}}{\frac{4}{5}+2\frac{1}{2}\div\frac{5}{9}}+\frac{7}{16}\times4\frac{4}{7}.$

20. Simplify $\frac{4\frac{4}{7}-2\frac{2}{3}}{4\frac{4}{7}+2\frac{2}{3}}+\frac{4\frac{4}{7}+2\frac{2}{3}}{4\frac{4}{7}-2\frac{2}{3}}+5\frac{17}{27}+57.$

21. Simplify (a) $\dfrac{13\frac{1}{3} \times 13\frac{1}{3} \times 13\frac{1}{3} - 1}{13\frac{1}{3} \times 13\frac{1}{3} - 1}$

(b) $\dfrac{5\frac{7}{8} + 11\frac{3}{4} + 3}{5\frac{7}{8} + 11\frac{3}{4} - \frac{1}{8}}$ of $1\frac{3}{7}$ of a guinea.

MISCELLANEOUS EXERCISES IN FRACTIONS.

XXVI.

1. $\frac{4}{11} + \cdot\dot{0}\dot{3} + 2\frac{9}{22} + 5\cdot35 + 4\cdot\dot{1}\dot{2}$.
2. $(8\cdot785 + 9\cdot125) - (3\frac{1}{4} + 7\frac{5}{16})$.
3. $2\frac{3}{4} \times \frac{1}{5} \times 1\cdot\dot{9} \times \cdot\dot{3}\dot{4} \times \cdot05$.
4. $(27\frac{9}{16} \div 7\frac{7}{8}) \div (29\cdot7402 \div \cdot097)$, give answer as a decimal.
5. Add $\frac{9}{13}$ of £2 1*s.* 8½*d.* to $\frac{1}{19}$ of £3 0*s.* 11½*d.*, and express the sum as the decimal of 5 guineas.
6. Simplify $\dfrac{\cdot002 \times 1\cdot75 \div \cdot00007}{1\frac{2}{3} \div \frac{1}{3}}$; and show whether is ·275 or ·276 nearer in value to $\cdot27\dot{5}$, and how much nearer.
7. Find the value of:—

$$\tfrac{2}{5}(6\tfrac{2}{3} + 2\tfrac{1}{2})\ £ + \frac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 3\frac{1}{3} + \frac{13}{36}} \text{ of } \tfrac{19}{20} \text{ of a crown.}$$

8. $\dfrac{\cdot0967 + 19\cdot4688}{7\cdot8398 - \cdot0136}$ of £2 14*s.* 2½*d.*

$+ \dfrac{\cdot96\dot{5} + \cdot04\dot{4}}{965\cdot\dot{1} \div 9\cdot\dot{5}}$ of £12 16*s.* 9¼*d.*

9. Multiply and bring them to their proper values:— £54·32 by 4·7; ·038765 of 2 cwt. 1 qr.; and $\cdot4\dot{4}$ of a guinea.
10. What number added to $1\frac{7}{10}$, $3\frac{9}{16}$, $2\frac{1}{20}$, $\frac{5}{24}$, will make the sum total 10; and what will make it $15\frac{4}{15}$?
11. Reduce 3·45 of half-a-guinea to the decimal of half-a-crown, and find the value of the result as the decimal of £120.
12. Simplify, giving the answer as a decimal:—

$$\frac{\cdot04275}{3\cdot0\dot{5}} \times \frac{4\cdot21\dot{6}}{\cdot\dot{3}4\dot{2}} \times \frac{2\cdot\dot{7}}{1\cdot5318}.$$

13. Reduce to circulators $\frac{11}{13}$, $\frac{19}{73}$, $\frac{14}{37}$, and $\frac{16}{43}$, and prove the accuracy of the first and last results by a reverse operation.
14. A young man lost $\frac{1}{5}$ of his money in betting on races, $\frac{1}{8}$ of the remainder in stock jobbing, $\frac{3}{5}$ of what was left by investing in foreign bonds, having only £970 4*s.* left; find the amount of his property at first.

The following have been proposed in Various Departments.

15. Add together $\frac{2}{5}$, $\frac{4}{9}$, $\frac{3}{10}$, and $\frac{7}{18}$, and divide the result by $7\frac{5}{8}$.

16. Find the value of $\frac{2}{3}$ of $\frac{5}{8}+\frac{3}{7}$ of $\frac{14}{15}$ of a shilling.

17. Reduce $\frac{23205}{46436}$ to its lowest terms.

18. Add together $\frac{2}{3}+\frac{1}{5}+\frac{2}{15}+1\frac{3}{7}+\frac{4}{105}$ and subtract the result from $3\frac{1}{15}$.

19. Multiply 21·5839 by ·00158.

20. Reduce £8 9*s.* 3*d.* to a fraction of £21 12*s.* 9*d.*, expressing the result in its lowest terms.

21. Reduce $\frac{1}{7}$ of $2\frac{1}{3}$ of $6\frac{7}{8}\div 4\frac{2}{5}$ to a simple fraction, and multiply the result by $1\frac{23}{25}$.

22. Add together $\frac{5}{12}$ of a pound, $\frac{3}{7}$ of a guinea, $\frac{4}{13}$ of a shilling, and [illegible]/13 of a half-penny.

23. Reduce $\frac{1}{13}$ to a decimal showing the whole of the circulating period.

24. Reduce $\cdot 13\dot{0}\dot{9}$ to a vulgar fraction in its lowest terms.

25. Divide 69·814 by ·00521 and by 52100.

26. Reduce $\frac{3}{4}$ of half-a-crown to the decimal of a ˋ.

27. Find the value of ·8478 of a cwt.

28. Subtract 423·79283 from 1857·23.

29. What number added to $3\frac{2}{3}$, $1\frac{9}{20}$, $2\frac{7}{12}$, $1\frac{8}{15}$, will make the sum total 12?

30. Add together $\frac{5}{13}$ of a pound, $\frac{3}{16}$ of 6*s.* 8*d.*, $\frac{1}{40}$ of a crown, and $\frac{9}{13}$ of a penny.

31. What number multiplied by $\frac{2}{5}$ will give $15\frac{3}{5}$ as the product?

32. Add together $3\frac{5}{8}$ pounds, $9\frac{3}{7}$ shillings, and $2\frac{4}{5}$ pence.

33. Add together ·130055, 900, 57·1, 13·34, and ·00000397 and subtract the result from $1314\cdot\dot{9}$.

34. Multiply 95·376 by ·0002.

35. Express $\frac{241}{15424}$ as a decimal.

36. Reduce $\frac{7}{8}$ of half-a-crown to the decimal of a guinea.

37. Reduce $\cdot 18\dot{1}5\dot{3}$ to a vulgar fraction in its lowest terms.

38. Express 7*s.* $8\frac{1}{2}$*d.* as the decimal of a £.

39. Find the value of ·03234 of a guinea.

40. What fraction of 5 mls. 5 f. 7 pls. $11\frac{1}{7}$ in. is $\frac{8}{17}$ of a league?

41. Divide 76·57 by ·0019, and multiply the quotient by $\frac{3}{7}$ of ·0008568.

42. What fraction of 6 mls. 2 f. 7 p. 11 y. 1 ft. 6 in. is $\frac{7}{15}$ of a league?

43. Find value of ·525 of £366 6s. 8d. + $\cdot\dot{0}2\dot{7}$ of £2 18. 7d.

44. Express 12·57 hrs. + ·42 days + ·3 wks. as the decimal of $365\frac{1}{4}$ days.

45. If $1\frac{2}{3}$ of a sum of money = $\frac{3}{7}$ of 5s. 10d., find the sum.

46. Find value of 12·55 ÷ ·01004 + 1255 ÷ 10·04 + ·001255 ÷ 1004.

47. Add together $\frac{1}{2}$ of 2s. $6\frac{3}{4}$d. + $\frac{1}{3}$ of £3 2s. $6\frac{1}{4}$d. + $\frac{1}{6}$ of £5 7s. $3\frac{1}{4}$d., and take the result from $\cdot\dot{4}6153\dot{8}$ of £4 12s. $4\frac{1}{4}$d.

48. Add together $\cdot\dot{2}\dot{7}$ of an acre, $\cdot\dot{5}4$ of a rood, and 7·5625 of a sq. yd., giving the result in sq. ft.

49. If $12\frac{15}{28}$ cwts. cost £$37\frac{1}{15}$, find the price of 1 cwt.

50. Bring 3 ac. 2 rds. 17 pls. to the decimal of a square mile.

PRACTICE.

109. Practice is the process of finding the value of a quantity by means of aliquot parts when the price of a unit of any denomination of that quantity is given.

An ALIQUOT PART of a quantity or number is one which when taken a certain number of times makes up that quantity or number exactly. 4 is an aliquot part of 8, because 4 when taken twice makes up 8 *exactly*; 6*d.* is an *aliquot part* of a shilling, or of a pound, because when taken twice it is =1*s.*, and when taken 40 times it is =£1 *exactly*. We say 6*d.*= $\frac{1}{2}$ of a shilling; 6*d.*= $\frac{1}{40}$ of a pound, and so on.

To find the *aliquot parts* of any quantity or number, divide by 2, 3, 4, &c., rejecting any divisors which produce *fractions* in the quotient.

ALIQUOT PARTS OF £1.

s.	*d.*		*s.*	*d.*		*d.*		*d.*	
10	0	= $\frac{1}{2}$	2	6	= $\frac{1}{8}$	10	= $\frac{1}{24}$	4	= $\frac{1}{60}$
6	8	= $\frac{1}{3}$	2	0	= $\frac{1}{10}$	8	= $\frac{1}{30}$	2	= $\frac{1}{120}$
5	0	= $\frac{1}{4}$	1	8	= $\frac{1}{12}$	$7\frac{1}{2}$	= $\frac{1}{32}$	$1\frac{1}{2}$	= $\frac{1}{160}$
4	0	= $\frac{1}{5}$	1	3	= $\frac{1}{16}$	6	= $\frac{1}{40}$	1	= $\frac{1}{240}$
3	4	= $\frac{1}{6}$	1	0	= $\frac{1}{20}$	5	= $\frac{1}{48}$	$\frac{1}{2}$	= $\frac{1}{480}$

CASE I.

110. *When the price and quantity are each of only one denomination.*

RULE.—The product of the price and quantity will be the answer in the *same name* as the price.

Ex. 1. 324 yards of cloth, at 7*s.* per yard.

Here, 324 × 7 = 2268*s.* = £113 8*s.* *Ans.**

REASON.—324 yards, at a shilling per yard, will cost 324*s.*; therefore at 7*s.* the cost will be 7 *times* as much.

If the price be an aliquot part of a higher denomination of money.

RULE.—Take a like part of the quantity for the *answer.*

Ex. 2. Find the price of 324 yards, at 5*s.* per yard.

Here, 5*s.* = ¼ of a pound; every 4 yards will cost a pound; hence we take one-fourth of £324 for the answer.

yds.
324 at 5s.
5s. = ¼ = £81 *Ans*

Ex. 3. Find the value of 326 yards, at 4*d.* per yard.

Here, 4*d.* = ⅓ of a shilling; the price of every 3 yards being a shilling, we take the third part; we have 2 remaining; as each yard costs 4*d.*, two yards will cost 8*d.*, that is ⅓ of 2*s.*

yds.
326 at 4*d.*
d.
4 = ⅓ = 10,8*s.* 8*d.*
1/20 = £5 8*s.* 8*d.* *Ans.*

Post Office.

Ex. 4. Find the value of 653 articles at ¾*d.* each.

In this example we multiply the number of articles by ¾*d.*, and have the answer in farthings, which we reduce to £ *s.* *d.*

653 at ¾*d.*
3
4)1959 farthings.
12)489 ¾*d.*
2,0)4,0 9¾
£2 0 9¾ *Ans.*

By First Principles.—Cost of 653 at ¼*d.* = 653*f.* = 13*s.* 7¼*d.* At ¾*d.* it will be three times as much; then 13*s.* 7¼*d.* × 3 = £2 0*s.* 9¾*d.* as before.

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* Strictly speaking, this operation is not Practice; but, as it is better always to select the most convenient method (which this frequently is), it has been inserted. In fact, all the other rules given under Practice are modifications of this one.

Ex. 5. Find the value of 1245 things at 4*d.* each.

At 1*s.* each 1245 will be 1245*s.*, and at 4*d.* each, since 4*d.* = $\frac{1}{3}$ of a shilling, the value will only be one third as much.

1245 @ 4*d.*

4*d.* = $\frac{1}{3}$ of a *s.* = 415*s.* = £20 15*s.* *Ans.*

By First Principles.—Cost of 1245 at 1*d.* each = 1245*d.* = £5 3*s.* 9*d.*; hence, at 4*d.*, it will be £5 3*s.* 9*d.* × 4 = £20 15*s.* as before.

XXVII.

Post Office.

1. Find the value of 846 articles at $\frac{3}{4}$*d.* each.
2. " 352 " 1$\frac{1}{2}$*d.* "
3. " 729 " 11*d.* "
4. " 89379 " $\frac{1}{4}$*d.* "
5. " 979 " 9*d.* "

Case II.

111. ***When the quantity is of one denomination and the price of several denominations.***

Rule.—1°. Multiply the quantity by the highest denomination in the price.

2°. Take aliquot parts of *one* of this highest denomination and add the results.

Post Office.

Ex. 6. Find the value of 144 reams of paper at 13*s.* 4*d.* per ream.

144 @ 13*s.* 4*d.*
13
1872*s.* value at 13*s.*
4*d.* = $\frac{1}{3}$ of a *s.* = 48*s.* " at 4*d.*
1920*s.* = £96 *Ans.*

By First Principles.

Cost of 144 at 1*s.* per ream . . . . . = 144*s.* = £7 4
" 144 at 13*s.* " . . . . . = £7 4 × 13 = 93 12
" 144 at 1*d.* = 12*s.*, and at 4*d.* = 12*s.* × 4 = 2 8
£96 0*s.* *Ans.*

Or by the principle explained (**31**) divide one and multiply the other by 3, and we have $\frac{144}{3}$ @ 13*s.* 4*d.* × 3 = 48 @ £2 = £96.

Various Departments.

Ex. 7. What is the dividend on £2734 16*s.* 8*d.* @ 9*s.* 4$\frac{1}{2}$*d.* in the £ ?

	2734	16*s.*	8*d.* @ 9*s.* 4$\frac{1}{2}$*d.*
5*s.* = $\frac{1}{4}$ of £1	683	14	2
4*s.* = $\frac{1}{5}$ of £1	546	19	4
4*d.* = $\frac{1}{12}$ of 4*s.*	45	11	7$\frac{1}{3}$
$\frac{1}{2}$*d.* = $\frac{1}{8}$ of 4*d.*	5	13	11$\frac{5}{12}$
	£1281	19*s.*	0$\frac{3}{4}$*d.* *Ans.*

This question may be worked more briefly by taking parts thus: 10*s.* = $\frac{1}{2}$ of £1 and 7$\frac{1}{2}$*d.*, the difference between 10*s.* and 9*s.* 4$\frac{1}{2}$*d.*, = $\frac{1}{16}$ of 10*s.*

By applying the principle explained in (**31**) we have £2734 16*s.* 8*d.* × 6 = £16409 ; then 9*s.* 4$\frac{1}{2}$*d.* ÷ 6 = 1*s.* 6$\frac{3}{4}$*d.* ; whence we have—

	16409 @ 1*s.* 6$\frac{3}{4}$*d.*	
6*d.* = $\frac{1}{2}$ =	8204	6
$\frac{3}{4}$ = $\frac{1}{8}$ =	1025	6$\frac{3}{4}$
	2,0)2563,9	0$\frac{3}{4}$
	£1281 19*s.*	0$\frac{3}{4}$*d.* *Ans.*

Post Office.

6. Find the value of 3107 sheep, each sheep being worth £1 14*s.* 7$\frac{1}{2}$*d*

7. Find the dividend on £3762 10*s.* at 8*s.* 2$\frac{1}{2}$*d.* in the pound.

Various Departments.

8. Find the value of 204 at £3 12*s.* 5*d.*

Admiralty Office.

9. Find the value of 4562$\frac{1}{4}$ things at £3 15*s.* 9$\frac{1}{2}$*d.* each.

Various Departments.

10. Find the value of 6723 pieces of cloth, each piece being worth £1 8*s.* 8$\frac{1}{4}$*d.*

11. What is the dividend on £2468 at 15*s.* 6*d.* in the £ ?

12. Find the value of 373 articles at 9*s.* 7$\frac{3}{4}$*d.* each.
13. „ 875 „ at 5*s.* 11*d.* each.
14. „ 327 „ at £1 2*s.* 5$\frac{1}{2}$*d.*
15. „ 373 „ at £2 16*s.* 10$\frac{1}{4}$*d.*
16. „ 217$\frac{1}{4}$ yards of lace at £2 17*s.* 7$\frac{1}{2}$*d.* each.
17. „ 136 articles at £1 7*s.* 2*d.*
18. „ 372 „ at 3*s.* 7$\frac{1}{2}$*d.*
19. „ 205 „ at £3 12*s.* 5*d.*

Admiralty Office.

20. Find the value of 3875 articles at £8 18*s*. 6¾*d*.
21. „ 7394¾ „ at £12 8*s*. 8¼*d*.

Various Departments.

22. Find the value of 27 cwts. of sugar at £2 18*s*. per cwt.
23. „ 473 lbs. of coffee at 1*s*. 5½*d*. per lb.
24. „ 132½ tons of iron at £7 15*s*. 6*d*. per ton.
25. Find the cost of 716 acres at £44 11*s*. 3¼*d*. per acre.
26. Find the value of 6214 sheep, each sheep being worth £1 14*s*. 7½*d*.

CASE III.

112. *When both the commodity and price are of several denominations.*

RULE.—Multiply the price by the highest denomination in the quantity, and take aliquot parts of the price for the remainder of the quantity.

Prisons, Science and Art, &c.

Ex. 8. Find the price of 13 cwts. 3 qrs. 11 lbs. of sugar at £2 6*s*. 8*d*. per cwt.

The cost of 13 cwts. is found by multiplying £2 6*s*. 8*d*. by 13. Then aliquot parts are taken for the 3 qrs. 11 lbs., as will be clearly understood from the annexed work.

cwts. qrs. lbs.			
13 3 11 @	£2	6	8
			13
	30	6	8
2 qrs. = ½ of a cwt. =	1	3	4
1 qr. = ½ of 2 qrs. =		11	8
7 lbs. = ¼ of 1 qr. =		2	11
4 lbs. = ⅐ of 1 qr. =		1	8
	£32	6	3 *Ans.*

27. Find the price of 17 acres 3 rds. 15 pls. at £37 10*s*. per acre.

Various Departments.

28. Find the price of 19 cwts. 1 qr. 13 lbs. at £2 11*s*. 4*d*. per cwt.
29. Find the value of 11 oz. 6 dwts. 15 grs. of gold, at £3 17*s*. 10*d* per oz.

Admiralty.

30. Find the value of 38 cwts. 1 qr. 16 lbs. at £2 12*s*. 6*d*. per cwt.
31. Find the value of 16 cwts. 1. qr. 21 lbs. at £3 18*s*. 9*d*. per cwt.

Various Departments.

32. Find the value of 13 cwts. 3 qrs. 17 lbs. of tea at £22 8s. per cwt.

33. Find the cost of 14 cwts. 3 qrs. 19 lbs. at £5 16s. 8d. per cwt.

113. If the price be not that of a *unit* of the highest denomination, reduce the given quantity to that denomination of which the price is given. Or, find from the given price, what the price of *one* of the highest denomination would be at the same rate.

Post Office.

Ex. 9. Find the price of 3 cwts. 2 qrs. 17 lbs. at £1 5s. 8d. per quarter.

It is evident that £1 5s. 8d. × 4 = £5 2s. 8d. the price per cwt. at same rate; hence,

	£	s.	d.
3 cwts. 2 qrs. 17 lbs. @	£5	2	8
			3
cost of 3 cwts. =	15	8	0
2 qrs. = $\frac{1}{2}$ of 1 cwt. =	2	11	4
14 lbs. = $\frac{1}{4}$ of 2 qrs. =	0	12	10
2 lbs. = $\frac{1}{7}$ of 14 lbs. =	0	1	10
1 lb. = $\frac{1}{2}$ of 2 lbs. =	0	0	11
	£18	14	11 *Ans.*

Or since 3 cwts. 2 qrs. 17 lbs. = 14 qrs. 17 lbs., we have

	£	s.	d.
	£1	5	8
			14
price of 14 qrs. =	17	19	4
14 lbs. = $\frac{1}{2}$ of a qr. =	0	12	10
2 lbs. = $\frac{1}{7}$ of 14 lbs. =	0	1	10
1 lb. = $\frac{1}{2}$ of 2 lbs. =	0	0	11
	£18	14	11 *Ans.*

Customs and Excise.

34. Find the value of 7 cwts. 3 qrs. 11 lbs. at £2 13s. 1d. per quarter.

Constabulary (Cadetships).

35. Find (by Practice) the price of 8 tons 13 cwts. 18 lbs. of coal at 3½d. per quarter.

36. Find the dividend on £1760 5s. at 10s. 4d. in the £.

Post Office, Board of Trade, Fisheries, &c.

37. A rate of 1*s.* 5*d.* in the pound is levied on a parish where the rateable rental is £360817 10*s.* Find the amount produced.

38. Find the value of 105 pockets of hops, each weighing 1 cwt. 1 qr. 8 lbs., at £4 12*s.* 6*d.* per cwt.

CONTRACTIONS.

114. *For tons, cwts., qrs.*

RULE.—Multiply the tons by 1, and consider the product as pounds; the cwts. by 1, and consider the product as shillings; the qrs. by 3, and consider the product as pence.

£1 per ton is 1*s.* per cwt. and 3*d.* per qr.: hence the rule.

Various Departments.

EX. 10. Find the value of 9 tons 4 cwts. 3 qrs. 21 lbs. at £14 15*s.* 9*d.* per ton.*

When a qr. costs 3*d.* a lb. will cost $\frac{3}{28}d.$; hence, the reason of multiplying the lbs. by $\frac{3}{28}$.

	tons	cwts.	qrs.	lbs.
	9	4	3	21
	1	1	3	$\frac{3}{28}$
	9	4	$11\frac{1}{4}$	
			14	
	£129	9	$1\frac{1}{2}$	
10*s.* = $\frac{1}{2}$	4	12	$5\frac{5}{8}$	
5*s.* = $\frac{1}{2}$	2	6	$2\frac{13}{16}$	
6*d.* = $\frac{1}{10}$		4	$7\frac{77}{160}$	
3*d.* = $\frac{1}{2}$		2	$3\frac{237}{320}$	
	£136	14	$9\frac{51}{320}$ *Ans.*	

Admiralty.

39. Find the price of 56 tons 4 cwts. 2 qrs. at £58 7*s.* 6*d.* per ton.

40. „ 38 tons 1 cwt. 16 lbs. at £2 12*s.* 6*d.* per ton.

War Department.

41. Find the price of 56 tons 15 cwts. 2 qrs. 21 lbs. at £24 6*s.* 8*d.* per ton.

115. *For cwts., qrs. and lbs.*

RULE.—Multiply the cwts. by 1, the qrs. by 5, and the lbs. by $2\frac{1}{7}$, and consider the products as in the preceding.

---

* Examples 10, 11, and 12 may be worked very conveniently by the rule given in page 119.

Customs.

Ex. 11. Calculate the duty on 353 cwts. 2 qrs. 7 lbs. of foreign sugar at 17*s*. 4*d*. per cwt.

In this example, and most others, in which the shillings are nearly £1, it is better to find for the difference and subtract. Thus the difference between £1 and 17*s*. 4*d*. is $=$ 2*s*. 8*d*., and the difference of the values at these two rates will be the answer.

	cwts.	qrs.	lbs.	
	353	2	7	@ 17*s*. 4*d*.
	1	5	$2\frac{1}{7}$	
	353	11	3	at £1 per cwt.
2*s*. $= \frac{1}{10} =$	35	7	$1\frac{1}{2}$	
8*d*. $= \frac{1}{3} =$	11	15	$8\frac{1}{2}$	
	47	2	10	at 2*s*. 8*d*.
	£306	8	5	*Ans*.

116. *For acres, roods, and perches.*

Rule.—Multiply by 1, 5, $1\frac{1}{2}$, respectively, and consider the products as in the preceding.

Various Departments.

Ex. 12. Find the price of 25 acres 1 rood 10 poles at £42 2*s*. 4*d*. per acre.

When by multiplying by 1, 5, $1\frac{1}{2}$, the value at £1 per acre is obtained, it is easy to find the value at any other rate, as in the annexed work.

	acrs.	rd.	pls.	
	25	1	10	@ £42 2*s*. 4*d*.
	25	1	10	
	1	5	$1\frac{1}{2}$	
	25	6	3	price at £1 per acre
			6	
	151	17	6	price at £6 „
			7	
	1063	2	6	price at £42 „
2*s*. $= \frac{1}{10}$ of a £ $=$	2	10	$7\frac{1}{2}$	price at 2*s*. „
4*d*. $= \frac{1}{6}$ of 2*s*. $=$	0	8	$5\frac{1}{4}$	price at 4*d*. „
	£1066	1	$6\frac{3}{4}$	*Ans*.

Ex. 13. Find the price of 13 oz. 8 dwts. 16 grs. of gold at £3 17*s*. 6*d*. per oz.

In this example we multiply by 1, 1, and $\frac{1}{2}$ respectively; for when an oz. costs £1 a dwt. will cost 1*s*. and a grain will cost $\frac{1}{2}$*d*.

	oz.	dwts.	grs.	
	13	8	16	
	1	1	$\frac{1}{2}$	
$\frac{1}{8}$	13	8	8	price at £1 per oz.
			4	
	53	14	8	price at £4 per oz.
2*s*. 6*d*. $= \frac{1}{8}$ of £1	1	13	7	„ 2*s*. 6*d*.
	52	1	1	*Ans*.

Ex. 14. Find the cost of making a road 7 miles 3 fur. 44 yds. long, at £30 3*s.* 4*d.* per mile.

	£	s.	d.	
	£30	3*s.*	4*d.*	
			7	
	211	3	4	cost of 7 miles
2 fur. = $\frac{1}{4}$ of a mile	7	10	10	„ 2 furlongs
1 fur. = $\frac{1}{2}$ of 2 fur.	3	15	5	„ 1 „
44 yds. = $\frac{1}{5}$ of a fur.	0	15	1	„ 44 yds.
	£223	4	8	*Ans.*

Ex. 15. Find the value of 689 c. ft. 5′ 6″ 9‴ at £13 13*s.* 4*d.* per cubic foot.

Here 689 = 700 − 11 = 10 × 10 × 7 − 11

£ 13 13*s.* 4*d.* × 10 = £ 136 13*s.* 4*d.*

	£	s.	d.		
136 13 4 × 10 =	1366	13	4		
			7		
	9566	13	4	value of	700
13 13 4 × 11 =	150	6	8	„	11
	9416	6	8	„	689
4′ = $\frac{1}{3}$ of a foot	4	11	$1\frac{1}{3}$		
1′ = $\frac{1}{4}$ of 4	1	2	$9\frac{1}{3}$		
6″ = $\frac{1}{2}$ of 1′	0	11	$4\frac{2}{3}$		
9‴ = $\frac{1}{8}$ of 6″	0	1	$5\frac{1}{12}$		
	£9422	13	$4\frac{5}{12}$	*Ans.*	

Various Departments.

42. What is the rent of 245 acres 3 rds. 30 pls. of land at £2 14*s.* per acre?

43. What will be the price of 35 acres 2 rds. 30 pls. at £30 1*s.* 8*d.* per acre?

44. Find the value of 2 rds. 19 pls. 12 yds. of land at £80 13*s.* 4*d.* per acre.

117. Sometimes certain deductions have to be made before the true weight of goods, called the *net* or *neat* weight, is found.

The weight of both goods and box, &c., which contains them is called the *gross* weight.

TARE is the weight of the bag or barrel, which contains the goods, or an *allowance* made for it, usually a per-centage, that is, a deduction per 100 lbs.

TRETT is an allowance of 4 lb. in every 104 lb., or $\frac{1}{26}$ part, allowed for dust, &c.

BEAMAGE or CLOFF is an allowance of 2 lb. in every 3 cwt., or $\frac{1}{168}$ part, considered necessary to turn the beam.

Trett and *cloff*, however, are falling into *disuse*.

First tare is deducted; trett is taken from what remains, and so on. When all deductions are taken we have the *net* weight.

Customs and Excise.

Ex. 16. What is the net weight of the following:

			cwts.	qrs.	lbs.	
J. D.	1	....	17	2	9	Tare at 18 per cent.
	2	....	18	2	14	
	3	....	19	3	0	
	4	....	20	1	0	
	5	....	16	3	26	
	5 butts		93	0	21	gross.
Tare $\frac{18}{100}=\frac{9}{50}=$			16	3	$2\frac{33}{50}$	
			76	1	$18\frac{17}{50}$	net weight. *Ans.*

In finding the tare we multiply by 9 and divide by 50.

Ex. 17. Raisins.

	Boxes.		cwts.	qrs.	lbs.	
C. L.	5	....	2	1	24	Tare 6 lbs. each box, what is the net weight?
	5	....	2	2	1	

	cwts.	qrs.	lbs.	
	2	1	24	
	2	2	1	
	4	3	25	gross.
Tare 10 × 6 = 60 lbs. =		2	4	
	4	1	21	net weight. *Ans.*

45. What is the net weight of

			cwts.	qrs.	lbs.	
F. B.	1	....	7	2	19	Tare at 17 per cent.
	2	....	6	3	21	
	3	....	7	0	14	

XXVII.

Customs.

46. Return for duty 60 chests of liquorice paste, weighing 160 cwts. 2 qrs gross, allowing for tare 42 lbs. each chest, and for leaves 7 per cent.

118. FLOORING, PAINTING, &c., are frequently calculated by Practice: read DUODECIMALS in connection with this subject.

Science and Art, Prisons, Junior Clerks, &c.

Ex. 18. What will the painting of a room cost at 2*s*. 3*d*. the square yard, whose height is 10 feet, width 15 feet, and length 19 feet?

19 × 10 × 2 = 380 the area of the two sides.
15 × 10 × 2 = 300 the area of the two ends.

680 sq. feet, area of the whole room to be painted, which ÷ 9 = $75\frac{5}{9}$ sq. yards. Then $75\frac{5}{9}$ at 2*s*. 3*d*. = (multiply the first quantity and divide the 2nd by nine) 680 at 3*d*.

680
3*d*. = $\frac{1}{4}$ of a *s*. = 170*s*. = £8 10*s*. *Ans.*

47. What will the painting of a room cost at 2*s*. 3*d*. the sq. yard, whose height is 15 feet, width 16 feet, and length 18 feet?

Constabulary and Education Offices.

48. Find how many yards of paper $\frac{7}{8}$ of a yard wide will paper a room, whose length is 26 ft. 4 in., breadth 18 ft. 8 in., and its height 12 ft. 3 in.

Various Departments.

49. What is the expense of digging a ditch of which the solid content is 5755 cubic yards at the rate of 15*s*. $7\frac{3}{4}$*d*. per yard?

50. Find the cost of papering a room 16 ft. long, 11 ft. wide, and 10 ft. high, with paper 30 in. broad, at $7\frac{1}{2}$*d*. a yard.

Post Office, Customs, War Department, &c.

51. Find the cost of papering a room, whose height is 12 ft., width 16 ft., and length 20 ft., the price of paper being $2\frac{1}{4}$*d*. the sq. yard.

MISCELLANEOUS EXERCISES IN PRACTICE.

XXVIII.

Post Office.

1. If an officer's pay is 12*s*. 3*d*. per day, what is that per year?

2. A bankrupt owes £2468, and he could pay 15*s*. 6*d*. in the £, what are his effects worth?

3. The charge of carrying the mails by railway to Birmingham is £28 4*s*. 4*d*. per day; how much is it in 365 days?

4. Find the value of 328 articles at 8*s.* 5½*d.* each.
5. „ 1530 „ at 15*s.* 9*d.* each.
6. „ 674 „ at £4 8*s.* 4*d.* each.
7. „ 638 „ at £2 17*s.* 4*d.* each.

Clerks in Post Office, Police Courts, Science and Art, Board of Trade, Fisheries, and Prisons Departments.

8. Find the dividend on £2574 15*s.* at 13*s.* 5*d.* in the pound.

9. Find the cost of making a road, length 9 miles 5 fur. 44 yds., at £25 8*s.* 4*d.* per mile.

10. Find the value of 15 silver plates, each weighing 7 oz. 11 dwts. 6 grs., at 6*s.* 8*d.* per ounce.

11. Find the value of 70 bales of cotton, each weighing 1 cwt. 1 qr. 21 lbs., at £2 3*s.* 1½*d.* per cwt.

12. Find the value of 3 acres 1 rd. 14 pls. at £125 per acre.

13. Find the value of 14 oz. 8 dwts. 20 grs. of gold at £3 17*s.* 6*d.* per ounce.

14. Find the dividend on £3854 10*s.* at 8*s.* 2½*d.* in the pound.

15. Find the value of 16 oz. 6 dwts. 20 grs. of gold at £3 17*s.* 6*d.* per ounce.

16. Find the income-tax on £2382 7*s.* 6*d.* at 16*d.* in the pound.

17. Find the value of 983 articles at 3*s.* 10½*d.* each.

18. Find the value of 969 „ at 19*s.* 11½*d.* each.

19. What is the duty on 85960 lbs. of hops at 1*d.* $\frac{12}{25}$ of a farthing per lb.?

20. Find the value of 2654¾ at £7 15*s.* 4*d.* each.
21. „ 7394¼ at £12 8*s.* 8¼*d.* each.
22. „ 37 tons 3 cwts. 22 lbs. at £12 17*s.* 7*d.* per cwt.
23. „ 5 tons 3 cwts. 1 qr. 5 lbs. at £3 13*s.* per cwt.

War Department.

24. Find the price of 56 tons 10 cwts. 2 qrs. 21 lbs. at £24 6*s.* 8*d.* per ton.

Surveyor of Taxes.

25. How much income-tax at 10½*d.* in the £ must a man pay who has £1875 per annum?

26. Find (by Practice) the price of 35 bars of gold, each weighing 3 oz. 11 dwts. 9 grs., at £2 12*s.* 6*d.* per oz.

Proposed where Candidates are to know 'Vulgar and Decimal Fractions.'

27. A room is 42 ft. long, 28 ft. broad, and 12 ft. high, what will be

the cost of covering the walls with a paper 2 ft. 3 in. wide, at 9*d.* per yard?

28. Find the cost of 4 cwts. 3 qrs. 22½ lbs. at £1 9*s.* 2*d.* per cwt.

29. If a person's estate is worth £1384 16*s.* per annum, and the rent-charges upon it amount to 14*s.* 9½*d.* in the pound, what is the net annual income?

30. What is the rent of 67 acres of land for 2 years 9 months 11 days, the yearly rent of an acre being £2 5*s.*?

31. A bankrupt whose liabilities are £3236 17*s.* 6*d.* can pay 13*s.* 10*d.* in the pound, what are his assets?

32. Find the value of 6 qrs. 3 bushels 1¾ pecks of wheat at £3 18*s.* 8*d.* per quarter.

33. Find the price of 3 qrs. 20 lbs. 12 oz. at £11 13*s.* 4*d.* per cwt.

34. Find the amount of income-tax on £875 15*s.* at 14*d.* in the £.

35. Find the value of 39 cwts. 1 qr. 20 lbs. at £6 2*s.* 6*d.* per cwt.

36. What is the dividend on £2485 at 17*s.* 9½*d.* in the pound?

37. Find the value of 1001 articles at 8*s.* 7½*d.*

38. Find the value of 5 lbs. 9 oz. 7 dwts. of gold at £3 17*s.* 10*d.* an oz.

39. Find the value of 197 articles at £4 12*s.* 5½*d.*

40. Find the income-tax at 16*d.* in the pound on £1405 15*s.* 11¼*d.*

41. Find the cost of 11 miles 3 fur. and 55 yds. of railway at £32500 per mile.

42. Find the price of 0 tons 11 cwts. 2 qrs. 21 lbs. at £10 11*s.* 4*d.* per cwt.

43. Find the price of 17 acres 3 rds. 15 pls. at £60 8*s.* 4*d.* per acre.

44. Find the value of 6943 sheep, each sheep being worth £1 13*s.* 4½*d.*

45. Find the cost of 7 cwts. 3 qrs. 20½ lbs. at £3 2*s.* 5*d.* per cwt.

46. What is the dividend on £4970 at 8*s.* 10¾*d.* in the pound?

47. Find the value of 3 tons 18 cwts. 3 qrs. 12 lbs. at £2 17*s.* 9*d.* per cwt.

Proposed also in Several Departments.

48. Find the value of 194 articles at £4 12*s.* 5¾*d.* each.

49. Find the value of 3546 pieces of cloth, each piece being worth £1 6*s.* 7¾*d.*

50. What is the price of 329 yards 3 qrs. 2 nls. at 5*s.* 2½*d.* per qr.?

51. Find the value of 821 articles at 13*s.* 10¾*d.*

52. Find the value of 3 lbs. 5 oz. 14 dwts. 12 grs. at 17*s.* 6*d.* per oz.

Education Office, London.

53. Find the price of 8 cwts. 1 qr. 12 lbs. at £4 12*s.* 5½*d.* per quarter.

Various departments.

54. Find how much a man saves per annum who has a salary of £200 a year, his daily outlay being 9s. 9½d.

55. Find the value of 5899 articles at £4 12s. 9d. per gross.

56. Find the value of 9765 articles at £4 16s. 8d. per dozen.

57. Find the value of 457 articles at £14 17s. 9½d. each; then prove the accuracy of the result by multiplication, after expressing 17s. 9½d. as the decimal of a pound.

58. Find the cost of $\cdot 3\dot{2}\dot{7}$ of 1 cwt. 2 qrs. 7 oz. + $\cdot\dot{7}0\dot{3}$ of 9 cwts. 1 qr. 2 lbs. 6 oz. + ·24 of 7 cwts. 2 qrs. 10 lbs. 12½ oz. at £8 13s. 4d per cwt.

DUODECIMALS.

119. DUODECIMALS (Latin, *duodecim*, twelve) are so called because they increase by twelves from right to left. Although the various quantities in this rule increase in a twelve-fold ratio, yet the figures by which they are expressed are in the ordinary decimal notation, and are multiplied in the usual way.

In DUODECIMALS a foot is divided first into 12 equal parts called inches or primes, marked thus ($'$); and each of these again into 12 equal parts called seconds, marked thus ($''$) In like manner each second is divided into 12 equal parts called thirds ($'''$), each third into 12 equal parts called fourths ($''''$) and so on.

Hence $1'$ (inch or prime) $= \frac{1}{12}$ of a foot.
$1''$ (second), $\frac{1}{12}$ of $\frac{1}{12}$ $= \frac{1}{144}$ „
$1'''$ (third), $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ $= \frac{1}{1728}$ „
$1''''$ (fourth), $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ $= \frac{1}{20736}$ „

DUODECIMALS are added, subtracted, multiplied, and divided, just as other compound numbers, 12 of a less denomination making 1 of a greater, as in the following table:—

12 fourths ($''''$) = 1 third ($'''$)
12 thirds ($'''$) = 1 second ($''$)
12 seconds ($''$) = 1 inch or prime ($'$)
12 inches or primes ($'$) = 1 foot.

The marks ′, ″, ‴, &c., are called *indices*, because they indicate or point out the denomination of the quantity to which they are attached.

Since feet stand in the place of units, feet multiplied by feet will give square feet as product: feet multiplied by inches or primes will give inches or primes, and so on, as follows:

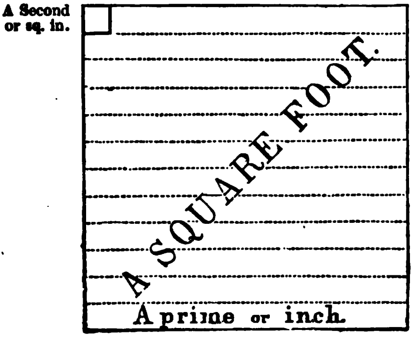
Feet multiplied by feet give feet as product.
Feet „ primes „ primes „
Feet „ seconds „ seconds „

Primes multiplied by primes give seconds.
Primes „ seconds „ thirds.
Primes „ thirds „ fourths.

Seconds multiplied by seconds give fourths.
Seconds „ thirds „ fifths.
Seconds „ fourths „ sixths.

In other words, the product will always be the denomination indicated by the *sum of the indices* of the multiplicand and multiplier: thus $7'' \times 5'''' = 35''''''$, thirty-five sixths.

The following geometrical figure will best illustrate what has just been said.



DUODECIMALS are principally used by workmen and artificers, in ascertaining the superficial and solid contents of their work.

The superficial content of a square or rectangular figure is found by multiplying the length by the breadth. The solid content or volume of a rectangular parallelopiped is found by multiplying the area of the end by the length.

RULE FOR MULTIPLYING DUODECIMALS.

1°. Write the multiplier under the multiplicand, feet under feet, primes under primes, &c.

2°. Commencing with the highest denomination of the multiplier, multiply each denomination of the multiplicand by it separately, beginning at the lowest, setting down the products and carrying one for every twelve.

3°. Multiply by the next highest denomination in the multiplier in the same manner, setting down the products under the denominations of same kind, which will generally be effected by falling back one place to the right.

4°. The sum of the partial products of all the figures in multiplier, will be the total produced.

Ex. 1. In a log of timber 20 ft. 9 in. long, 2 ft. 5 in. broad, and 2 ft. 3 in. thick, how many cubic or solid feet are there?

ft.				
20	9′			
2	5′			
41	6′			=product by 2ft.
8	7′	9″		=product by 5′
50	1′	9″		=product of length and breadth.
2	3′			
100	3′	6″		
12	6′	5″	3‴	
112ft.	9′	11″	3‴	*Ans.*

In the above example we first say, $2\text{ft.} \times 9' = 18' = 1 \text{ ft. } 6'$, we set down the 6′ and carry the 1 ft.: then twice 20 ft. + 1 = 41 ft., and so on.

War Department.

Ex. 2. Multiply by the method of duodecimals 46 ft. 5 in. 6 parts by 9 ft. 7 in. 8 parts.

$$\begin{array}{rrrrl}
\text{ft.} & & & & \\
46 & 5' & 6'' & & \\
9 & 7' & 8'' & & \\ \hline
418 & 1' & 6'' & & =\text{product by 9 ft.} \\
27 & 1' & 2'' & 6''' & =\text{product by } 7'. \\
2 & 6' & 11'' & 8''' & =\text{product by } 8''. \\ \hline
447\text{ft.} & 9' & 8'' & 2''' & \textit{the answer.}
\end{array}$$

Ex 3. The side of a solid cube is 7 ft. 2$\frac{1}{2}$ in., find the content in solid yards, feet, and inches.

The content will be 7 ft. $2\frac{1}{2}' \times 7$ ft. $2\frac{1}{2}' \times 7$ ft. $2\frac{1}{2}'$.

$$\begin{array}{rrrrrr}
\text{ft.} & & & & & \\
7 & 2' & 6'' & & & \\
7 & 2' & 6'' & & & \\ \hline
50 & 5' & 6'' & & & \\
1 & 2' & 5'' & & & \\
 & 3' & 7'' & 3''' & & \\ \hline
51 & 11' & 6'' & 3''' & & \\
7 & 2' & 6'' & & & \\ \hline
363 & 8' & 7'' & 9''' & & \\
8 & 7' & 11'' & 0''' & 6'''' & \\
2 & 1' & 11'' & 9''' & 1'''' & 6''''' \\ \hline
374 & 6' & 6'' & 6''' & 7'''' & 6''''' \\
 & 12 & & & 12 & \\ \cline{2-2} \cline{5-5}
 & 78 & & & \frac{90}{144} & =\frac{15}{24}=\frac{5}{8} \\
 & 12 & & & & \\ \cline{2-2}
 & 942 & & & &
\end{array}$$

∴ final result = 374 c. ft. 942$\frac{5}{8}$ in. = 13 c. yds. 23 c. ft. 942$\frac{5}{8}$ in.

In the multiplication of duodecimals, it must be distinctly recollected that when we perform one multiplication, the product is in square measure, and the *seconds* correspond to square inches. It is equally necessary to remember, that in cubic measure the denomination in the third place, *thirds*, corresponds to cubic inches.

XXVIII. (*a*)

Various Departments.

1. How many square yards, feet, and inches, are there in the top of a table, the sides of which are 10 ft. 7½ in., and 9 ft. 5 in.?

2. How many square yards are there in a rectangular plot, the length of which is 6 ft. 9 in., and the breadth 4 ft. 10 in.?

3. The side of a cube is 17 in.; find its content in solid yards, feet, and inches?

4. How many square yards, &c., are there in a table 6 ft. 8 in. long, by 4 ft. 9 in. broad?

5. The area of a plot of ground is 101 yds. 3 ft. 90 in., and the length is 36 ft. 9 in.; find the breadth.

6. The length of a French metre being 39·37 in., find the number of solid feet and inches in a cube the length of whose side is 2 metres.

7. Multiply by the method of duodecimals 7 ft. 5 in. 8 parts by 9 ft. 4 in. 11 parts, and express the result obtained in square inches and a fraction of a square inch.

Ex. 4. Multiply by the method of duodecimals 7 ft. 5 in. 9 pts. 9 ft. 6 in. 9 pts., and the result by 4 in. 7 pts., giving the answer in cubic inches.

$$
\begin{array}{rrrrrrr}
\text{ft.} & & & & & & \\
7 & 5^{\prime} & 9^{\prime\prime} & & & & \\
9 & 6 & 9^{\prime\prime} & & & & \\
\hline
67 & 3^{\prime} & 9^{\prime\prime} & & & & \\
3 & 8^{\prime} & 10^{\prime\prime} & 6^{\prime\prime\prime} & & & \\
 & 5^{\prime} & 7^{\prime\prime} & 3^{\prime\prime\prime} & 9^{\prime\prime\prime\prime} & & \\
\hline
71 & 6^{\prime} & 2^{\prime\prime} & 9^{\prime\prime\prime} & 9^{\prime\prime\prime\prime} & & \\
 & 4^{\prime} & 7^{\prime\prime} & & & & \\
\hline
23 & 10^{\prime} & 0^{\prime\prime} & 11^{\prime\prime\prime} & 3^{\prime\prime\prime\prime} & 0^{\prime\prime\prime\prime\prime} & \\
3 & 5^{\prime} & 8^{\prime\prime} & 7^{\prime\prime\prime} & 7^{\prime\prime\prime\prime} & 8^{\prime\prime\prime\prime\prime} & 3^{\prime\prime\prime\prime\prime\prime} \\
\hline
27\ \text{c. ft.} & 3^{\prime} & 9^{\prime\prime} & 6^{\prime\prime\prime} & 10^{\prime\prime\prime\prime} & 8^{\prime\prime\prime\prime\prime} & 3^{\prime\prime\prime\prime\prime\prime}
\end{array}
$$

$$
\begin{array}{rl}
27 & \\
12 & \\
\hline
327 & \text{primes} \\
12 & \\
\hline
3933 & \text{seconds} \\
12 & \\
\hline
47202 & \text{thirds or c. inches.}
\end{array}
\qquad
\begin{array}{r}
10 \\
12 \\
\hline
128 \\
12 \\
\hline
\dfrac{1539}{1728} = \dfrac{171}{192} = \dfrac{57}{64}
\end{array}
$$

$\therefore$ *Ans.* $47202\frac{57}{64}$ c. in.

8. Multiply by duodecimals 2 ft. 7 in. 11 parts by 3 ft. 5 in. 7 parts, and explain clearly each term of the result.

9. The sides of a rectangular plot are 57 ft. 7 in. and 43 ft. 5 in.; find its area in square yards, feet, and inches.

10. What length must be cut off a board 6¾ inches wide, that the area cut off may be a square foot?

11. Find the thickness of a solid whose length is 2 yards, breadth a yard and a-half, and solid content 1 cubic yd. 6 c. ft. 1296 c. in.

12. How many square feet of deal 1 in. thick, will be required to make a box, the external dimensions of which are 2 ft. 6 in. 1 ft. 3½ in., and 1 ft. 1 in.?

13. France contains 128 millions of English acres, and the Pyrenees spread over it would cover it to the depth of 115 feet, find the bulk of the Pyrenees in cubic yards and feet, and find the depth to which they would cover Ireland, which contains 20 millions of acres.

14. What is the height of a closet 8⅓ ft. by 6⅔ ft., which will exactly contain 12 boxes 4⅙ feet long, 3⅓ ft. wide, and 2½ ft. deep?

PROPERTIES OF NUMBERS.

In working arithmetic it is of the utmost importance to be able to resolve those numbers which admit of it into their component parts or prime factors. The most convenient mode of finding the prime factors of a number is by commencing with small numbers, and trying which of them will divide evenly into the given number. We can place the divisors on the left, as below—

Resolve 462 and 15950 into their prime factors.

2	462
3	231
7	77
11	11

2	15950
5	7975
5	1595
11	319
29	29

$\therefore 462 = 2 \times 3 \times 7 \times 11$ and $15950 = 2 \times 5 \times 5 \times 11 \times 29$.

Now should it be required to find the G. C. M. of these two numbers, we take the common factors and multiply them together.

$\therefore$ G. C. M. of 462 and 15950 $= 2 \times 11 = 22$.

The learner should re-read pp. 77–9, and add to the *Multiplication Table*, already learned, the following table of prime numbers:—

1	2	3	4	5	6	7	8	9	10	11	12
13	26	39	52	65	78	91	104	117	130	143	156
17	34	51	68	85	102	119	136	153	170	187	204
19	38	57	76	95	114	133	152	171	190	209	228
23	46	69	92	115	138	161	184	207	230	253	276

To find the L. C. M. *of Fractional Quantities.*

Rule. Find the L. C. M. of their numerators when the quantities are written in a fractional form.

SIMPLE PROPORTION.

120. RATIO is the relation which exists between any two quantities of the same kind, and is found by dividing the former by the latter: thus, the *ratio* of 6 to 3 is 2, and the ratio of 12 to 4 is 3, for $6 \div 3 = 2$ and $12 \div 4 = 3$.

RATIO is another name for the quotient of *two quantities of the same kind.* Therefore, the quantities may be expressed as the terms of a fraction, the *ratio* being what we have called the real value of the fraction. We may say $\frac{6}{3} = 2$, and $\frac{12}{4} = 3$.

What were formerly called the *terms* of a *fraction*, are now called the *terms* of a *ratio*; what we have called the *numerator* of a fraction, we now call the *antecedent* of a ratio, being usually written before the other term. What we termed the *denominator* of a fraction, we now call the *consequent* of a ratio.

The terms of a ratio are generally written thus, 6 : 3 (read, 6 to 3) instead of $\frac{6}{3}$, or $6 \div 3$; and 12 : 4 is the same as $\frac{12}{4}$, or $12 \div 4$.

As has been shown in *Fractions*, the numerator corresponds to the dividend. and the denominator to the divisor; so with the terms of a ratio, the *antecedent* corresponds to the *dividend*, and the *consequent* to the *divisor*; and hence the *quotient* corresponds to, and is, the ratio.

The term *quotient* is used in a more *general* sense than the term ratio, for quotient means, not only the number of times one quantity contains another of the same kind, but is applied to *compound quantities* arising by taking the half, or any other part of a quantity of several denominations. *Ratio* is always expressed by an abstract number, and has reference to the number of times or parts of a time the antecedent contains the consequent. We may say every ratio is a quotient, but we cannot say every quotient is a ratio.

The terms of a ratio may both be *multiplied* or both *divided* by the same number, and the ratio of the results will be the same as that of the original numbers, just as we have seen with respect to the terms of a fraction. For example, 6 : 3 = 18 (6 × 3) : 9 (3 × 3); and 12 : 5 = 60 (12 × 5) : 25 (5 × 5).

121. When ratios are equal, their terms form what is called a PROPORTION, written thus, 6 : 3 :: 18 : 9, and read 6 to 3, as 18 to 9. These four numbers are *proportionals*, that is, 6 bears the same relation to 3, that 18 does to 9; or, that the quotient of 6 divided by 3 is equal to the quotient of 18 divided by 9. Hence, the terms of any two equal fractions will form a proportion, as they imply equal quotients. Thus, $\frac{5}{7}=\frac{20}{28}$, becomes, 5 : 7 :: 20 : 28, the numerator of one is, to its denominator, as the numerator of the other, to its denominator.

It follows that any four numbers are proportionals, when the first divided by the second is equal to the third divided by the fourth. They are not proportionals when this is not the case; therefore, by this test, we can determine if any four numbers are proportionals.

Take the four numbers 12, 6, 10, and 4, and we see that 12 ÷ 6 = 2, but 10 ÷ 4 = $2\frac{1}{2}$, therefore these four numbers are not proportionals. Again, 15, 9, 10, and 6; now 15 ÷ 9 = $1\frac{2}{3}$, and 10 ÷ 6 = $1\frac{2}{3}$, these four numbers are proportionals, since they have *equal* ratios; and they may be written 15 : 9 :: 10 : 6. The second and third terms of a proportion are called *means* (9 and 10); the first and fourth *extremes* (15 and 6). When four numbers form a proportion, *the product of the means is equal to the product of the extremes.* Thus, 6 : 3 :: 8 : 4; here, 6 × 4, the extremes, = 8 × 3, the means, = 24.

If the product of any two numbers be equal to the product of any other two numbers, the four numbers will form a proportion, the factors of one product forming means, and those of the other forming extremes. This is another test by which we can determine if four numbers be proportionals.

We may make any change in writing four proportionals, provided the same two terms still form means or extremes, thus:—

We may say,

1°. *The first term : to the second term : : the third term : the fourth term.*

2°. *The first term : the third term : : the second term : the fourth term.*

3°. *The second term : the first term : : the fourth term : the third term.*

4°. *The second term : the fourth term : : the first term : the third term.*

By adding,

5°. *The first term + the second : the second term : : the third term + the fourth : the fourth term.*

In the above explanations, we have considered the terms *abstract* numbers. The first and second terms, when compound quantities are considered, must be of the same kind, and also the third and fourth of the same kind, since we cannot compare quantities of *different* kinds so as to have any *ratio* or relation between them. It has been shown that we cannot express one quantity as a fraction of another quantity of a different kind.

From the product of the means being equal to the product of the extremes, the rule for finding a *fourth proportional* to three given numbers may be easily deduced. There are always *three terms* given; which must be, either *two extremes* and *one mean*, or *two means* and *one extreme*. If two extremes and one mean be given. From the two extremes, say 10 and 6, we find the product of the extremes, 60; now, as the product of the means is also 60, and we have one mean, say 4, which is *one factor* of 60, considered as product of means; hence, $60 \div 4 = 15$, the other mean. Therefore we can say, 10 : 15 : : 4 : 6; or, 15 : 10 : : 6 : 4, &c.

It follows that —*product of means* ÷ *one extreme = the other extreme*; and—*product of extremes* ÷ *one mean = the other mean.*

In this rule there are always three numbers given to find a fourth, hence it has been called 'The Rule of Three.' It has also been called 'The Golden Rule,' from its great importance.

122. *To find a fourth proportional to three quantities.*

RULE.—Divide the product of the second and third terms by the first.

Ex. Find a fourth proportional to 8, 6, and 24.

$$\begin{array}{r} 8 : 6 : : 24 \\ 6 \\ \hline 8)144 \\ \hline 18 \text{ } \textit{Ans.} \end{array}$$

Proof.

By **121.**

$8 \div 6 = 1\frac{1}{3}$

$24 \div 18 = 1\frac{1}{3}$

Proof.

The extremes $8 \times 18 = 144$

The means $6 \times 24 = 144$

Find fourth proportionals to :

1.* 16 : 18 : : 24.

2. 12 : 30 : : 50.

3. 10 : 8 : : 80.

4. 24 : 100 : : 168.

5. 36 : 12 : : 720.

6. 7 : 14 : : 28.

7. 23 : 69 : : 177.

8. 25 : 125 : : 1025.

123. GENERAL RULE FOR PROPORTION.

1°. Set down the term which corresponds to the answer in the third place.

2°. If, from the nature of the question, the answer is to be *more* than the third term, set down the *greater* of the other two terms in the second place. If the answer is to be

---

* *Answers.*

(1.) 27. (2.) 125. (3.) 64. (4.) 700.
(5.) 240. (6.) 56. (7.) 531. (8.) 5125.

less than the third term, write the *least* of the other two in the second place.

3°. Then divide the product of the second and third terms by the first; the quotient will be the fourth term or answer.

Case I.

124. When the three given terms are each of one denomination.

Various Departments.

Ex. 1. A coach goes from London to Liverpool at the rate of 9 miles per hour in 24 hours; in what time would the distance be travelled by a train travelling 32 miles per hour?

We see at once that the answer is to be in hours; therefore 24 is written in third place. Anything going 32 miles an hour will take less time than 9 miles an hour; hence 32 will be the first term.

$$\begin{array}{l} \text{mls.} \quad \text{mls.} \quad \text{hrs.} \\ 32 \;:\; 9 :: 24 \\ \qquad\quad \underline{\quad 9} \\ 32)216\,(6\tfrac{3}{4} \text{ hrs. } \textit{Ans.} \\ \qquad \underline{192} \\ \qquad \tfrac{24}{32} = \tfrac{3}{4} \end{array}$$

By First Principles.

24 hours at rate of 9 miles $= 24 \times 9 = 216$, the number of miles travelled; whence $216 \div 32 = 6\frac{3}{4}$ the number of hours necessary to travel the distance at 32 miles per hour.

XXIX.

Post Office.

1. How much land at 16*s*. an acre shall be given in exchange for 480 acres at 15*s*. an acre?

2. If a pasture can maintain 7 horses 6 weeks, how long can it maintain 35 horses?

3. If £100 gain £10 in 12 months, how much will gain the same in 9 months?

4. If 100 men can finish a work in 12 days, how many men are sufficient to do the same in 3 days?

Where 'Vulgar and Decimal' Fractions are required.

5. If 5 men can do a piece of work in 27 days, how long would 6 men be in doing double the quantity?

6. If by working 9 hours a day I can finish a piece of work in 12 weeks, how long shall I take to finish it if I work 8 hours per day?

7. A man by working 6¼ hours a day does a piece of work in 12 days; how many hours a day must he work in order to do it in 7½ days?

CASE II.

125. ***When the first two terms are of different denominations, and the third term of one denomination.***

RULE.—Reduce the first and second terms to the lowest denomination mentioned in either, then proceed as in *Case I.*

Ex. 2. If 17 tons of coal cost £19 19*s*. 6*d*., how much may be bought for £10 11*s*. 6*d*.?

The reason of the statement of this example is precisely similar to that given in Ex. 1. We reduce, according to the Rule, the first and second terms to pence, and then multiply the second and third terms together, and divide the product by the first term, which gives the *Ans.* in the same denomination as the third term.

$$\begin{array}{rrr}
 & & \text{tons.}\\
\text{£19 19}s.\ 6d.: & \text{£10 11}s.\ 6d.:: & 17\\
20 & 20 & \\
\hline
399 & 211 & \\
12 & 12 & \\
\hline
4794 & 2538 & \\
 & 17 & \\
\cline{2-2}
\end{array}$$

$$4794)43146\ (9 \text{ tons } Ans.$$
$$43146$$

It is much better to write the terms as below and try to contract; thus, dividing the first and third by 17 we have

$$Ans. = \frac{\text{£10 11}s.\ 6d. \times 17}{\text{£19 19}s.\ 6d.}$$
$$= \frac{\text{£10 11}s.\ 6d. \times 1}{\text{£1 3}s.\ 6d.} = 9.$$

Find a fourth proportional to :—

8. As £1648 6*s*. 9*d*. : £505 10*s*. 4*d*. :: £52.

9. If a horse cost 24*s*. 11*d*. for 13 days, how many days will £15 16*s*. 3*d*. keep him?

10. How much is due for 95 days, the salary being £28 7*s*. per annum?

11. What must £92 pay towards a tax when £83 7*s*. 4*d*. is assessed on £650 13*s*. 4*d*., and how much is the poundage rate?

War Department, Science and Art, Police Courts, &c.

12. If a train going 25 miles an hour performs a certain distance in 4½ hours, how long would a train going 30 miles an hour take to perform three times the distance?

13. Find the income-tax on £17030 5*s*. at 7*d*. in the pound.

14. Find the value of a cargo, to insure which it cost £194 8*s*. 8*d*., at 12*s*. 9½*d*. per cent.

15. If an acre cost £161 6*s*. 8*d*., how much can be purchased for £534 4*s*. 6*d*.?

India Office.

16. If the yearly profits of an investment be £11 9*s*. 6*d*. per cent., how much must be invested in order to produce an annual return of £640 13*s*. 9*d*.?

CASE III.

126. *When the third term is of more denominations than one.*

RULE.—Reduce it to the lowest denomination, and then proceed as before. The answer will be in that denomination to which the third has been reduced.

Note.—If the third term can be conveniently multiplied by the second, it will be unnecessary to reduce it.

India Board.

Ex. 3. If a pocket of hops weighing 1 cwt. 3 qrs. 12 lbs. cost £7 13*s*. what is the price per cwt.?

```
cwts. qrs. lbs.    cwt.
  1    3   12  :    1    : :   £7 13s.
  4               112          20
 ---              ---         ---
  7               112          153
 28                            112
 ---                          ----
208                           1836
                             153
                             ---- ---
                        208)17136(82s. 4d.
                             1664 ------
                             ----  (£4 2s. 4 8/13d. Ans.
                              496
                              416
                              ---
                               80
                               12

                              960
                              832
                              ---
                          128/208 = 8/13
```

In the annexed operation the first and second terms are reduced to lbs., and the third term to shillings, before multiplying, and the answer is consequently found in shillings.

One of the most common errors into which many persons fall in working Proportion consists in introducing unnecessary reductions. For instance, the learner, in this example, may, instead of bringing £7 13s. into shillings, multiply £7 13s. by 112, by means of compound multiplication, and by this operation save a multiplication and a division by 20. When the third term consists of several denominations, the reduction process is, if possible, be avoided.

By First Principles.

1 cwt. 3 qrs. 12 lbs. $=208$ lbs

$\therefore$ Cost of 208 lbs. $=$ £7 13*s* $=153$*s*.

$\therefore$ Cost of 1 lb. $=\frac{153}{208}$*s*.

Cost of 112 lbs. $=\dfrac{153\times112}{208}=\dfrac{153\times7}{13}=\dfrac{1071}{13}$*s*.

$=82\frac{5}{13}$*s*. $=$ £4 2*s*. $4\frac{3}{13}$*d*. *Ans.*

Post Office.

17. A bankrupt owes £4968, he has only money sufficient to pay 9*s*. 7*d*. in the £; how much money has he to pay his debts?

18. What is the value of 19½ chaldrons of coals at £1 11*s*. 6*d*. per chaldron?

19. If £70 16*s*. 4*d*. is the cost of 16 cwts. 4 lbs. what would 1 ton 2 cwts. 3 qrs. cost?

20. What is the half year's rent on 547 acres of land at 15*s*. 6*d*. per acre?

Audit Office.

21. If 33 cwts. 2 qrs. 8 lbs. cost £1315 8*s*. 9¾*d*. what will 3½ cwts. cost?

22. If 37 cwts. 3 qrs. 22 lbs. cost £477 16*s*. 8*d*. what will 1 cwt. cost?

Where Candidates are to know 'Arithmetic, including Vulgar and Decimal Fractions.'

23. What weight of sugar may be bought for £93 12*s*. when the cost of 6 cwts. 2 qrs is £27 14*s*. 8*d*.?

24. What is the tax on a house rented at £327 12*s*. 6*d*. if the tax on a house rented at 35 guineas is £6 8*s*. 7½*d*.?

CONTRACTIONS.

127. I. The operation may be often contracted by dividing the first and second, or first and third terms by any number that will *measure* them, and using the quotients instead of these terms themselves.

NOTE.—We can never contract the second and third terms

Ex. 4. What is the price of 17 cwts. 2 qrs. 14 lbs. at £44 16*s*. per cwt.?

After reducing the three terms, we contract the first and third. It will be found most convenient to write the dividends above and the divisors under a line, when the example has been properly stated.

$$\begin{array}{rcccc} \text{cwt.} & & \text{cwts. qrs. lbs.} & & £\ \ \text{s.} \\ 1 & : & 17\ \ 2\ \ 14 & :: & 44\ \ 16 \\ \underline{112} & & \underline{4} & & \underline{20} \\ 112) & & 70 & & 896 \\ & & \underline{28} & & \\ & & 1974 & & \end{array}$$

$$\frac{1974 \times \cancel{896}}{\cancel{112}} = \frac{1974 \times 8}{1} =$$

15792*s.* = £789 12*s.* ***Ans.***

25. The house rent of a labourer for 4 weeks is 9*s.* 4*d.*, what is it for 2 years ?

26. When 10 cows can be bought for £115 3*s.* 9*d.*, how many, at the same rate, will £403 3*s.* $1\frac{1}{2}$*d.* buy ?

27. When 24 cwts. of hay cost £2 15*s.*, how many cwts. can I buy for £8 17*s.* 6*d.* ?

II. 1°. Having stated the example, multiply the ***first*** and ***second*** or ***first*** and ***third*** terms by any number which will produce products, wanting the ***low denominations*** of these terms.

2°. Use these products instead of the terms themselves.

Ex. 5. How much linen may be bought for £41 12*s.* 6*d.*, if 405 yards cost £69 7*s.* 6*d.*

Looking at 12*s.* 6*d.* and 7*s.* 6*d.* we see at once that multiplying by 8 will give pounds, without either shillings or pence in the product.

$$\begin{array}{ccccc} £\ \ \text{s.}\ \ \text{d.} & & £\ \ \text{s.}\ \ \text{d.} & & \text{yds.} \\ 69\ \ 7\ \ 6 & : & 41\ \ 12\ \ 6 & :: & 405 \\ \underline{\quad\quad 8} & & \underline{\quad\quad 8} & & \\ 555 & & 333 & & \end{array}$$

$$\frac{333 \times \cancel{405}}{\cancel{555}} = \frac{\cancel{333} \times 81}{\cancel{111}} = \frac{3 \times 81}{1} =$$

243 yds. ***Ans.***

28. What will be the price of 8 cwts. 1 qr. 7 lbs. of cheese, when 3 cwts. 2 qrs. 14 lbs. can be bought for £29 ?

29. How much land can be rented for £768 3*s.* 4*d.* when 4 acres are rented at £6 13*s.* 4*d.* ?

30. When 59 yards can be bought for £7 7*s.* 6*d.* how many can be bought for £10 6*s.* 3*d.* ?

31. When 73 lbs. of beef can be had for £1 19*s.* $6\frac{1}{2}$*d.* what will be the price of 7006 lbs. ?

Science and Art, Prisons, Fisheries, &c.

32. If 16 Napoleons are worth £12 14*s.* 8*d.* what is the value of 105 Napoleons ?

33. If 17 cwts. 3 qrs. 14 lbs. of barley cost £8 18*s.* 9*d.*, how much may be bought for £2 16*s.* 3*d.* ?

Various Departments.

34. When the income-tax was 7*d.* in the pound a person paid £24 7*s.* 6*d.*, what additional sum does he pay now that the income-tax is 16*d.* in the pound ?

128. *If fractions be in some, or in all the terms.*

RULE.—Reduce them to improper fractions, if necessary, and proceed as in the general rule ; that is, invert the terms of the fraction in the first place, and then multiply the three terms of the proportion together.

NOTE.—The first and second terms before being multiplied must be fractions of the *same* denomination.

Various Departments.

Ex. 6. A ship is worth £16000, and a person possessed of $\frac{5}{16}$ of it sells $\frac{3}{8}$ of his share ; what has he remaining, and what is it worth ?

$\frac{3}{8}$ of $\frac{5}{16}=\frac{15}{128}$, what he sold ; then take $\frac{15}{128}$ from $\frac{5}{16}=\frac{40}{128}-\frac{15}{128}=\frac{25}{128}$, what he has remaining.

The whole ship being $\frac{128}{128}$, we have

By decimals.

$$\begin{array}{l} 1 : \cdot 1953125 :: £16000 \\ \qquad 16000 \\ \hline £3125\cdot 0000000 \ \textit{Ans.} \end{array}$$

$$\frac{128}{128} : \frac{25}{128} :: £16000$$

$$128 : 25 :: 16000$$

$$\frac{\cancel{16000}\times 25}{\cancel{128}}=\frac{\cancel{1000}\times 25}{\cancel{8}}$$

$$= 125\times 25 = £3125 \ \textit{Ans.}$$

Again,

Since he possesses $\frac{25}{128}$, we may find $\frac{1}{128}$ thus: £16000 ÷ 128 = £125 ; hence $\frac{25}{128}$ = £125 × 25 = £3125 *Ans.*

Or, one-sixteenth = £1000 ; hence, $\frac{5}{16}$ = £5000, what he had at first of the ship; then $\frac{3}{8}$ of £5000 = £1875, what he sold ; whence £5000 − £1875 = £3125 *Ans.* as before.

Various Departments.

35. If $\frac{3}{5}$ of a lb. cost 8*s.* 2*d.* what must I pay for $5\frac{1}{7}$ lbs. ?

36. If $\frac{11}{15}$ of a cwt. be worth £4 19*s.* what will be the price of $\frac{5}{9}$ of a ton ?

37. If $\frac{13}{15}$ of a cwt. be worth £7 3*s.* what will be the value of $\frac{6}{11}$ of a ton ?

38. If $2\frac{3}{4}$ lbs. of tea cost 12*s.* 9*d.* what will $\frac{3}{12}$ of a pound cost ? (Solve by decimals.)

39. If $2\frac{3}{8}$ lbs. of tea cost 9*s.* 6*d.* what will $\frac{5}{12}$ of a pound cost ?

129. *To divide any quantity or number into parts, proportional to the parts of another quantity or number.*

Ex. 7. Three labourers earn £2 13*s.* 4*d.*, one working 8 days, the second 5, and the third 3 days ; how much is each to receive ?

Here 8 + 5 + 3 = 16, the total number of days worked, and £2 13*s.* 4*d.* ÷ 16 = 3*s.* 4*d.* = the value of a day's work ; hence,

3*s.* 4*d.* × 8 = £1 6*s.* 8*d.* the first man's share.
3*s.* 4*d.* × 5 = 16*s.* 8*d.* the second „
3*s.* 4*d.* × 3 = 10*s.* 0*d.* the third „

Ex. 8. Suppose the three labourers to work the same number of days, and to earn £4 17*s.* 6*d.* ; but the first only works $4\frac{1}{2}$ hours per day, the second 6 hours, and the third 8 hours, how will the money be divided ?

$$\left.\begin{array}{l}8\times4\frac{1}{2}=36\\5\times6\ \ =30\\3\times8\ \ =24\end{array}\right\}\text{90 total number of hours worked by all; then}$$

£4 17 6*d.* ÷ 90 = 1*s.* 1*d.* the wages per hour.
36 × 1*s.* 1*d.* = £1 19*s.* first man's share.
30 × 1*s.* 1*d.* = £1 12*s.* 6*d.* second „
24 × 1*s.* 1*d.* = £1 6*s.* third „

40. There is a certain work completed by three men for £2 17*s.* 6*d.* ; one man worked 50 hours, a second, 40 hours, and a third, 10 hours, how much should each receive ?

41. Three farmers hold 30, 40, and 50 acres, respectively ; the tax on the whole is £77 16*s.* 8*d.* How much has each man to pay ?

42. A bankrupt owes one man £800, and another, £900 ; his effects are worth £272 : how much should each creditor receive ?

43. Two men bought an ox weighing 5 cwts. 2 qrs. 20 lbs., and sold it at $7\frac{1}{2}$*d.* per lb., what is each man's share of the selling price ; one gave half-a-crown for every 2*s.* of the other in the purchase money ?

44. If 10 men working $10\frac{1}{2}$ hours a day can complete a certain work in 9 days ; if 3 of them can only work 6 hours each day, how many hours a day must the remainder work in order that the work may be finished in 10 days ?

45. A piece of work can be done by 144 men in 36 days ; but a certain number took ill ; it was done by the remainder in 48 days : how many workmen took ill ?

COMPOUND PROPORTION.

130. When *more* than two ratios can be formed; or, when an example in *Simple Proportion* is modified by one or more conditions, it is *Compound Proportion.*

Various Departments.

Ex. 1. If 21 men take 8 days to dig 20 acres, how many acres will 16 men dig in 12 days?

Had this question been thus expressed: If 21 men dig 20 acres, how many acres will 16 men dig? it would have been worked thus:—

$$\text{As } 21 : 16 :: 20 : \overset{\text{acres}}{15\tfrac{5}{21}} \text{ acres.}$$

But the 16 men are to work 12 days, whilst the 21 men are only to work 8 days; hence we have,

$$\overset{\text{days}}{8} : \overset{\text{days}}{12} :: \overset{\text{ac.}}{15\tfrac{5}{21}} : \overset{\text{ac.}}{22}\ \overset{\text{r.}}{3}\ \overset{\text{p.}}{17\tfrac{1}{7}}\ \textit{Ans.}$$

Instead of giving the two statements in full, as above, it is much preferable to *abridge* them in the following manner:—

Rule.—1°. Set the term which is the same name as the answer in the third place.

2°. Compare the other terms by pairs, and consider what effect each ratio will have on the answer, and set them down according to the General Rule for Simple Proportion.

3°. Then multiply all the second terms together, and multiply this product by the third term.

4°. Divide this last product by the product of all the first terms, for the answer.

For Contractions.—Divide *any* term in the *first* place and any in the *second* place by common factors; or, any of the *first* terms and any of the *third* terms by common factors.

Having stated the same question according to the rule, we multiply 21 and 8, the two first terms, and 16 and 12, the two second terms, and then proceed as in Simple Proportion.

$$\begin{array}{rcrcr}
 & & & & \text{acres.} \\
21 & : & 16 & :: & 20 \\
8 & : & 12 & :: & \\
\hline
168 & & 192 & & 20 \\
 & & & & 192 \\
\hline
 & & & 168) & 3840 \\
\hline
\end{array}$$

22 ac. 3 rds. $17\frac{1}{7}$ pls. *Ans.*

Contracted.

The dividends are written above, and the divisors below, a line; then they are contracted by dividing by common factors.

$$\begin{array}{rcr}
21 & : & 16 \quad :: \quad 20 \\
8 & : & 12 \\
\hline
\end{array}$$

$$\frac{\overset{2}{\cancel{16}} \times \overset{4}{\cancel{12}} \times 20}{\underset{7}{\cancel{21}} \times \cancel{8}} = \frac{2 \times 4 \times 20}{7}$$

$$= \frac{160}{7} = 22 \text{ ac. } 3 \text{ r. } 17\tfrac{1}{7} \text{ p.}$$

By First Principles.

21 men in 8 days dig 20 acres

1 man in 8 days will dig $\frac{20}{21}$ acre

1 man in 1 day will dig $\frac{20}{168}$ acre

whence 16 men in 1 day will dig $\dfrac{20 \times 16}{168} = \dfrac{20 \times 2}{21}$

„ 16 „ 12 days „ $\dfrac{\overset{4}{\cancel{12}} \times 20 \times 2}{\underset{7}{\cancel{21}}} = \dfrac{8 \times 20}{7} = \dfrac{160}{7}$ = 22 acres 3 rds. $17\frac{1}{7}$ pls. *Ans.*

Education and Constabulary Offices.

Ex. 2. If 20 horses and 196 sheep can be kept 18 days for £151 10*s.*, what sum will keep 15 horses and 72 sheep for 8 days, supposing 5 horses to eat as much as 76 sheep?

When 5 horses = 76 sheep, 20 horses = 304 sheep; therefore 20 horses and 196 sheep = 304 + 196 = 500 sheep. Again, 15 horses = 3 × 76 = 228 sheep, and 15 horses and 72 sheep = 300 sheep. Hence the statement.

By First Principles.	£	*s.*	*d.*
500 sheep for 18 days =	151	10	
100 sheep for 18 days =	30	6	
100 sheep for 1 day =	1	13	8
300 sheep for 1 day =	5	1	
300 sheep for 8 days = £40 8*s.* *Ans.*			

$$\begin{array}{rcrcrr}
 & & & & \text{£} & s. \\
5\cancel{00} & : & 3\cancel{00} & :: & 151 & 10 : \\
18 & : & 8 & & & 4 \\
\hline
\cancel{90} & : & \cancel{24} & & & \\
15 & : & 4 & & 15)606 & \\
\hline
 & & & & \text{£}40 & 8s. \\
\end{array}$$

XXX.

Post Office, War Department, Prisons, &c.

1. If the carriage of 21 cwts. for 4 miles costs 11*s.* 8*d.*, find the cost of carrying 7 cwts. 47 miles?

2. If 7 men can mow 34 acres in 12 days of 8¼ hours each, how many can be mown by 30 men in 11 days of 7⅘ hours each?

Post Office (Provincial Clerks).

3. If £600 gain £45 in 18 months how much will £100 gain in a year?

Customs.

4. If £100 in 12 months gain £7 interest, what is the interest of £571 for 6 years?

Various Departments.

5. If 12 men working 8 hours a day can build a house in 70 days, in how many days might it be built by 42 men working 10 hours a day?

6. If 6 iron bars 4 ft. long, 3 inches broad, and 2 inches thick, weigh 288 lbs., how much will 15 weigh, each 6½ ft. long, 4 in. broad, and 3 in. thick?

7. If 5 men receive £18 15*s.* wages for 12 weeks, what will be the wages of 16 men for 20 weeks?

8. If 49 men can do a piece of work in 130 days of 8 hours each, how many hours a day must 196 men work to do as much in 26 days?

9. If 780 men on full rations consume 91 quarters of wheat in 49 days, how many on half rations will consume 69 quarters in 9 days?

10. If 3 men can mow 7 acres of grass in 5 days of 9 hours each, in how many days of 10 hours each will 5 men mow 35 acres?

11. If 200 men in 12 days of 8 hrs. each can dig a trench 160 yds. long 6 wide and 4 deep, in how many days of 10 hours will 90 men dig a trench 450 yards long 4 wide and 3 deep?

12. If the carriage of 65 lbs. for 120 miles cost 6*s.*, how much may I have carried 750 miles for a guinea?

XXXI.

MISCELLANEOUS EXERCISES IN PROPORTION.

Audit Office.

1. If 3 cwts. cost £11 1*s.* 3*d.* what will be the price of 16 cwts. 1 qr. 21 lbs.?

2. If 3 cwts. 13 lbs. cost £14 2*s.* 9*d.* what will 7 cwts. 2 qrs. 25 lbs. cost?

3. If 3875 cwts. cost £34596 9*s.* 8¼*d.* what will 1 cwt. cost?

4. If 124 tons 16 cwts. 2 qrs. 16 lbs. cost £1613 19*s.* 6*d.* what will 2 tons cost?

Miscellaneous Questions.

5. How long will 40 men take to build a wall 10 feet high, if 11 men require 30 days to build one half as long, but only 7½ feet high?

6. If the carriage of 18 tons 13 cwts. 3 qrs. 14 lbs. cost 18*s*. 7¾*d*., what will be the cost of the carriage of 256 tons 19 cwts. 2 qrs. 26 lbs.?

7. If a penny loaf weighs 4 oz. when flour is at 4*s*. the peck, what should it weigh when flour is 5*s*. 4*d*. the peck?

8. If either 5 oxen or 7 horses will eat up the grass of a field in 87 days, in what time will 4 oxen and 6 horses eat up the same?

9. If 120 men make an embankment ¾ of a mile long, 30 yds. wide, and 7 yds. high, in 42 days, how many men would it take to make an embankment 1000 yds. long, 36 yds. wide, and 22 feet high, in 30 days?

10. What would be the charge for fuel for 800 men for 12 months, provided the ration for each man per diem consisted of $1\frac{1}{5}$ foot of wood from 1st May to 31st October, and half that quantity for the remainder of the year, the price of fuel being $6\frac{5}{9}$*d*. per foot?

11. What sum of money will be required per week of 7 days to pay 8 companies, each consisting of 95 men, at 1*s*. 1½*d*. a day for each man?

12. If 11 men build a wall in 9 days, how many should build a wall twice as high in 6 days?

13. If 12 men, working 8 hours a day, take 15 days to complete a piece of work, how long will 16 boys, working 10 hours a day, take to do the same work, the work of one man being equal to that of 2 boys in the same time?

14. What ought a loaf to weigh when wheat is 4*s*. a bushel, if when at 5*s*. 3*d*. a bushel it weighs 2 lbs. 8 oz.?

15. If the carriage of 8 cwts. of coals 128 miles cost 48*s*., what would be the charge for 4 cwts. 32 miles at the same rate?

16. What is the income of a person who pays £22 7*s*. 5*d*. for income tax at the rate of 7*d*. in the pound?

17. 44½ guineas used to be coined out of 1 pound standard gold, how many sovereigns are now coined out of this weight?

18. If a bankrupt, whose effects are worth £1181 12*s*. 6*d*., owes £4726 10*s*., how much will he be able to pay in the £?

19. If 24 writers who can copy 80 words a minute get through 1000 pages in 10 days, how many should 72 writers, at the rate of 100 words a minute, copy in 16 days?

20. If 20 English navvies, each earning 3*s*. 6*d*. a day, can do the same piece of work in 15 days that it takes 28 foreign workmen, each earning 3 francs a day, to complete in 20 days, taking the value of the franc at 10*d*., determine which class of workmen it is most profitable to employ? If a piece of work done by navvies cost £3000, what would be the cost of the same work done by foreign workmen?

Customs, Junior Clerks, War Department, Post Office, Department of Science and Art, &c.

21. If a 4*d.* loaf weighs 24 oz. when wheat is at 50*s.* per quarter, what is the price of wheat when the 6*d.* loaf weighs 30 oz.?

22. If it cost £18 5*s.* 9*d.* to carpet a room 22 ft. long and 19 ft. wide, how much will a carpet of the same kind cost for a room 27 ft. long and 16 ft. wide?

23. Find the cost of carpeting a room that is 25 ft. long by 20 ft. wide, the price of a yard of carpet 2 ft. 6 in. wide being 4*s.* 3*d.*?

24. A person who values his property at £3500 insures half of it at 5*s.* 6*d.* per cent., and half at 4*s.* 6*d.* per cent; what does it cost him?

25. Reduce £125 10*s.* into francs and centimes, exchanged at 23 francs 6 centimes per £ (100 centimes = 1 franc).

26. If 17 cwts. 3 qrs. 14 lbs. of barley cost £8 18*s.* 9*d.*, how much may be bought for £5 12*s.* 6*d.*?

27. How much will a creditor lose on a debt of £5342 5*s.* when a bankrupt can only pay 13*s.* 6*d.* in the pound?

28. Find the income-tax on £7530 14*s.* $4\frac{1}{2}$ at 16*d.* in the pound.

29. If 12 men can mow a field of grass in 5 days, in how many days could 8 men mow the same?

30. How many yards of carpet, 2 ft. 6 in. broad, will cover a room which is 32 ft. long and 15 ft. broad?

31. If 5 gallons of oil cost 18*s.* 4*d.* find the cost of 13 gals. 3 qrts. 1 pint.

32. What is the income corresponding to an income-tax of £108 1*s.* $4\frac{1}{2}$*d.* at the rate of 9*d.* in the pound?

Customs and Excise.

33. Required the cost of 17 hogsheads of tobacco, each weighing 11 cwts. 2 qrs. 19 lbs. at £19 8*s.* per cwt.

War Department.

34. If the price of 1 oz. of gold is £3 10*s.* what is the price of 14 ingots, each weighing 3 lbs. 7 oz. 14 dwts. 21 grs.?

35. If 1 yard of cloth cost 15*s.* $6\frac{3}{4}$*d.* what will be the price of $32\frac{1}{2}$ yards at the same rate?

Various Departments.

36. If 780 men on full rations consume 221 quarters of wheat in 119 days, how many on half rations will consume 437 quarters in 57 days?

37. If the shares of a railway are at $76\frac{1}{2}$ when consols are at $93\frac{1}{2}$, what should be their price when consols are at $89\frac{1}{4}$?

38. How many yards of matting $7 \cdot \dot{3}$ ft. broad will cover a floor $27 \cdot \dot{3}$ ft. long and $10 \cdot 08\dot{3}$ ft. broad?

39.* If $3\frac{2}{5}$ lbs. of tea cost 15*s.* 3*d.* how many pounds can I buy for £4 3*s.* $10\frac{1}{2}$*d.*

40. A tobacconist mixed together 80 lbs. of tobacco at 14*d.* per lb., 100 lbs. at 20*d.*, 60 lbs. at 4*s.* 10*d.*, and 20 lbs. at 2*s.* 10*d.* per lb.; what will be the value of 3 oz. of this mixture?

41. Find the income-tax on £8313 5*s.* at 7*d.* in the £.

42. If a bar of gold weighing 7 lbs. 1 oz. 14 dwts. is worth £257 2*s.* what is that per oz.?

43. If 12 men can dig a trench 15 yds. long and 4 broad in 3 days of 12 hours each, in how many days of 9 hours each can 8 men dig a trench 20 yards long and 8 broad?

44. If £59 10*s.* will buy 25 cwts. 2 qrs. of sugar, what quantity will £6 14*s.* 2*d.* buy?

45. If a penny loaf weighs 5 oz. when wheat is at 62 shillings the quarter, what ought it to weigh when wheat is 64*s.* the quarter?

46. If by paying down £70 17*s.* 6*d.* a person becomes entitled to £2 9*s.* 7*d.* a year, what income should he gain by paying £2151 17*s.* 8*d.* in the same way?

47. If a tax on an income of £1132 amount to £125 1*s.* $5\frac{7}{8}$*d.* what is that in the pound?

48. If the carriage of 107 cwts. 10 lbs. cost £37 9*s.* $7\frac{1}{2}$*d.* what would the carriage of a ton cost?

49. The shares of a given railway are at $59\frac{1}{2}$ when consols are at $93\frac{1}{2}$, what should be their value when consols are at $91\frac{1}{2}$?

50. If the price of 3 bushels of wheat is 16*s.* 9*d.* find the price of 12 quarters 2 bushels 1 peck.

51. Find the income corresponding to an income-tax of £50 7*s.* 1*d.* at 5*d.* in the pound.

52. Gold of the value of £423284 arrives from Australia; what is its weight, the price being £3 18*s.* per oz.?

53. Find the value of 14 cwts. 3 qrs. 17 lbs. of sugar at £2 17*s.* per cwt.

54. If $3\frac{1}{2}$ lbs. of sugar cost 2*s.* what will be the price of 54 cwts. 2 qrs. 7 lbs.?

---

* 15*s.* 3*d.* $= 15\frac{1}{4} = 15 \cdot 25$, and $3\frac{2}{5} = 3 \cdot 4$, and £4 3*s.* $10\frac{1}{2}$*d.* $= 83\frac{21}{24}\left(\frac{7}{8}\right)$*s.* $= 83 \cdot 875$: then

$$\begin{array}{rcl|l} \cancel{1}\cancel{5}\cdot\cancel{2}\cancel{5} & : & \cancel{8}\cancel{3}\cdot\cancel{8}\cancel{7}\cancel{5} :: 3\cdot 4 \text{ lbs.} & \cdot 61 \text{ will divide 1st and 2nd, and} \\ \cancel{3}\cdot\cancel{0}5 & : & \cancel{1}\cancel{6}\cdot\cancel{7}\cancel{7}\cancel{5} :: & \text{we have } 1 : 5\cdot 5. \text{ Then } 5\cdot 5 \times 3\cdot 4 \\ \cdot\cancel{6}\cancel{1} & : & \cancel{3}\cdot\cancel{3}\cancel{5}\cancel{5} :: & = 18\cdot 7 \text{ lbs. } \textit{Ans.} \end{array}$$

55. If 8 tons 4 lbs. of sugar cost £466 15*s.* 5*d.*, find the price per lb.

56. A person fails for £1078, and his effects are worth £370 18*s.* 5½*d.* What will be the dividend on £78 10*s.* ?

57. If 77$\frac{3}{16}$ cwts. cost £119 12*s.* 9¾*d.*, what is the value of 18 cwts. 1 qr. 21 lbs. ?

58. Find what is the income of a person who pays an income-tax of £77 1*s.* 3¾*d.* when the rate is 10*d.* in the pound.

59. If 16 pioneers, in 5 days of 12⅓ hours long, can dig a trench 139·75 yards long, 4½ yards wide, and 2⅓ yards deep, how many hours per day must 90 pioneers work during 42 days, in order to dig a trench 4910$\frac{1}{16}$ yards long, 4⅞ yards wide, and 3⅛ yards deep?

60. If 248 navvies, in 5½ days of 11 hours each, dig a trench 232⅓ feet long, 3 feet 8 inches wide, and 2 feet 4 inches deep, in how many days of 9 hours each should 24 navvies dig a trench 337⅓ feet long, 5⅔ feet wide, and 42 inches deep, if the degree of hardness of the ground in the first case be to that in the second as 7:4?

THE CHAIN RULE.

CONJOINED PROPORTION OR CHAIN RULE.

131. Conjoined Proportion or Chain Rule is the combination of several ratios or proportions, through which the ratio of the first and last term is discovered.

132. This rule, which relates principally to the exchanges between nations, is frequently applied to other mercantile transactions, and will be understood from the following examples.

Ex. 1. If 3 lbs. of tea be worth 4 lbs. of coffee, and 12 lbs. of coffee worth 40 lbs. of sugar, how many pounds of sugar are worth 27 lbs. of tea?

We just write *Ans.* = the term to which, from the question, it must be equal; then a term of the same name as that already written is placed on the left under *Ans.*,

Ans. = 27 lbs. tea,
3 lbs. tea = 4 lbs. coffee,
12 lbs. coffee = 40 lbs. sugar;

$$\text{Hence } \frac{\overset{\overset{3}{\cancel{9}}}{\cancel{27}} \times \cancel{4} \times 40}{\cancel{3} \times \underset{\cancel{3}}{\cancel{12}}} = 120 \text{ lbs. } \textit{Ans.}$$

and its equal opposite to it on the left, and so on; and having written all the terms in this manner, we divide the product of the consequents by the product of the antecedents.

133. Care must be taken when cases of Reduction arise to have the terms of the same name in the same denomination.

Ex. 2. A can do as much work in 3 days as B can do in 4, B can do as much in 10 days as C in 12, and C can do as much in 9 days as D in 15; how many days' work of D are equal to 18 of A?

Ans. = 18 of A
3 of A = 4 of B
10 of B = 12 of C
9 of C = 15 of D

$$\text{Hence } \frac{\overset{\cancel{2}}{\cancel{18}} \times 4 \times 12 \times \overset{\cancel{5}}{\cancel{15}}}{\cancel{3} \times \underset{\cancel{2}}{\cancel{10}} \times \cancel{9}} = 48 \textit{ Ans.}$$

XXXII.

1. If 75 thalers = 9 francs, 15 scudi = 81 francs, 270 gulden = 124 scudi, and 10 gulden = £1, find the number of thalers = £4 15s.

2. Russian thalers are exchanged at Rome at the rate of 4 for 3 scudi; 4 scudi are given in Paris for 1 Napoleon, and 15 Napoleons in London for £11 15s.; what will a person lose on exchanging in Rome 3,600 thalers worth 3s. each?

3. The exchange between London and Paris is 25·5 francs for £1 sterling; between Paris and Amsterdam is 117 francs for 55 florins, and between Amsterdam and Hamburg 11 florins for 13 marks; find the exchange between London and Hamburg.

4. The first of six boys can copy 8 lines while the second copies 2, and the second 5 while the third is copying 6, and the third 7 while the fourth copies 8, and the fourth 9 while the fifth copies 10, and the fifth 15 while the sixth copies 14; how many lines should the first copy while the sixth copies 128?

5. A London merchant owes a sum of £3,500 to another in Paris; what is his gain, if, instead of sending direct, he remits the money from London to Venice, from Venice to Hamburg, and from Hamburg to Paris, the exchange being £1 = 24·6 francs, 19 francs = 10 Hamburg merks, 1 Hamburg merk = $4\frac{1}{4}$ lire of Venice, and $55\frac{1}{4}$ lire = £1?

6. The pound Troy of British standard silver of which 11 ounces 2 dwts. are pure silver, makes 66 shillings, and 100 francs weigh 461 grams, of which $\frac{9}{10}$ are pure silver: find the value in francs and decimal

of a franc, of 20 shillings, the gram being = 15·43235 grains, neglecting the value of the metal with which the silver is alloyed.

7. If 10 gallons malt spirits be worth 6 gallons of brandy, and 8 gallons of brandy worth 12 gallons of sherry, and 18 gallons of sherry worth 14 of claret, find how many gallons of malt spirits 21 gallons of claret are worth.

8. If 14 lbs. of butter are worth 10 stones of oats, and 5 sts. of oats = $3\frac{1}{2}$ sts. of flour, and 2 sts. of flour = 10 sts. of salt, and 15 sts. of salt = 6 doz. of eggs, how many doz. of eggs are 7 lbs. of butter worth?

9. If 7 lbs. of beef be worth 8 lbs. of pork, and 9 lbs. of pork = 7 lbs. of bacon, and 20 lbs. of bacon = 18 lbs. mutton, and 12 lbs. mutton = 10 lbs. cheese; how many pounds of cheese are equivalent to 8 pounds beef?

PARTNERSHIP.

134. When several persons join in trade the profits or losses will be proportional to the capital subscribed by each and the time it was employed in business.

Partnership is usually divided into two cases: 1. When the stocks have been in trade the *same* time. 2. When the stocks have been in trade for *different* periods of time.

Ex. 1. Two persons enter into partnership, one puts in £450, the other £330; how are they to divide their first year's profits, which amount to £487 10*s.*?

The sum of the stocks £450 + £330 = £780; then, we have £780 : 450 :: £487 10*s.* : £281 5*s.* the first partner's share.

Again, £780 : £330 :: £487 10*s.* : £206 5*s.* the share of the other partner.

By First Principles.

$$\text{Gain on £780} = \text{£487 10}s.\text{, and hence gain on £1} = \frac{\text{£487 10}s.}{780}$$

$$\therefore \text{gain on £450} = \frac{\text{£487 10}s. \times \cancel{450}}{\cancel{780}} = \frac{\text{£487 10}s. \times 15}{26} = \text{£281 5}s.$$

$$\text{and gain on £330} = \frac{\text{£487 10}s. \times \cancel{330}}{\cancel{780}} = \frac{\text{£487 10}s. \times 11}{26} = \text{£206 5}s.$$

India Board.

Ex. 2. A tradesman starts with a capital of £960, and after 3 years takes another into partnership with £2100. After 4 years more the whole profits amount to £2304; how ought this to be divided between them?

First man's capital × its time in trade = £960 × 7 = £6720
Second man's capital × its time in trade = £2100 × 4 = £8400
15120

Whence, we have 15120 : 6720 :: £2304 : £1024 share of first; and 15120 : 8400 :: 2304 : £1280 share of second. The reason for the above operation is obvious, for £6720 for one year is the same as £960 for 7 years, so far as gain or loss is concerned.

XXXIII.

1. If I commence business with a capital of £2000, and at the end of 2 years take a partner who has £6000. At the end of 2 years more, how much should my partner receive, our profits being £1000?

2. A, B and C enter into partnership: A puts in £450 for 4 months: B £360 for 6 months: and C £260 for 9 months. What should be each of their shares in a gain of £210?

3. Four persons hold a pasture farm in common, for which they are to pay £80 17*s*., and into which A puts 8 cows for 3 months; B, 9 cows for 4 months; C, 16 cows for 3 months; and D, 13 cows for 3 months. How much of the rent should each pay?

SIMPLE INTEREST.

135. INTEREST is the price paid for the use of money.

The money borrowed is called the PRINCIPAL.

The sum given for the use of £100 for a year is called the *rate per cent.*

The PRINCIPAL and *interest*, when added together, are called the AMOUNT, that is, the whole amount of money due by the borrower.

If I lend my friend £100 for a year, and if he pay me £5 for the use of it; the £100 lent to him is the *principal*; the £5 which he pays me is the *rate*; and the whole money due at the end of the year, £100 + £5 = £105, is the *amount.*

If the interest be *added* to the principal at the end of certain periods, and this amount considered as *new* principal, it is called *Compound Interest.*

When the interest is *not* so added, it is Simple Interest.

Almost all the examples in finding the interest of money will be understood from the following *proportional* consideration:—

If £100 (cent.) produce a certain sum (the rate) in one year, what will any other sum of money (the given principal) produce, in any other time (the given time)?

136. To find the Interest.—General Rule.

Multiply the principal by the rate, and that product by the time in years, months or days, and divide by 100, 1200, or 36500, respectively, for the answer.

FOR YEARS.

War Department.

Ex. 1. What is the simple interest on £675 18*s*. for 5 years at 3½ per cent.?

In the annexed example we first multiply by 3½, the rate, then this product by 5, the time in years. We divide this last product by 100, as in (**57.**) and find the answer, £118 5*s*. 7⅘*d*.

Many prefer the following Method:—Multiply the rate by the time, and take aliquot parts of 100.

$3\frac{1}{2} \times 5 = 17\frac{1}{2}$	£675 18 0
$10 = \frac{1}{10}$	67 11 9·6
$5 = \frac{1}{2}$	33 15 10·8
$2\frac{1}{2} = \frac{1}{2}$	16 17 11·4
(or $20 = \frac{1}{5}$ and $2\frac{1}{2} = \frac{1}{8}$).	£118 5*s*. 7·8

£675 18
3½
2027 14
337 19
2365 13
5
1,00)118,28 5
20
5,65
12
7,80
$\frac{7,80}{100} = \frac{4}{5}$
£118 5*s*. 7⅘*d*. *Ans.*

Note.—From a long experience, I can with confidence recommend the student who intends to thoroughly understand arithmetical calculations, to reject all rules that are merely approximations. Nowhere is this more necessary than in calculating interest.

Ex. 2. Find the simple interest on £775 16*s.* 8*d* for $3\frac{3}{4}$ years at $2\frac{1}{2}$ per cent. per annum.

NOTE.—*I recommend the following method.*

$$\frac{£775\ 16\ 8 \times 3\frac{3}{4} \times 2\frac{1}{2}}{100} = \frac{£775\frac{5}{6} \times 15 \times 5}{100 \times 4 \times 2}$$

$$= \frac{£4655 \times \overset{3}{\cancel{15}} \times \cancel{5}}{\underset{2}{\cancel{6}} \times \underset{\underset{4}{\cancel{20}}}{\cancel{100}} \times 4 \times 2} = \frac{£4655}{8 \times 8} = \frac{£581\ 17\ 6}{8}$$

$$= £72\ 14s.\ 8\tfrac{1}{4}d.\ \textit{Ans.}$$

or $\underline{3\frac{3}{4} \times 2\frac{1}{2} = 9\frac{3}{8}}$ product of rate and time.

$$\begin{array}{rr} & £775\ 16\ \ 8 \\ \hline 10 = \frac{1}{10} \text{ of } 100 & 77\ 11\ \ 8 \\ 10 - 9\frac{3}{8} = \frac{5}{8} = \frac{1}{16} \text{ of } \ 10 & 4\ 16\ 11\frac{3}{4} \\ \cline{2-2} & 72\ 14\ \ 8\frac{1}{4}\ \textit{Ans.} \end{array}$$

Ex. 3. Find the simple interest on £533 6*s.* 8*d.* for 10 months at $4\frac{1}{2}$ per cent. per annum.

$$\frac{£533\frac{1}{3} \times 1\cancel{0} \times 4\frac{1}{2}}{12 \times 10\cancel{0}} = \frac{\overset{40}{\cancel{£1600}} \times \overset{1}{\cancel{9}}}{\cancel{3} \times \underset{3}{\cancel{120}} \times 2} = £20\ \textit{Ans.}$$

Ex. 4. Find the amount at simple interest of £575 17*s.* 6*d.* for 1 year and $10\frac{1}{2}$ months at 5 per cent. per annum.

By the rule:

$$\frac{£575\frac{7}{8} \times 22\frac{1}{2} \times 5}{1200} = \frac{£4607 \times \overset{\overset{3}{\cancel{9}}}{\cancel{45}} \times \cancel{5}}{\underset{\underset{\underset{16}{\cancel{48}}}{\cancel{240}}}{\cancel{1200}} \times 8 \times 2}$$

$$= \frac{£13821}{16 \times 16} = \frac{£863\ 16\ 3}{16} = £53\ 19\ 9\tfrac{3}{16} \text{ the interest.}$$

$$\therefore £575\ 17\ 6 + £53\ 19\ 9\tfrac{3}{16} = £629\ 17\ 3\tfrac{3}{16}\ \textit{Ans.}$$

Various Departments.

Ex. 5. Find the interest on £1158 17*s.* 6*d.* for 1 year 115 days at $2\frac{1}{2}$ per cent.

We first multiply the principal by 2½, and then this product by 480, the number of days (365 + 115) = 480 = 10 × 8 × 6. We divide the last product by 36500 according to the rule.

$$
\begin{array}{rrrl}
£ & s. & d. & \\
1158 & 17 & 6 & \\
 & & 2\frac{1}{2} & \\
\hline
2317 & 15 & 0 & \\
579 & 8 & 9 & \\
\hline
2897 & 3 & 9 & \\
 & & 10 & \\
\hline
28971 & 17 & 6 & \text{prod. by } 10 \\
 & & 8 & \\
\hline
231775 & 0 & 0 & \text{prod. by } 80 \\
 & & 6 & \\
\hline
365\cancel{00})13906\cancel{50} & 0 & 0 & \text{prod. by } 480 \\
10950 & & & \\
\hline
29565 & \multicolumn{3}{l}{(£38\ 2s.\ \textit{Ans.}} \\
29200 & & & \\
\hline
365 & & & \\
20 & & & \\
\hline
7300 & & & \\
7300 & & & \\
\hline
\end{array}
$$

The work of the above more briefly.

£1158 17*s.* 6*d.* = $1158\frac{7}{8}$: then $\frac{1158\frac{7}{8}\times 2\frac{1}{2}\times 480}{365\times 100}=\frac{9271\times\cancel{5}\times 48\cancel{0}}{8\times 2\times 365\times\cancel{100}}=$ $\frac{9271\times\cancel{48}}{\cancel{8}\times\cancel{2}\times 365\times 2}=\frac{9271\times 3}{365\times 2}=\frac{27813}{730}=£38\ 2s.\ \textit{Ans.}$

By Decimals.

In multiplying by 2·5 we use the rule (36).

$$
\begin{array}{rr}
£1158\frac{7}{8}= & 1158{\cdot}875 \\
2\frac{1}{2}= & 2{\cdot}5 \\
\hline
 & 2897{\cdot}1875 \\
 & 480 \\
\hline
 & 2317750000 \\
 & 115887500 \\
\hline
 & 36500)1390650{\cdot}0000 \\
\hline
 & £38\ 2s.\ \textit{Ans.}
\end{array}
$$

By First Principles.

Interest of £100 for 1 year = 2½ = £2 10*s.*

hence int. of £1 for 1 year $=\frac{2\frac{1}{2}}{100}=\frac{5}{200}=\frac{1}{40}$

„ int. of £1 for 1 day $=\frac{1}{40\times 365}$

„ int. of £1 for 480 days $=\frac{480}{40\times 365}=\frac{12}{365}$

Whence int. of £1158 17*s*. 6*d*. for 480 days $= \frac{£1158\ 17\ 6 \times 12}{365} =$ $\frac{£13906\ 10}{365} = £38\ 2s.$ *Ans*.

Ex. 6. Find the interest and amount of £225 10*s*. from 23rd September to 11th January following, at 5 per cent.

	days.
September	7
October	31
November	30
December	31
January	11
Total no. of days	110

Then $\frac{£225\frac{1}{2} \times 110 \times \cancel{5}}{\cancel{36500}} = \frac{451 \times 11\cancel{0}}{730\cancel{0} \times 2} = \frac{4961}{1460} =$ £3 7*s*. $11\frac{37}{73}$*d*. *interest*.

£225 10 + £3 7 $11\frac{37}{73}$ = £228 17 $11\frac{37}{73}$. *amount*

XXXIV.

Constabulary Office.

1. Find the simple interest on £507 10*s*. for 3¼ years at 4 per cent.

Customs, Post Office, &c.

2. Find the interest on £1257 15*s*. for 5¾ years at 4 per cent. per annum.

Various Departments.

3. Find the interest on £3000 for 8 months, at 4 per cent., less income-tax at 14*d*. in the pound.

4. Find the interest on £260 10*s*. for 3⅓ years at 4¼ per cent.

5. Find the simple interest on £4164 10*s*. for 6 years at 2¾ per cent. per annum.

6. Find the interest on £1250 in 7½ years at 3¼ per cent.

7. Find the interest on £1257 15*s*. for 5¾ years, at 4¼ per cent. per annum.

8. Find the interest on £194 from 3rd March to the 18th December following at 5 per cent. per annum.

9. Find the interest on £260 10*s*. for 3½ years at 4¼ per cent.

10. Find the amount of £322 13*s*. for 10¼ years at 3½ per cent. simple interest.

11. Find the interest at 4¾ per cent. on £924 12*s.* 8*d.* lent on 18th March and repaid on the 11th August, the former day being excluded and the latter included.

137. *The following class of questions has been frequently proposed.*

Various Departments.

Ex. 7. At what rate per cent. will the interest on £1250 for 5 years amount to £265 12*s.* 6*d.* ?

		£	*s.*	*d.*
Interest of £1250 for 5 years	=	265	12	6
Interest of £1250 for 1 year =one-fifth of £265 12*s.* 6*d.*	=	£53	2	6

$$\text{Therefore interest of £1 for 1 year} = \frac{£53\ \ 2\ \ 6}{1250}$$

$$\text{Whence interest of £100 for 1 year} = \frac{£53\ \ 2\ \ 6 \times \cancel{100}}{\cancel{1250}}$$

$$= \frac{£53\ \ 2\ \ 6 \times 2}{25} = \frac{£106\ \ 5}{25} = 4\tfrac{1}{4}\ \textit{Ans.}$$

Various Departments.

12. At what rate per cent. per annum will the interest on £1270 18*s.* 9*d.* amount to £76 5*s.* 1½*d.* simple interest in 2 years?

13. At what rate per cent. simple interest will £320 amount to £360 16*s.* in 3 years ?

14. At what rate per cent. simple interest will £951 7*s.* 6*d.* amount to £1141 13*s.* in 5 years ?

War Department.

15. At what rate per cent. per annum simple interest will £300 amount to £414 in 8 years?

138. *Science and Art, Prisons, Post Office, and other Offices.*

Ex. 8. In what time will £2000 amount to £2857 10*s.* at 5¼ per cent. simple interest ?

	£	*s.*
The amount =	2857	10
The principal =	2000	
Therefore the interest is	£857	10*s.*

Now, interest of £2000 for 1 year at 5¼ = £105; then £857 10 ÷ 105 = 8 yrs. 2 months. *Ans.*

Various Departments.

16. In what time will £171 7*s.* 6*d.* amount to £217 1*s.* 6*d.* at 3⅓ per cent?

17. In what time will £1337 12*s.* 1*d.* amount to £1698 15*s.* 1¾*d.* at 6 per cent. simple interest?

Various Departments.

Ex. 9. What sum of money put out to interest for 3½ years at 4 per cent. will amount to £296 8*s.*?

The interest of £1 for 3½ years at 4 per cent. $=\frac{3\frac{1}{2}\times 4}{100}=\frac{\cdot 14}{100}=\frac{7}{50}$, therefore the amount of £1 at the same rate and for same time $=1\frac{7}{50}$; whence £296 8 ÷ $1\frac{7}{50}=\frac{£\cancel{296}\ \cancel{8}\times 50}{\cancel{57}}=£\cancel{98}\ \cancel{16}\times\frac{50}{\cancel{19}}=£5\ 4\times 50$ = £260. *Ans.*

18. What sum will amount to £45 0*s.* 9¾*d.* in a year at 6½ per cent.?

19. What sum of money put out at simple interest for 3 years, at 4½ per cent., will amount to £1602 0*s.* 1¼*d.*?

Education and Constabulary Offices.

20. Find the simple interest on £2175 10*s.* for 3 years at 4¼ per cent

India Office.

21. Find the simple interest on £1923 15*s.* for 2 years and 8 months at 3⅜ per cent.

Proposed in Departments where 'Arithmetic including Vulgar and Decimal Fractions' is to be known.

22. What principal lent at 3½ per cent. simple interest, will amount in 5 years to £1884 19*s.* 10½*d.*?

23. Find the interest at 4¾ per cent. on £1835 8*s.* 4*d.*, lent on 11th of March and repaid on 4th August following?

24. At what rate will the interest on £4127 10*s.* amount to £92 17*s.* 4½*d.* in a year?

25. What sum of money put out to interest for 6 years at 3½ per cent. will amount to £259 7*s.* 5*d.*?

26. At what rate per cent. will £1303 6*s.* 8*d.* amount to £1687 14*s.* 10*d.* in 10 years at simple interest?

27. Find the simple interest on £588 6*s.* 8*d*, at 3¼ per cent. for 5 yrs.

28. In what time will £527 10*s.* amount to £602 13*s.* 4½*d.* at 4¾ per cent. simple interest?

29. Find the simple interest on £1248 12*s.* at 3⅛ per cent. for 5 yrs.

30. At what rate per cent. (simple interest) will £2063 15*s.* amount to £2249 9*s.* 9*d.* in 2½ years?

139. (1). When principal, rate per cent. and time are given:

$$\text{I. Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$

(2). When interest, rate and time are given to find the principal:

$$\text{II. Principal} = \frac{100 \times \text{Interest}}{\text{Rate} \times \text{Time}}$$

(3). When principal, interest and time are given to find the rate:

$$\text{III. Rate} = \frac{100 \times \text{Interest}}{\text{Principal} \times \text{Time}}$$

(4). When principal, interest and rate are given to find the time:

$$\text{IV. Time} = \frac{100 \times \text{Interest}}{\text{Principal} \times \text{Rate}}$$

(5). When amount, rate and time are given to find the principal:

$$\text{V. Principal} = \frac{100 \times \text{Amount}}{100 + \text{Rate} \times \text{Time}}$$

Ex. 8. What principal will produce £45 10*s.* interest in 3½ years at 4 per cent.?

By II. we have the *principal* $= \frac{100 \times \pounds 45\ 10s.}{3\frac{1}{2} \times 4} = \frac{\pounds 4550}{14} = \pounds 325$ *Ans.*

Ex. 9. At what rate per cent. will £325 produce £45 10*s.* in 3½ years?

By III. we have, *rate* $= \frac{\cancel{100} \times \pounds 45\ 10s.}{\cancel{325} \times 3\frac{1}{2}} = \frac{4 \times \pounds 45\ 10s.}{13 \times 3\frac{1}{2}} = \frac{4 \times \cancel{\pounds 45\ 10s.}}{\cancel{45\frac{1}{2}}} =$ 4 *Ans.*

Ex. 10. What principal will amount to £370 10*s.* at 4. per cent. in 3½ years?

By V. we have, *principal* $= \frac{100 \times \pounds 370\ 10s.}{100 + 4 \times 3\frac{1}{2}} = \frac{\pounds 37050}{\cancel{114}} = \frac{\pounds 6175}{19} =$ £325 *Ans.*

DISCOUNT.

140. Discount is an allowance made for the payment of money before it becomes due.

If I owe my friend £105, which I am to pay him at the end of a year; he cannot expect me to pay before the proper time, except he pays or abates me a certain sum in consideration of my paying him before he is legally entitled to payment. Supposing that he requires his money presently, he will very probably be satisfied with £100 instead of waiting until one year elapse for £105.

The £100 he gets from me is called the *Present worth* of the £105; the £5 which my friend abates, is called the *Discount*; and, like Interest, it is calculated at a per centage.

Properly speaking the *Present worth* is that sum which, if put to interest at the given rate and for the given time, would amount to the sum due. £100 put to interest for a year, at 5 per cent., will amount to £105; therefore £100 is the *true* present worth of £105 due 1 year hence at 5 per cent per annum.

141. *To find the present worth accurately.*

Rule.—I. Say £100 + its interest at the given rate and for the given time : the given sum :: £100 : the present worth.

Ex. 1. What is the present value of a bill of £550 payable in 2 yrs., at 5 per cent.?

The amount of £100 in 2 yrs., at 5 per cent., is £110, consequently £100 is the *present worth* of £110 due in two years; hence the reason of the statement.

£110 : £550 :: £100

$$\frac{\cancel{5}\cancel{5}\cancel{0} \times 10\cancel{0}}{\cancel{1}\cancel{1}\cancel{0}} = £500 \textit{ Ans.}$$

Or, since the present worth of £110 is £100, the present worth of £1 is $=\frac{100}{110}=\frac{10}{11}$; hence £550 × $\frac{10}{11}$ = £500 *Ans.*

A bill is not *legally* payable in Great Britain and Ireland until 3 days, called 'Days of Grace,' after the time mentioned in it. These days must be included in the calculation. When a bill becomes *nominally* due on the 29th, 30th, or 31st, of a month which has not so many days, the last day of the month is taken; therefore the bill will be legally due on the 3rd of the following month. Bills falling legally due on Sunday are paid in Great Britain and Ireland on Saturday.

There is another kind of discount or abatement sometimes made to the purchaser of goods, in consideration of ready money payment. This gives the seller the advantage of a cash payment, while it brings a small reduction in the cost to the buyer.

142. Bankers and merchants invariably calculate the present worth of bills by the following rule :—

RULE.—II.—1°. Find the time from the day on which the bill is discounted, until the day it will be *legally* payable.

2°. The interest of the amount of the bill for this time, and at the given rate, is the discount charged.

Ex. 2. A bill of £120 is drawn on 14th May, at 4 months, and discounted on 3rd August; what ready money must be given for it, at 5 per cent. ?

The time from 3rd August till 17th (14th + 3 Days of Grace) September, is 45 days; now the interest of £120 for 45 days is 14*s*. 9$\frac{39}{73}$*d*., the *discount*. Hence £120 − 14*s*. 9$\frac{39}{73}$*d*. = £119 5*s*. 2$\frac{34}{73}$*d*., the *present worth*.

Since by this rule the discount of £105 for a year, at 5 per cent., is £5 5*s*., and £5 by Rule I.; it follows that the discount charged by merchants and bankers is too great. On comparing both results, it will be evident that the interest on any bill, as found by Rule II., exceeds the *true* discount by the interest upon the *true* discount.

Ex. 1. What is the discount on, and the present worth of, a bill of £250 drawn on 13th March at 6 months, discounted on 5th July at 5½ per cent.

This bill will be legally due on 16th September; and from 5th July to 16th September is

July 26
Aug. 31
Sept. 16
73 days

Then the interest of £100 for 73 days at $5\frac{1}{2}$ per cent. $= \frac{1\cancel{00}\times\cancel{73}\times 5\frac{1}{2}}{\cancel{36500}} = \frac{11}{5\times 2} = \frac{11}{10} = £1\frac{1}{10}$; therefore $101\frac{1}{10} : 1\frac{1}{10} :: 250 :$ £2 14*s.* $4\frac{276}{337}$*d.* discount; wherefore £250 − £2 14*s.* $4\frac{276}{337}$*d.* = £247 5*s.* $7\frac{61}{337}$*d.* the present worth.

Ex. 2. Find the difference between the interest and discount on a bill of £363 2*s.* 6*d.* due 9 months hence, at 5 per cent. per annum.

Since interest on £100 for 9 m. at 5 p. c. = £$3\frac{3}{4}$, we have true discount $(a) = \frac{£363\frac{1}{8}\times 3\frac{3}{4}}{103\frac{3}{4}}$. Now, the interest on this sum (see p. 163) is the *ans.* $(b) = \frac{£363\frac{1}{8}\times 3\frac{3}{4}\times 9\times 5}{103\frac{3}{4}\times 1200}$

$$=\frac{\overset{7}{\overset{\cancel{581}}{\cancel{2905}}}\times\cancel{15}\times\cancel{4}\times 9\times\cancel{5}}{8\times\cancel{4}\times\underset{\cancel{83}}{\cancel{415}}\times\underset{\underset{16}{\cancel{240}}}{\cancel{1200}}} = \frac{63}{128}\text{ £} = 9s.\ 10\tfrac{1}{8}d.$$

Ex. 3. The difference between the interest and discount on a certain sum due $1\frac{1}{4}$ year hence, at 4 per cent. is 7*s.* 6*d.*; find the sum.*

Since discount on £105 at the same rate and for the same time is £5, and interest on same sum

$$=\frac{\overset{21}{\cancel{105}}\times\cancel{4}\times\cancel{1\tfrac{1}{4}}}{\underset{\underset{\underset{4}{\cancel{16}}}{\cancel{20}}}{\cancel{100}}} = £5\tfrac{1}{4}$$

Now since the difference on £105 = 5*s.*, we have
5*s.* : 7*s.* 6*d.* : : £105 : £157 10*s.* *Ans.*

Ex. 4. The discount on a bill of £470 17*s.* 10*d.* due 4 years hence is £85 12*s.* 4*d.*; find the rate per cent. calculated.

Here £470 17*s.* 10*d.* — £85 12*s.* 4*d.* = £385 5*s.* 6*d.* the present worth; again, £85 12*s.* 4*d.* ÷ 4 = £21 8*s.* 1*d.* discount for 1 year; ∴ £385 5*s.* 6*d.* : 100 : : £21 8*s.* 1*d.* : $5\frac{5}{9}$ *Ans.*

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* Given Dec. 30. 1876.

XXXV.

1. Find the discount on, and the present worth of, a bill of £743 10*s.* drawn on 25th Sept. 1862, at 10 months, discounted 15th March following at 5 per cent.

2. What is the difference between the simple interest and discount on a bill of £1000 drawn at 9 months at 8½ per cent.?

3. How should a hatter sell hats for ready money, which he sells at 18*s.* giving a year's credit, reckoning discount at 8 per cent.?

4. Explain the difference between interest and discount, and find the true present worth of £553 15*s.* due 2 years hence, at 5⅜ per cent.

5. Find the present worth of £2674 6*s.* due 3 years hence at 4⅗ p. c.

6. What is the true discount on £1186 5*s.* 11*d.* due 8 months hence, the rate of interest being 5¾ per cent. per annum?

7. Find discount on £594 14*s.* 9*d.* due 8 months hence, at 5¾ per cent.

8. Find the discount on £1511 7*s.* 1*d.* payable 3¼ years hence, at 4 per cent. simple interest.

9. Find the discount on £2267 0*s.* 7½*d.* payable 3¼ years hence, at 4 per cent. simple interest.

10. Required the present value of £430 due 4 months hence, at 4 per cent. per annum.

11. The discount on a sum due one year hence at 5 per cent. per annum interest is £15: what is the sum?

12. The retail price of a book is 24*s.*, and 25 are sold for £20 8*s.*: what rate per cent. of discount is allowed?

13. A man buys goods at noon every day in the year, and pays for them at noon the following New Year's Day, what was the average length of his credit? If money be worth 5 per cent. per annum, find the ratio between the price paid by him, and that by a ready money customer.

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143. COMMISSION, INSURANCE, BROKERAGE, &c., like *Interest* and *Discount*, are usually calculated at a per-centage.

COMMISSION is paid to an agent for transacting business.

Life Insurance secures a sum at death or some fixed age, by paying an annual premium.

The *Policy of Insurance* is the legal deed or document of agreement between the parties concerned. The whole annual payment is called the PREMIUM.

Insurance from *sea risk*, or *Marine Insurance*, is paid on the value of the cargo or vessel for one voyage only.

144. *To find the commission, &c.*

RULE.—Just proceed as in Simple Interest.

Ex. 1. What is the insurance on £853 12*s.*, at £4 6*s.* 8*d.* per cent.?

In this example we multiply by the rate, and then divide by 100.

£853 12*s.*
4
s. *d.* 3414 8
6 8 = ⅓ = 284 10 8
1,00)36,98 18 8
£36 19 $9\frac{11}{25}$ *Ans.*

Ex. 2. What is the brokerage on £7360 16*s.* 8*d.*, at 5*s.* per cent.?

Here $\frac{5s.}{£100}=\frac{5}{2000}=\frac{1}{400}$; hence we divide by 400 for the answer. £7360 16*s.* 8*d.* ÷ 400 = £18 8*s.* 0½*d.* *Ans.* Or, since 5*s.* = £¼, we have,—

$$\begin{array}{lrrr} & £ & s. & d. \\ & 7360 & 16 & 8 \\ \hline s. & & & \\ 5=\frac{1}{4}= & 1840 & 4 & 2 \\ \hline \frac{1}{1,00}= & £18 & 8 & 0\frac{1}{2}\ \textit{Ans.} \end{array}$$

XXXVI.

1. What premium must be paid for insuring goods to the amount of £6500, at 6 per cent.?

2. What is the brokerage on £3964 14*s.*, at 15*s.* per cent.?

3. What is the commission on goods worth £767 14*s.* 7*d.*, at 7½ per cent.?

4. At 2*s.* 6*d.* per cent., what will be the brokerage on £4500?

5. Calculate the brokerage on £3907 10*s.*, at 2*s.* 6*d.* per cent.

6. What is the premium of insurance, at 5 per cent., on goods worth £410 10*s.*?

7. A person aged 26 insures his life for £2340, at £2 9*s.* 6*d.* per cent.; what annual premium must he pay?

8. A merchant insures a cargo whose value is £4500, at 5 per cent.; for what sum should he insure so as to cover both the loss of the cargo and premium?*

9. How much must be insured on goods worth £1805, at 5 per cent., so that the value of the goods and the premium may be repaid in case of loss?

10. What is the premium on a policy of £2475, on a life of 24 years, at £2 2*s.* 4*d.* per cent. for that age?

11. What is the commission on £998 16*s.* 8*d.*, at £7 12*s.* 6*d.* per cent.?

EQUATION OF PAYMENTS.

145. When a person owes several debts payable at different times, to find the time at which the whole might be paid without loss to either party.

Rule.—1°. Multiply each debt by the time which should elapse before it will become due.

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* If £100 were insured, only £95 would be actually received, since £5 premium was paid; therefore the insurance for £100 will cover a loss of £95 only. Now, if an insurance for £100 covers a loss of £95, what insurance will cover a loss of £4500?

2°. Then divide the sum of the products by the sum of the debts.

Various Departments.

Ex. 1. Find by any rule with which you are acquainted the sum which, paid one year hence, will be an equivalent for £200 due now, and £600 due at the end of two years, the rate of interest being £5 per cent.

The present value of £600 due two years hence is £$545\frac{5}{11}$: then the interest of £200 + £$545\frac{5}{11}$ for a year = £$37\frac{3}{11}$: therefore 200 + $545\frac{5}{11}$ + £$37\frac{3}{11}$ = £782 14*s.* $6\frac{6}{11}$*d.* *Ans.*

Ex. 2. One person owes another £50, payable in 6 months; £60 payable in 8m.; and £90, payable in 4m.: at what time may all be paid without loss or gain?

Here the sum of the products is 1140, which, divided by 200, the sum of the debts, gives 5·7 months.

$$\begin{array}{r} 50 \times 6 = 300 \\ 60 \times 8 = 480 \\ 90 \times 4 = 360 \\ \hline 2{,}00)11{,}40 \\ \hline \end{array}$$

Ans. 5·7 months

XXXVII.

Various Departments.

1. Explain any rule with which you are acquainted for determining 'Equated Time of Payment,' reckoning either simple or compound interest, and apply it to find what sum paid two years hence will be an equivalent for £100 due one year hence, and £500 due three years hence, the rate of interest being 5 per cent.

Not Proposed.

2. If I owe my friend £80, payable in 4m.; £70, payable in 6m.; £90, payable in 8m.; and £120, payable in 10m.: in what time may I pay him the whole debt at once?

3. Required the equated time for the payment of three debts: one, £700, payable in 6m.; another of £800, payable in 9m.; and a third of £600, payable in 10m.?

4. A debt is to be paid: $\frac{1}{3}$ at the end of 2m.; $\frac{1}{3}$ at 4m.; and the remainder at 12m.: what is the equated time for the payment of the whole?

5. One-half of a debt of £2000 is payable in 10m.; $\frac{1}{4}$, in 12m.; $\frac{1}{8}$, in 16m.; and the remainder, in 20m.; when might the whole be paid at one payment?

THE STOCKS.

At any time when Government cannot obtain money by ordinary taxation, equal to the exigencies of the State, it contracts a loan, at some *fixed rate* of interest, making an entry in the Bank books that each lender owns so much of the National Debt as stands opposite his name. When raising a new loan Government advertises that it is willing to borrow the amount in certain specified sums, at a certain fixed rate of interest; and intending lenders forward tenders of the sum in cash they are willing to pay to become a national creditor, for each £100. The amount of Stock one possesses can be sold or transferred like any other property, the purchaser having his name entered in the Bank books, and receiving the usual *half-yearly* interest, or dividend, as it falls due. The whole amount of money borrowed by Government, and on which it guarantees interest, is called the NATIONAL DEBT.*

Borrowed at various rates of interest much inconvenience arose in consequence, and an average rate (3½) was made in 1757 by the "Consolidated Annuities Act;" hence the term "Consols." In 1822 the late Sir R. Peel reduced the interest on £140,000,000 of the NATIONAL DEBT from 5 to 4 per cent.; and in 1824 a further reduction was effected on a portion of this Stock from 4 to 3½ per cent. At the present time the rates are 3½, 3, and 2½ per cent.

Exchequer Bills are a kind of Promissory Notes issued by the Exchequer to meet some temporary expenditure unprovided for in the Annual Estimates, the interest being usually 2*d.* per cent. per day. When afterwards it is found inconvenient to pay them off, they are "Funded," *i.e.*, converted from a floating to a permanent debt. Hence a holder of Government Stock is said to have money in the *Funds*. It is obvious that if the rate of interest for money in mercantile transactions, and the stability of the State remained invariable, the price of £100 Stock would remain the same; but as the value of money, like the value of any other property fluctuates, and as the most stable Governments are subject to political perturbations affecting their credit more or less, the price of Stocks varies from time to time. Notwithstanding the enviable stability of the British Empire, the announcement of the Mutiny at the Nore, May

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* The Bank of England, projected by William Paterson in 1793, was established the following year, and lent at 8 per cent. its capital, consisting of £1,200,000, to the Government, then greatly embarrassed. This sum, with 3 millions to which the £1,300,000 belonging to the London merchants, which Charles II. seized and appropriated in 1672, constituted the original *National Debt*, which increased from time to time. For instance, in the terrible struggle against the first Napoleon, we expended about 550 millions; and the Russian War, 1854-5, cost the Government about 30 millions.

1798, was followed by an extreme depression of the 3 per cents. to 47½. When trade is dull, there is a disinclination to employ capital in new enterprises, money is likely to lie in Bank, the rate of interest to become very low, and the Funds to rise. On the contrary, when trade is flourishing, a speculative spirit pervades society, money is usually invested with cheerfulness, and the Funds are likely to fall. If, however, dull and depressed markets continue for some time, money will gradually become crippled in circulation and even scarce, giving rise to a feeling of discontent, which results in a general fall in the price of all Stocks. As a seller of Government Stock or of shares in public companies is not likely to know a purchaser, and as certain formalities are necessary to render the transfer legal, a number of persons called *Stockbrokers** devotes their time to this class of agency, charging 2*s.* 6*d.* for the sale or purchase of £100 Government Stock, 10*s.* per cent. on the price of Railway Stock, and a sum fixed by the Stock Exchange for the sale or purchase of shares depending on their cash value. In most companies, with the exception of Railways, only a portion of the capital is actually paid by the Shareholders, say £10 or £20 out of £50 or £100, and on this sum the interest is of course always calculated. Every shareholder, however, is liable, in the event of the company becoming insolvent, for the remainder of the unpaid capital on his own shares, or at least for so much of it proportionately as will liquidate the company's debts. It is evident that such a practice, within certain restrictions, combined with a wealthy and efficient directorate, is calculated to inspire confidence in a company. When a company has been some time in operation, and when its business has become moderately developed, an increase of capital, in most instances, becomes necessary. This increase can be effected by making a "call" on the shareholders to pay a certain sum on each share, or by issuing a number of new shares at a price considerably below what they would bring in the open market, to each shareholder, giving him one new share for every three or four old shares in his name. When Railway and some other companies have expended all their capital, and require more money, they usually raise it by what are called *preference* shares, *i.e.*, they guarantee the payment of their interest by a first charge on their earnings. Additional capital is often raised by giving the lenders still greater security, such money being usually termed *debentures*, *obligations*, *&c.* Hence it frequently happens that the capital or paid-up Stock of a company consists of several distinct portions, some of which are valuable, and others almost worthless. Irrespective of the causes for variation in price of Stock, above enumerated, may be mentioned the approach of the time for the payment of the half-yearly (or in the case of some good mines, &c., the quarterly) dividend to which the purchaser is entitled if he buy before a certain date. Dividends are usually paid to the shareholders by cheques or orders on the company's bankers; and in case of *Foreign Loans*, which are usually payable after a certain period, there are *coupons* attached to the *bond*, one of which becomes payable half-yearly, according to date.

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* The wisdom of our forefathers rendered "Stockjobbing" illegal, *i.e.*, buying and selling for a rise or a fall, without contemplating an *actual* purchase or sale, but merely receiving or paying the "difference" on the "account" day, which occurs twice in every month. But the restrictions were removed by a five-line Act of Parliament passed several years ago. A buyer for a rise is termed a "bull," and one for a fall a "bear." The "account" in Consols is monthly.

DIVIDENDS.—The *dividend* is the amount of interest paid on stock or shares: hence when the time of its payment approaches the stocks will rise: hence also the meaning of the letters '*Ex. Div.*' or 'x. d.' (exclusive of dividend), which are attached to the price of shares in newspapers. When £100 stock sells for £100 cash, it is said to be at *par;* when it sells for more than £100 cash, it is at a *premium;* and when it sells for less than £100 cash, it is at a *discount.*

In Ireland, when a person buys Government Stock, he must pay the seller the portion of dividend that has accrued since the previous time of payment. On the 3 per cent. consols, the dividends are paid on 5th January and 5th July, and on new threes on 8th of April and 13th October.

148. PURCHASES AND SALES OF STOCK.

Ex. 1. How much stock can a person purchase in the 3 per cents. at $92\frac{1}{8}$ for £6457 10*s.*, brokerage included?

Here £$92\frac{1}{8}+\frac{1}{8}$ (the brokerage) = £$92\frac{1}{4}$, the sum which will purchase £100 stock: then $\frac{\text{£}6457\ 10 \times 100}{92\frac{1}{4}}$ = the amount which can be purchased for £$6457\frac{1}{2}$ (10*s.* = $\frac{1}{2}$) = $\frac{6457\frac{1}{2} \times 100}{92\frac{1}{4}} = \frac{\cancel{12915} \times \cancel{4} \times 100}{\cancel{369} \times \cancel{2}}$. (This is found by reducing the two mixed numbers, $6457\frac{1}{2}$ and $92\frac{1}{4}$ to improper fractions, and writing the denominator of the former below and that of the latter above the line) = $\frac{\cancel{1435} \times 2 \times 100}{\cancel{41}} = 35 \times 2 \times 100 =$ £7000. *Ans.*

By statement, $92\frac{1}{4}$: £6457 10 :: £100 : £7000.

Ex. 2. How much will a person receive for £7000 in the 3 per cents. at $92\frac{5}{8}$, brokerage $\frac{1}{8}$ per cent.?

Every £100 stock will realise £$92\frac{5}{8}-\frac{1}{8}$ (the brokerage) = £$92\frac{1}{2}$: then $\frac{\text{£}7000 \times 92\frac{1}{2}}{100} = \frac{\cancel{7000} \times 185}{\cancel{100} \times \cancel{2}} = 35 \times 185 =$ £6475. *Ans.*

XXXVIII.

Find the amount of stock which can be purchased by investing:—

1. £6802 in the 3 per cents., at 76.
2. £6129 in the 4 per cents., at $85\frac{1}{8}$.
3. £5048 in the $3\frac{1}{2}$ per cents., at $78\frac{7}{8}$.

4. How many shares can be purchased in the Midland railway for £14701 10*s.*, at $148\frac{1}{2}$.

5. How many Bank shares can be purchased for £4600 at $28\frac{3}{4}$, brokerage $\frac{1}{8}$ per share?

Find how much sterling money will be realised by selling:—

6. £1850 in the 3¾ per cents. at 87⅛.
7. £1200 in the 4 per cents. at 92⅝.
8. £1550 in the 3 per cents. at 75⅜.
9. £4770 Bank stock (£10 shares) at 35½, brokerage (⅛) included.
10. £6850 in railway shares, at 76¼ per cent.

149. INCOME OF INVESTMENTS.

Ex. 3. What yearly income will a person secure by investing £1991 15*s.* in the 3 per cents., at 96⅜?

Here 96⅜ gives £3 income, whence 96⅜ : 1991¾ : £3

$$\frac{771}{8} : \frac{7967}{4} :: 3.$$

$$\frac{\cancel{8}}{771} \times \frac{7967}{\cancel{4}} \times 3 = \frac{47802}{771} = £62. \quad \textit{Ans.}$$

Ex. 4. How much should a person invest in the 3½ per cents. at 78, to secure an income of £154?

Every £78 invested secures an income of £3 10*s.*: hence,

$$\begin{array}{ll} £3\ 10 : & £154 :: 78 \\ \underline{\qquad 2} & \underline{\quad 2} \\ \cancel{7} & \cancel{308} \\ 1 & 44 \times 78 = £3432. \quad \textit{Ans.} \end{array}$$

What income will arise from investing:—

11. £2560 8*s.* 9*d.* in the 3 per cents., at 72⅛.
12. £2392 16*s.* 3*d.* in the 3 per cents., at 73⅝.
13. £3963 4*s.* 9½*d.* in the 4 per cents., at 98⅞.
14. £4374 12*s.* 6*d.* in Railway shares, paying 5 per cent., at 110¾.
15. £1126 8*s.* 9*d.* in Union Banks, at 28⅛, including brokerage, and paying £2½ per share.

150. RATES OF INTEREST.

Ex. 5. What is the rate of interest when the 3 per cents. are at 80?

Here £80 gives £3: hence £100 will give

$$\frac{100 \times 3}{80} = \frac{300}{80} = 3\tfrac{3}{4} \quad \textit{Ans.}$$

What rate of interest per cent. per annum will be received by investing:—

16. In the 3 per cents. at 84: 4 per cents. at 90.
17. In the 3½ per cents. at 88: 4 per cents. at 96.
18. In railway 5 per cent. preference shares, at 105⅝.
19. In £10 shares Bank stock, paying 12 per cent., at 30.

151. The transfers of stock may be worked by Compound Proportion, as below.

Ex. 6. A person transfers his capital from a $3\frac{1}{2}$ per cent. stock at 77 to a 4 per cent. stock at $93\frac{1}{2}$: find the increase or decrease per cent. in his income.

$$\begin{array}{l} 3\frac{1}{2} : 4 :: 100 \\ 93\frac{1}{2} : 77 \end{array}$$

$$\frac{4 \times \overset{\cancel{7}}{\cancel{77}} \times 100 \times 2 \times 2}{\cancel{7} \times \underset{17}{\cancel{187}}} = \frac{1600}{17} = 94\tfrac{2}{17}.$$

Here we take £100 as his first income. Now, by transferring from a $3\frac{1}{2}$ to a 4 per cent. stock, a person must increase his income if the prices of both be the same; but he pays $93\frac{1}{2}$ for the 4 per cents. and only gets 77 for the 3 per cents., consequently his income will be less in the ratio of $93\frac{1}{2} : 77$; hence the reason of the statement: in every case the income will be in an inverse ratio to the prices of the stock. We find $94\frac{2}{17}$ his income after the transfer, therefore he loses $100 - 94\frac{2}{17} = 5\frac{15}{17}$ per cent.

Ex. 7. A person sells £7300 out of the 3 per cents. at 96, and invests his money in railway 5 per cent. stock at £106 13s. 4d.: find how much his income is increased.

$$\text{First income} = \frac{£7300 \times 3}{100} = £219$$

$$\begin{array}{l} 3 : 5 :: £219 \\ 106\frac{2}{3} : 96 \end{array}$$

$$\frac{\overset{3}{\cancel{96}} \times \cancel{5} \times 219 \times \cancel{3}}{\cancel{3} \times \underset{\underset{2}{10}}{\cancel{320}}} = \frac{657}{2} = £328\ 10s.$$

∴ Increase in income = £328 10s. − £219 = £109 10s.

The reason for this statement is precisely the same as that given for the statement of the previous example.

Ex. 8. What must be the market value of 5 per cent. stock in order that after deducting the income-tax of 6d. in the pound it may yield $4\frac{1}{2}$ per cent. interest?

$$4\tfrac{1}{2} : 5 :: 100$$
$$240 : 234$$

$$\frac{5 \times \overset{26}{\cancel{234}} \times 10\cancel{0} \times \cancel{2}}{\underset{12}{\cancel{240}} \times \underset{1}{\cancel{9}}} = \frac{1300}{12} = 108\tfrac{1}{3}. \quad \textit{Ans.}$$

In this instance, if we bought £100 stock for £100 cash, we would receive 5 per cent. interest; but since we only receive 4½ per cent. interest, the price of £100 stock must be more than £100 cash; hence the reason of the first ratio 4½ : 5. But in order to be able to afford to pay the income-tax we must pay less for the stock in the ratio of £1 to 19s. 6d. or 240 : 234.

Ex. 9. "Compare the rates of interest obtained by a person investing in the 3 per cents. at 82½ with that obtained by investing the same sum in the 3½ per cents. at 93½."

$$\text{Interest of £1 in the 3 per cents.} = \frac{3}{82\frac{1}{2}}$$

$$\text{,, £1 ,, } 3\tfrac{1}{2} \text{ ,,} = \frac{3\frac{1}{2}}{93\frac{1}{2}}$$

$$\therefore \text{ Required ratio} = \frac{3}{82\frac{1}{2}} : \frac{3\frac{1}{2}}{93\frac{1}{2}} = \frac{6}{165} : \frac{7}{187} = 6 \times \overset{17}{\cancel{187}} : 7 \times \overset{15}{\cancel{165}} =$$
$$\overset{2}{\cancel{6}} \times 17 : 7 \times \overset{5}{\cancel{15}} = 34 : 35.$$

Ex. 10. In a certain railway the total capital is £700,000 ; in addition to which the company borrows £98000 preference shares at 5 per cent., and £60000 debentures at 4¾ per cent. per annum. The annual receipts are £95208 6s. 8d., and the expenses are 60 per cent. of the receipts: what dividend will the shareholders receive after laying aside £1,000 as a reserve fund?

$$100 : (100 - 60) :: 95208\tfrac{1}{3} : \frac{\overset{2}{\cancel{40}} \times \overset{57125}{\cancel{285625}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{100}} \times 3} = \frac{114250}{3}$$

= 38083⅓ cash after deducting the expenses.

Interest of £98000	at 5	per cent.	=	£4900
,, 60000	at 4¾	,,	=	2850
Reserve	. . . .		=	1000
				£8750

Now £700000 : 100::(£38083⅓ − £8750) : $\frac{\cancel{100} \times 88\cancel{000}}{7\cancel{00000} \times 3} = \frac{88}{21} = 4\frac{4}{21}$. *Ans.*

Education and Constabulary Offices.

20. When the 3 per cents. are at 89¾, how much can be bought for £540, allowing for commission ⅛ per cent. upon the stock bought?

21. A person who has £2954 in the 3 per cents. at 80½, transfers his capital into the 4 per cents. at 98: find the alteration in his income.

22. When the 3 per cents. are at 91¾, find how much can be bought for £540, allowing ⅛ per cent. for commission upon the stock bought.

23. A person sells £5000 consols at 94⅞, and on their rising he sells £5000 more at 95⅝; on their again rising, he buys back the whole £10000 at 96. What does he lose on the transaction?

24. A person sells out of the 3 per cents. at 96, and invests his money in railway 5 per cent. stock at par: find how much per cent. his income is increased?

Inland Revenue.

25. A person sells £7500 railway stock, which pays 6 per cent. at 120, and invests half the proceeds in the 3 per cent. consols at 96, and the other half in the South Eastern stock, which pays 5½ per cent. at 90. How is his annual income affected?

Various Departments.

26. A person who has £1475 in the 3 per cents. at 75¼, transfers it to the 5 per cents. at 110⅝: what is the alteration in his income?

27. A person invests £9075 in the 3 per cents. at £90¾, and on their rising to 91 transfers it to the 3¼ per cents. at 93½: how is his annual income affected?

28. If the 3½ per cents. be at 91, how much must I invest in them in order to have a yearly income of £932, after paying 7*d.* in the pound income-tax?

29. A person invests £9075 in the 3 per cents. at 90¾, and on their rising to 91 transfers it to the 3½ per cents. at 97½: what increase does he make thereby in his annual income?

30. A person sells £6000 3 per cent. consols at 92¼, and invests this sum in railway stock, paying 5½ per cent. at 103⅛. Find how his income is affected (neglecting fractions of a penny).

Admiralty.

31. A person sells £8000 3 per cent. consols at 92⅞, and invests the proceeds in railway shares (paying 5 per cent.) at 102: how much is his income increased?

Various Departments.

32. If a person buy 3 per cents. at 92, and after receiving 6 months' dividend, sell out at 93, paying $\frac{1}{8}$ per cent. brokerage in each case, what rate of interest per cent. per annum has he for his money?

33. A person invests in a company at such a price that when the annual dividend is 4s. 9d. he gets 5 per cent. for his money; find the rate of interest per cent. when the dividend is only 2s. 9d.

34. A person sells out of the 3 per cents. at 86$\frac{1}{4}$, and gets £3400, if he invest one-half this sum in the 4 per cents at 97, and the remainder in the 3$\frac{1}{2}$ per cents at 94, find the alteration in his income.

35. A person pays on 1st January, 1867, £100 for 50 shares of £5 each in a Joint Stock Company; on 1st February, 1868, he pays a call of 10s. per share, and on 30th June, 1868, the company is wound up, when he must pay a call of 15s. per share. Find how much he loses if, instead of this, he had placed his money at interest at 5 per cent. per annum.

36. What sum would be saved annually if the interest on a public debt of £8000000 were reduced from 3$\frac{1}{4}$ per cent. to 3 per cent.? If, in consequence, the price of this stock fell from 101 to 95$\frac{5}{8}$, how much would the whole property of the fundholders be diminished?

37. A company, whose capital is £2500000, pays 6$\frac{1}{4}$ per cent. per annum to its shareholders, clear of income-tax at 4d. in the pound; find the total annual receipts, the expenses being 60 per cent. of the gross receipts.

38. How much must a man invest in the 3 per cents. on the birth of a child, that the stock and accumulated dividends may amount to £10000 when the child comes of age, if he buy at 92$\frac{1}{2}$ and sell at same price?

39. "A landlord has an estate that brings him in £4,000 a year; but this income is liable to deductions for rates and repairs to the extent of 15 per cent.: if he sell the estate at 24 years' purchase on the gross income, and invest in the 3 per cents. at 97$\frac{1}{2}$, how will his income be affected?"

40. A person has £500 a year from shares in Water Works, which pay 10 per cent. dividend; he sells so many of these shares at a premium of 50 per cent. as when the proceeds are invested in the 3 per cents. at 90 bring him £200 a year; what income has he from the remainder of his shares?

41. The receipts of a Railway Company are apportioned as follows: 48 per cent. for working expenses, 10 per cent. for the reserve fund, a guaranteed dividend of 5 per cent. on one-fifth of the capital, and the remainder, £32000, for division amongst the holders of the rest of the stock, being a dividend at the rate of 4 per cent. per annum; find the capital and the receipts.

42. Compare the 3 per cents. at 89⅝, and railway shares paying 5 per cent. at 102⅝; compare also Midland Railway shares paying 7½ per cent. at 149⅝ and North-Easterns paying 9 per cent. at 180½.

PROFIT AND LOSS.

152. When a person sells an article for the same money which he paid for it, he has no profit: when he sells it for twice its cost he gains 'cent. per cent.' If an article be sold for half as much more as was paid for it, the gain is 50 per cent., that is, £50 on every £100 of outlay: if sold for one-fourth as much more than its cost the gain is 25 per cent., and so on. It is also evident that, if half the number of articles be sold for what the whole cost, the gain on them is 'cent., per cent.' If three-fourths the number of articles be sold for what the whole cost, the gain is 33⅓ per cent.; if ⅘ of the number, the gain is 25 per cent.: if ⅗, it is 66⅔ per cent., and so on.

The method of solving questions in this rule will be understood from the following examples.

Various Departments.

Ex. 1. A person having £5704 18*s.* 4½*d.* lays out one-third in goods, which he sells for £2316 5*s.* 10*d.*: how much has he at last, and how much has he gained?

	£	s.	d.	
From	5704	18	4½	his capital
Take ⅓ of it =	1901	12	9½	
Cash remaining =	3803	5	7	
Selling-price of goods =	2316	5	10	
First cost of same =	1901	12	9½	
His gain =	414	13	0½	
Original capital =	5704	18	4½	
	£6119	11	5	sum he has at last. Ans

Education Office (London).

Ex. 2. If 20 lbs. 4 oz. of tea at 5*s.* 10*d.* per pound be mixed with 46 lbs. 6 oz. at 4*s.* 3*d.* and 35 lbs. 6 oz. at 6*s.* 8*d.* and the mixture be sold at 6*s.* per pound, what will be the gain or loss, and what the gain or loss per cent.?

	£	*s.*	*d.*
20 lbs. 4 oz. @ 5*s.* 10*d.* =	5	18	$1\frac{1}{2}$
46 lbs. 6 oz. @ 4*s.* 3*d.* =	9	17	$1\frac{1}{8}$
35 lbs. 6 oz. @ 6*s.* 8*d.* =	11	15	10
102 lbs. prime cost of =	27	11	$0\frac{5}{8}$
Again, 102 lbs. @ 6*s.* =	30	12	
Subtract first cost and the gain is	£3	0	$11\frac{3}{8}$

For the gain per cent. we have,

£27 11 $0\frac{5}{8}$: 100 :: £3 0 $11\frac{3}{8}$

8   8

220 8 5   24 7 7

20   20

4408   487

12   12

52901   5851

100

52901)585100 (£11 1*s.* $2\frac{24746}{52901}$*d.* *Ans.*

581911

3189

20

63780(1

52901

10879

12

130548(2

105802

24746

52901

Customs and Excise.

Ex. 3. By selling cloth at the rate of 12*s.* per yard £15 per cent. was gained. What per cent. would be gained when it was sold at the rate of 10*s.* 6*d.* per yard?

Here we have 12 : 115 :: $10\frac{1}{2}$ ·

$$\text{or}\quad 8 : 115 :: 7 : \frac{115 \times 7}{8} = \frac{805}{8} = 100\tfrac{5}{8};$$

$$\text{then } 100\tfrac{5}{8} - 100 = \tfrac{5}{8} = 12s.\ 6d.\ \textit{Ans.}$$

In the above example 12*s.* and £115 form a ratio, because they both express selling price at the same rate per yard.

The learner will do well to remember that the gain or loss per cent. is always calculated upon the *prime* or *first cost.*

XXXIX.

1. How must butter be sold which cost £3 14*s.* per cwt. to gain 16 per cent.?

2. A merchant sold 200 yards of cloth for £75, and gained 15 per cent.; what did the cloth cost him per yard?

3. If 10 per cent. be gained by selling butter at £3 6*s.* 8*d.* per cwt., what will be gained per cent. by selling it at 7½*d.* per lb.?

4. Bought 4 packs wool, at £130 10*s.* each, and sold the whole at 20 per cent. profit; what was the selling price per pack?

5. A person sells a horse for £60, and loses 20 per cent., while he should have gained 12 per cent.; how much did he pay for the horse, and what is his entire loss?

6. If I sell cloth at 15*s.* per yard, and lose 10 per cent., what should be the selling price per yard to gain 18 per cent.?

7. What is gained by selling 178 cwts. 2 qrs. 14 lbs. of beef at 6½*d.* per lb., which was bought at £2 13*s.* 4*d.* per cwt.?

8. By selling 178 cwts. 2 qrs. 14 lbs. at 6½*d.* per lb., I gain 13¾ per cent.; what was the first cost of the whole?

9. How much goods bought at 7½*d.* per lb., must be sold at a gain of £6 13*s.* 4*d.* per cent., to gain £63 12*s.* 3*d.*?

10. Bought 500 stones of sugar upon which I gained £14 11*s.* 8*d.*, and found I had gained 12½ per cent.; what was my buying and selling price per stone?

11. If 23 lbs. at 2*s.* are mixed with 27 lbs. at 3*s.* and the mixture is sold at 2*s.* 9*d.* per lb., what is the total gain. and what is the gain per cent. on the money laid out?

12 (*a*). A merchant bought 80 yards of cloth, 1¼ yards wide, at 15*s.* per yard; but the cloth having been wet, shrunk 5 per cent. in length and 5 per cent. in width. At what rate per square yard must he sell the remainder to gain 12 per cent. on his outlay?

Various Departments.

12. By selling an article for £9 10*s.* the seller loses 5 per cent. on his outlay; what would be his loss or gain per cent., if he sold it for £11 17*s.* 6*d.* ?

13. A grocer buys 3 cwts. of sugar at 5*d.* per pound, and 7 cwts. at $6\frac{1}{2}$*d.*; he sells $5\frac{1}{2}$ cwts. at $5\frac{1}{2}$*d.* per pound; at what rate per pound must he sell the remainder in order to make 50 per cent. on his whole outlay ?

14. A person buys teas at 3*s.* and 4*s.* the pound, and mixes them in the proportion of 4 to 7; what will he gain per cent. by selling the mixture at 4*s.* 2*d.* per pound ?

AVERAGES AND PERCENTAGES.

153. 'An Average is a mean or medium of two or more quantities,' and is usually found by dividing the sum of the quantities by the number of them. A *percentage* is a comparison of the gains or losses in trade, the increase or decrease in population, &c., with 100 as a common standard.

Education and Constabulary Offices.

Ex. 1. Find the average of $17\frac{1}{2}$, $25\frac{1}{4}$, $96\frac{3}{8}$, 10, 0, $42\frac{3}{4}$, 56 ; and express the fractional part decimally.

$$\begin{array}{r} 17\frac{1}{2} \\ 25\frac{1}{4} \\ 96\frac{3}{8} \\ 10 \\ 0 \\ 42\frac{3}{4} \\ 56 \\ \hline 247\frac{7}{8} = 247\cdot875 \end{array}$$

Then 247·875 ÷ 7 the number of quantities = 35·410714285 *Ans.*

Ex. 2. In a certain office there is one person receiving £2000, two who receive £1100 each, six who receive £400, twelve who receive £200 each, and eighteen who receive £90 each ; what is the average income of the persons employed in the office ?

The total salaries of all is £10620, and the total number of persons employed is 39; hence the reason of the annexed operation.

$1 \times 2000 = 2000$
$2 \times 1100 = 2200$
$6 \times 400 = 2400$
$12 \times 200 = 2400$
$18 \times 90 = 1620$

39   39)10620
78   (£272 6*s.* $1\frac{11}{13}$*d. Ans.*
282
273
90
78
12
20
39)240(6
234
6
12
39)72(1
39
33
$\frac{33}{39} \left(=\frac{11}{13}\right)$

India Office.

Ex. 3. In 1858 the value of £100 tithe rent charge, reckoned on the average price of corn in the 7 years immediately preceding, was £105; in 1859, reckoned in the like way, it is 3 per cent. more. If it were reckoned on the price in 1851 only, it would be but £69. What would it be if reckoned on the price in 1858 only?

seven years.
1851
1852
1853
1854 } £105 average.
1855
1856
1857

Total for these seven years $= 105 \times 7 =$ £735. For the year 1851 it was £69; therefore for the other six years the total was £735 − £69 = £666 (1852—57 inclusive).

Again, total for 7 years (1852–58) $= 108 \times 7 = 756$;
∴ the average for 1858 $= 756 - 666 = 90$. *Ans.*

Ex. 4. The populations of 3 towns in the year 1841 were 20325, 41304, and 6117; and in the year 1851 they had increased respectively 9, 10, and 12 per cent.; find the average population of the 3 towns in 1851.

The population of the first in 1851 was $\frac{\cancel{2}\cancel{0}\cancel{3}\cancel{2}5 \times 109}{\cancel{1}\cancel{0}\cancel{0}} = \frac{813 \times 109}{4} =$ 22154·25.

The population of second was $\frac{41304 \times 110}{100} = 413\cdot04 \times 110 =$ 45434·40.

The population of the third was $\frac{6117 \times \cancel{1}\cancel{1}\cancel{2}}{\cancel{1}\cancel{0}\cancel{0}} = \frac{6117 \times 28}{25} =$ 6851·04.

$$
\begin{array}{rl}
\text{Then } 22154\cdot25 & \\
45434\cdot4 & \\
6851\cdot04 & \\
\hline
3)\overline{74439\cdot69} & \text{Total population in 1851.} \\
\hline
24813\cdot23 & \text{Average pop. in 1851. } \textit{Ans.}
\end{array}
$$

Ex. 5. "A merchant sells 49 quarters of corn at a profit of 7 per cent., and 84 quarters at a profit of 11 per cent.; and if he had sold the whole at a profit of 9 per cent. he would have received £2 10s. 9d. less than he actually did; what was the price he paid for the corn?"

We have seen (Art. 152) that an increase in price corresponds to an increase in quantity. Now, increasing 49 qrs. by 7 per cent., we have:—

$$\frac{49 \text{ qrs.} \times 107}{100} = 52\cdot43 \text{ qrs.}$$

Increasing 84 qrs. by 11 per cent., we have:

$$\frac{84 \text{ qrs.} \times 111}{100} = 93\cdot24 \text{ qrs.}$$

Again increasing (49 + 84), according to the conditions of the question, 9 per cent., we have:—

$$49 + 84 = 133, \text{ and } \frac{133 \times 109}{100} = 144\cdot97$$

Now, the sum of the first two results should exceed this result by a quantity corresponding to the decrease in price, that is

$(52\cdot43 + 93\cdot24) - 144\cdot97 = 145\cdot67 - 144\cdot97 = \cdot7 =$ £2 10s. 9d.

Therefore £2 10s. 9d. = $\frac{7}{10}$ of the price per qr.

$$\therefore \textit{Ans.} = \frac{\pounds 2\ 10\ 9 \times 10}{7} = \frac{\pounds 25\ 7\ 6}{7} = \pounds 3\ 12\text{s. } 6\text{d.}$$

Ex. 6. "If the increase in the number of male and female criminals be 1·8 per cent., while the decrease in the number of males alone is 4·6 per cent., and the increase in the number of females is 9·8 per cent., compare the number of male and female criminals, respectively."

Every 100 male and female criminals increased to 101·8; but every 100 males decreased to 100 − 4·6 = 95·4, that is 101·8 − 95·4 = 6·4 below the per-centage average. And every 100 females increased to 109·8, that is 8 above the per-centage average. In order to make the increase counterbalance the decrease, we must have more males than females in the ratio of the differences just found; that is males: females : : 8 : 6·4 = 80 : 64 or 5 : 4.

XL.

Education Office (Dublin)

1. If by selling wine at 15*s*. per gallon I lose 10 per cent., at what price must it be sold to gain 15 per cent. ?

The following have been proposed in Education Offices (England and Ireland) and the Constabulary (Ireland).

2. Find the average of 13, 27, 0, 32, 106, 86; and express the fractional part decimally.

3. The population of five parishes being 1236, 452, 364, 516, and 3430 respectively, find what the population of a sixth parish must be in order that the average population of the six may be 1256·5.

4. A person has $\frac{1}{5}$ of a ship worth £3484 which is insured for 91$\frac{2}{3}$ per cent. of its real value; what would he lose in case of the ship being lost ?

5. The population of 3 towns in the year 1841 were 21326, 42324, and 6706; and in the year 1851 it was found that the first two had increased 12 and 10 per cent. respectively, and the last decreased 18 per cent.; find the average population of the 3 towns in the year 1851.

6. If a person sells 22 articles for the same money which he paid for 36, what does he gain per cent. on his outlay ?

7. Find the average of 13, 27, 0, 46, 72, 86; and express the fractional part decimally.

8. If goods which were bought at £2 5*s*. 10*d*. per cwt. be sold at £2 11*s*. 4*d*. what is the gain per cent. ?

9. If by selling an article at £1 1*s*. 9*d*. a pound I gain 16 per cent. on my outlay, what was its prime cost ?

10. What is the premium on a policy of insurance for £9626 11*s.* 3*d.* at £2 12*s.* per cent.?

11. A grocer buys 3 cwts. of sugar at 5*d.* per pound and 7 cwts. at 6½*d.*; he sells 5½ cwts. at 5½*d.* per lb.; at what rate must he sell the remainder to make 25 per cent. on his outlay?

12. An army lost 18 per cent. of its strength by disease and desertion, and then lost 14 per cent. of the remainder in battle; the number then remaining was 84624; of how many did the army originally consist?

13. In 1841 the population of Great Britain was 21476000, and that of Ireland 7310000. In 1851 the former had increased 8·45 per cent. and the latter had decreased 12·5 per cent. Find the increase per cent. in the population of the whole kingdom.

14. A ship valued at £14500 is insured at £3 10*s.* per cent., and her cargo, valued at £32000, is insured at £5 per cent. Find the whole cost of insurance.

15. Having given the following table:

Length of Railway in England in 1849.	Number of		
Miles.	First Cls. Passengers.	Second Class.	Third Class.
4355·5	6186899	18844152	24848312

Find (1) the average number per mile of each class.

16. (2) the percentage proportion of each of the three classes.

17. What per centage is 25 of 35?

18. What per centage is 210·5 of 21·05?

INVOLUTION.

154. If a number be multiplied by itself, the product is called the *second power* or *square* of that number. Thus, 6 × 6 = 36; 36 is the *second power* or *square* of 6: and since 10 × 10 = 100; 100 is the square of 10. The numbers 6 and 10 are said to be *involved*, and are called *roots* in reference to their powers: 6 is the *square root* of 36.

If the *square* or *second power* of any number be multiplied by its *root*, the product is the *third power* or *cube* of that number. If the *third* power be multiplied by the root, the product is the *fourth* power, and so on.

Thus, $36 \times 6 = 6 \times 6 \times 6 = 216$, the *third* power of 6; and $4 \times 2 = 2 \times 2 \times 2 = 8$, the *third* power or *cube* of 2; also $8 \times 2 = 16$, the *fourth* power of 2.

The number of times a root is to be taken as factor is indicated by an *index*, which also points out the *power* or *degree* to which the root is to be involved. Thus we say $6^2 = 36$; $2^2 = 4$; and $6^3 = 216$.

155. To raise a number to any power, multiply the number by itself, and this product again by the number, and so on, until the number has been used as multiplier a time *less* than the index. The second power of a number when multiplied by itself gives the fourth power. The second power multiplied by the third gives the fifth, and so on according to the sum of the indices of the powers multiplied.

Ex. 1. Raise 8 to the fifth power.

Here, $8 \times 8 = 64$, second power; $64 \times 8 = 512$, the third power; $512 \times 8 = 4096$, the fourth power; $4096 \times 8 = 32768$, the fifth power.

With fractions, raise their terms to the required power, and write the result as a fraction. Or, reduce vulgar fractions to decimals, and raise the decimal to the required power. It will generally be most convenient to reduce mixed numbers to improper fractions, before multiplying.

Involve the following to the powers indicated by their indices:—

XLI.

1. 22^3, 30^2.
2. $\cdot04^4$, $3{\cdot}5^2$.
3. $1{\cdot}3^3$, $1{\cdot}05^3$,
4. 44^2, $\frac{2}{3}^3$.
5. 36^3, 20^4.
6. 367^2, 19^4.
7. $1{\cdot}\dot{3}^3$, $\cdot\dot{2}\dot{7}^4$, $1{\cdot}8\dot{3}^3$.
8. $\frac{11}{13}^2$, $\frac{6}{7}^3$, $\frac{13}{17}^3$.
9. $1{\cdot}\dot{4}^2$, $3\,\dot{2}3076\dot{9}^2$.
10. $\left\{\frac{7{\cdot}\dot{9}7\dot{2}}{\cdot7\dot{6}7\dot{5}}\right\}^3$

EVOLUTION.

156. *Evolution* is the finding of *roots* when any power of these roots is given.

The second powers of the first ten numbers are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100; therefore all *square* numbers. that is, those whose roots can be *exactly* found, must end either in 1, 4, 5, 6, 9, or 0.

The *sign* $\sqrt{}$, when placed before a number implies that the square root of that number is to be taken: thus, $\sqrt{36}=6$; $\sqrt{100}=10$.

The extraction of any *other root* is indicated by writing its index within this sign: thus, for the extraction of the third root, write, $\sqrt[3]{}$; $\sqrt[3]{8}=2$; and $\sqrt[3]{64}=4$.

157. *To find the square root of a number.*

RULE.—1°. Divide the number into periods of two figures each, commencing at the units' place, proceeding towards the *left* in whole numbers, and towards the *right* in decimals.

2°. Find a square number equal to, or next less than the left-hand period; write it under the period, and place its root as a quotient figure.

3°. From the left-hand period, take the number written under it, and annex the next period to the remainder for a dividend.

4°. For a divisor, write a 0 after twice that part of the root already found; find how often this number is contained in the dividend; write the quotient figure in the *place* of the 0 in the divisor, as well as in the quotient.

5°. Then multiply the divisor, thus changed, by the last figure of the quotient, and write the product under the dividend and subtract.

6°. Bring down the next period to the remainder for a new dividend, and double the quotient for a new divisor as before. In finding the *decimals* of a root which has no decimal figures in the power, annex 00 for period: in circulators, bring down two of the circulating figures.

War Department.

Ex. 1. Extract the square root of 106929.

10,69,29
Trial divisor, 60 9
—— ——(327 *Ans.*
Divisor, 62) 169
Trial divisor, 640 124
—— ——
Divisor, 647) 4529
4529
——

In this example, first divide the number into periods and we get 10 for the *left-hand* period. The next less *square* number to which is 9; the root of 9 is 3, which is placed in the quotient. Next take 9 from 10, and bring down 69, the *next period*, to the remainder. For a divisor double 3, which gives 6, to which annex 0; then try how often 60 is contained in 169, and place the next quotient figure, 2, both in quotient and divisor, multiply 62 by 2, subtract 124 from 169, and bring down the next period. For a new divisor, multiply 32 by 2 = 64, which write, 640 for trial divisor, as before.

Ex. 2. Find the square roots of 81830116 and 197·739844.

In both these examples there is a 0 in the root, which is written in *divisor* as well as in quotient.

81,83,01,16
81
———(9046 *Ans.*
1804) 8301
7216
———
18086) 108516
108516
———

1,97·73,98,44
1
——(14·062 *Ans.*
24) 97
96
———
28·06)17398
16836
———
28122) 56244
56244
———

Ex. 3. Extract the square root of 5 and that of $3\frac{2}{3}$.

5 (2·23606...
4
——
42) 100
84
——
443) 1600
1329
——
4466) 27100
26796
——
447206) 3040000
2683236
——
356764...

$3\frac{2}{3} =$
$3\cdot\dot{6}$
1
——(1·9148...
29) 266
261
——
381) 566
381
——
3824) 18566
15296.
——
38288) 327066
306304
——
...

Note (*Contraction*).—The work may be shortened by dividing the part found into half the given number, and from the quotient take half the part found for remaining figures one less than the part found. In extracting the square root of a fraction it is often more convenient to take the root of both numerator and denominator.

XLII.

India Board.

1. Extract the square root of 2854·7649.

Competitive Examinations in Science and Art, &c.

2. Extract the square root of 406278·76.
3. " " $262\frac{11}{25}$.

Various Departments.

4. Find the square root of $5\frac{21}{225}$, and of ·00028561.
5. " " $33\frac{144}{225}$, of ·0144, and of $\cdot\dot{1}$.
6. " " $514\frac{185}{256}$, and of 240168·6049
7. " " 49660209, and of $378\cdot3024\dot{9}$.
8. " " 7·0032, " $42\cdot0336\dot{1}$.
9. " " 1293785, " $\cdot\dot{6}0493827\dot{1}$.

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158. *Extraction of the third or cube root.*

Ex. 1. Find the cube root of 108190397125.

We first divide the number into *periods* of *three* figures each, just as in Numeration. Then look for the nearest cube root of 108, the left-hand period, which is 4; after subtracting its cube we have 44 as remainder. To this we bring down the *next* period, 190; then take 300 *times* the square of 4, that is, 4800, for the *trial divisor:* then, 44190 divided by 4800 gives 7, the *next* figure of the root. For the complete divisor take 30 times the previous part of the root and multiply it by 7 = 84; to this add

$$
\begin{array}{r}
300 \times 4^2 = 4800 \\
\left.\begin{array}{r} 30 \times 4 \times 7 = 840 \\ {}^2 = 49 \\ \hline 5689 \\ 49 \end{array}\right\} \\
\hline
662700 \\
\left.\begin{array}{r} 30 \times 47 \times 6 = 8460 \\ 6^2 = 36 \\ \hline 671196 \\ 36 \end{array}\right\} \\
\hline
67972800 \\
30 \times 476 \times 5 = 71400 \\
5^2 = 25 \\
\hline
68044225
\end{array}
\qquad
\begin{array}{r}
108,190,397,125 \\
64 \\
\underline{}(4765 \\
44190 \\
39823 \\
\hline
4367397 \\
4027176 \\
\hline
340221125 \\
340221125 \\
\hline
\end{array}
$$

$7^2=49$: these *three* numbers added give 5689, which multiply by 7, and placing the product under the dividend subtract, and bring down the next period, and so on. For the *next divisor* add 840 and 49 and 5689 and 49, the four numbers connected by the bracket, to which add 00, as before, for the next *trial divisor*, 662700. Every *successive* divisor is found in the same manner, and the work will stand as in the example.

In extracting the cube root of fractions we proceed in a similar manner as in finding the square root; add three cyphers for decimal figures, and three circulating figures if the fraction be a circulator.

XLIII.

Extract the cube root of:—

1. 356853433357.
2. 915215·787829.
3. 673834158663.
4. $5164\frac{309}{343}$.

Constabulary.

5. Extract $\sqrt[3]{25}$ to 3 places of decimals.

Various Departments.

6. Extract the cube root of 134217728.
7. " " 12167.
8. " " 228099131.
9. " " 493039.

India Office.

10. Extract the cube root of 1194389981.

11. Find the cube root of 233·744896; and also the cube root of the last-mentioned number multiplied by ·027.

12. If the solid content of a cube is 37 feet 64 inches, find its surface, and the cost of painting its outside at $1\cdot\dot{4}$*s.* per square foot.

13. Find the value of: $\sqrt{1\frac{7}{9}}+\sqrt[3]{37\frac{1}{27}}-\cdot\dot{9}+\sqrt[3]{\cdot124\dot{9}}+\frac{\sqrt{2+1}}{\sqrt{3+2\sqrt{2+1}}}$

14. Simplify the following:

$$\sqrt{1\cdot44}+\sqrt{30\tfrac{1}{4}}-\sqrt[3]{1607\frac{167}{343}}+\sqrt[3]{244\cdot140625}.$$

PROGRESSIONS.

159. When several numbers follow one another in regular order by the addition or subtraction of the same number, they form a *series* or *Arithmetical Progression.*

Thus 5, 7, 9, 11, &c., each successive number increasing by 2, form an increasing series: and 12, 9, 6, &c. each successive number being 3 less than the preceding, form a decreasing series. The number added or subtracted is called the *common difference.*

160. *To find the sum of an Arithmetical Series.*

Rule.—Multiply the sum of the extremes (the first and last terms) by half the number of terms.

Ex. 1. If the greatest term be 27, the least 3, and the number of terms 7, what is the sum of the series?

$$\begin{array}{r} 27 \\ 3 \\ \hline \end{array}$$

Sum of extremes $=30$; then $\frac{30 \times 7}{2} = 15 \times 7 = 105$ *Ans.*

161. *To find the Common Difference.*

Rule.—Divide the difference of the extremes by one less than the number of terms.

Ex. 2. The extremes are 2 and 39½ and the number of terms 13, what is the common difference?

$$\begin{array}{r} 39\frac{1}{2} \\ 2 \\ \hline \end{array}$$

Difference of extremes $= 37\frac{1}{2}$

$13 - 1 = 12$ one less than number of terms; then $\frac{37\frac{1}{2}}{12} = 3\frac{1}{8}$ the common difference, and the series will be 2, $5\frac{1}{8}$, $8\frac{1}{4}$, &c.

XLIV.

1. Find the sum of the series when the least term is 3, the greatest 30, and the number of terms 10.

2. If the extremes be 6 and 42 and the number of terms 7, what is the common difference?

3. The least term is 3, the greatest 9; find the sum of the series, the number of terms being 9?

4. In the last question what is the common difference?

5. The sum of the series is 54 and the extremes 7 and 11; what is the common difference?

6. Find the sum of 15 terms of the series $5\cdot2$, $6\cdot1$, 7, &c.

7. Find the sum of 22 terms of the series 1, $1\cdot\dot{3}$, $1\cdot\dot{6}$, $1\cdot\dot{9}$, &c.

8. Determine the common difference of a series the sum of which is $16\cdot\dot{7}14285$, the number of terms 9, and the last term 3.

162. In a *Geometrical Progression* the numbers increase or decrease by the multiplication or division by a number called the *ratio*. Thus 3, $6=3\times2$, $12=6\times2$, $24=12\times2$, is an increasing series. On the contrary 64, $32=64\div2$, $16=32\div2$, $8=16\div2$, is a decreasing series.

163. *To find any term of a Geometrical Series.*

RULE.—Raise the ratio to a power indicated by one less than the number of terms as index, and multiply (divide in a decreasing series) the least term by the product.

Ex. 2. The common ratio is 3, the number of terms is 7, and one extreme is 9; to find the other.

Here $3^6=729$; then $729\times9=6561$. *Ans.*

In this example the fourth term will be $3^3\times9=27\times9=243$: in like manner any other term may be found.

164. *To find the sum of a Geometrical Series.*

RULE.—Divide the difference of the extremes by one less than the ratio; then add that quotient to the greater extreme for the sum of the series.

Ex. 3. Given the extremes 7 and 54432, and the ratio 6; required the sum of the series.

Here $54432-7=54425$: then $54425\div5=10885$, and $10885+54432=65317$. *Ans.*

9. The extremes are 3 and $117\frac{3}{16}$, the common ratio $2\frac{1}{2}$, what is the sum of the series?

10. In the above what is the fourth term?

11. The first term is 4, the number of terms 6, and the common ratio 4; sum the series.

12. What is the fifth term in the last example?

13. The common ratio is ·3, the first term 27, and the number of terms 7, what is the sum of the series?

14. What is the fourth term in the last question?

COMPOUND INTEREST.

165. *To find the Compound Interest.*

RULE.—I.—1°. Add the interest due at the end of each year to the principal that produced it.

2. Consider this amount as principal for the next year, to which add its interest, as before; and so on, until the end of the given time.

Ex. 1. What is the amount of £760 in 4 years, at 5 per cent., compound interest?

By Decimals.	*£760 is the first principal.*
760	$\frac{1}{20}$= 38 *first year's interest.*
760 × ·05 = 38	——
——	798 *is the second principal.*
798	$\frac{1}{20}$= 39 18*s*. *second year's interest.*
798 × ·05 = 39·9	——
——	837 18*s*. *is the third principal.*
837·9	$\frac{1}{20}$= 41 17 10$\frac{4}{5}$ *third year's interest.*
837·9 × ·05 = 41·895	879 15 10$\frac{4}{5}$ *is the fourth principal.*
——	$\frac{1}{20}$= 43 19 9$\frac{27}{50}$ *fourth year's interest.*
879·795	——
879·795 × ·05 = 43·98975	£923 15*s*. 8$\frac{17}{50}$*d*. *amount in four years.*
——	
923·78475 =	
£923 15*s*. 8·34*d*. *Ans.*	

The interest being 5 per cent. we take $\frac{1}{20}$ of the principal. It is necessary to remark that when the payments are made *half-yearly* we must substitute the word 'half-year' for 'year' in the rule.

166. Rule.—II.—1°. Find the amount of £1 for a year, at the given rate, by simple interest.

2°. Involve this amount to the power indicated by the number of payments as index, and multiply the result by the principal for the compound interest.

The same example, £760 for 4 years, at 5 per cent.

Here, £1 amounts to £1·05 at the end of a year, for 1 × ·05 + 1 = 1·05. Then $1{\cdot}05^4$ = 1·21550625, and £760 (the principal) × 1·21550625 = 923·78475 = £923 15*s.* 8·34*d.* the *Ans.*, as above. The compound interest is £923 15*s.* 8·34*d.* – £760 = £163 15*s.* 8·34*d.*

Various Departments.

Ex. 2. What will be the amount of £825 at the end of two years, at 4 per cent. per annum compound interest, the interest accruing due half-yearly?

The amount of £1 in half a year, at 4 per cent. per annum = 1·02 : then by Rule II. $1{\cdot}02^4$ = 1·08243216 : then 1·08243216 × £825 = 893·006532 = £893 0*s.* 1·56768*d.* *Ans.*

Reason of Rule II.—1°. The amount of £1 for a year is 1·05, and since any two principals must be proportional to their corresponding amounts, for the same time and at the same rate,

£1 *principal* : 1·05 *principal for second year* :: 1·05 *amount of* £1 *in a year* : 1·05 × 1·05 the amount of £1·05 in a year. Again, 1 : 1·05 :: 1·05 × 1·05 : 1·05 × 1·05 × 1·05, the amount of £1 for three years. which is the product of three factors, each factor being the amount of £1 for a year.

2°. It is evident that £1 : its amount for any time :: any other principal : its amount for the same time.

167. *To find the present worth of any sum to be paid hereafter at compound interest.*

Rule.—Divide the given sum by the amount of £1 for the given time and at the same rate.

Ex. 1. What sum will amount to £347 5*s.* 9*d.* in 3 years, at 5 per cent. compound interest?

The amount of £1, at 5 per cent., for 3 years, is $1{\cdot}05^3 = 1{\cdot}157625$: then £347 5*s.* 9*d.* $\div 1{\cdot}157625 =$ £$347{\cdot}2875 \div 1{\cdot}157625 =$ £300. *Ans.*

Reason.—The reason is evident from 166.

Ex. 2. Find the compound interest on £4500 for 3½ years, at 4 per cent. per annum.

$$\begin{array}{r l}
1{\cdot}04 & \text{amount of £1 for a year} \\
\underline{1{\cdot}04} & \\
416 & \\
\underline{1040} & \\
1{\cdot}0816 & \text{amount of £1 for 2 years} \\
\underline{1{\cdot}04} & \\
43264 & \\
\underline{10816} & \\
1{\cdot}124864 & \text{amount of £1 for 3 years} \\
\underline{1{\cdot}02} & \text{,, \quad ,, \quad } \tfrac{1}{2} \text{ a year} \\
2249728 & \\
\underline{11248640} & \\
1{\cdot}14736128 & \text{amount of £1 for } 3\tfrac{1}{2} \text{ years} \\
\underline{45\cancel{0}\cancel{0}} & \\
73680640 & \\
\underline{58944512} & \\
663{\cdot}12576\cancel{0} & \\
\underline{2\cancel{0}} & \\
2{\cdot}5152 & \\
\underline{12} & \\
6{\cdot}1824 &
\end{array}$$

∴ *Ans.* Required = £663 2*s.* 6·1824*d.*

XLV.

Find the amount at compound interest of:—

1. £240 in 4 years at 3 per cent. per annum.
2. £550 in 4 years at 5 per cent. per annum.
3. £275 in 4 years at 5 per cent. per annum.
4. £8000 in 4 years at 3¼ per cent. per annum.
5. £4800 in 3 years at 3⅛ per cent. per annum.
6. £8600 in 4 years at 5½ per cent. per annum.

Find the compound interest on:—

7. £5000 for 4½ years at 5 per cent. per annum.
8. £875 for 3¾ years at 4 per cent. per annum.

9. £2000 for $4\frac{1}{4}$ years at 6 per cent. per annum.

10. £4000 for $2\frac{1}{2}$ years at 5 per cent. per annum, accruing due half-yearly.

11. What sum will amount to £405 3*s*. $4\frac{1}{2}$*d*. in 4 years at 5 per cent. per annum compound interest?

12. What sum will amount to £1104 14*s*. $11\frac{9}{10}$*d*. in 3 years at $3\frac{1}{3}$ per cent. per annum compound interest?

13. Find the compound interest on £4000 for $1\frac{1}{4}$ year at 4 per cent. per annum (payable quarterly).

14. Find the difference between the simple and compound interest on £945 16*s*. 8*d*. in 4 years at 6 per cent. per annum.

ALLIGATION.

168. The term alligation is usually applied to the mixing of different articles in different quantities or different ratios, and will be understood from the following.

Ex. 1. If a merchant has two kinds of tea, one of which cost him 2*s*. 6*d*. and the other 3*s*. per lb.; and if he wishes to mix them so as to sell at 2*s*. 8*d*. per lb., it is evident that on every lb. of the cheaper he will gain 2*d*. and on every lb. of the dearer he will lose 4*d*. Therefore, in order to make his gains counterbalance his losses, he must put 2 lbs. of the cheaper with 1 lb. of the dearer. This process is usually written

$$\left.\begin{array}{r}30d.\\36d.\end{array}\right\}\begin{array}{c}\text{mean}\\32d.\end{array}\left\{\begin{array}{l}4\text{ the lbs. of the cheaper}\\2\text{ the lbs. of the dearer}\end{array}\right.$$

that is, $36-32=4$ will represent the cheaper, and $32-30=2$ the dearer $\therefore 4:2$ or $2:1$ is the ratio required.

Ex. 2. A person buys wheat at 39*s*. per quarter, and some of a superior quality at 6*s*. per bushel: in what proportion must he mix them so that he may gain 25 per cent. by selling the mixture at 57*s*. 6*d*. per quarter?

$\frac{39\times125}{100}=48\frac{3}{4}s.$ the price at which he must sell the cheaper.

$\frac{48\times125}{100}=60s.$ the price at which he must sell the dearer.

$$\text{Then }\left.\begin{array}{r}48\frac{3}{4}\\60\end{array}\right\}\begin{array}{c}\text{mean}\\57\frac{1}{2}\end{array}\left\{\begin{array}{l}2\frac{1}{2}\\8\frac{3}{4}\end{array}\right.$$

$$\therefore 2\frac{1}{2}:8\frac{3}{4}=10:35=2:7\ \textit{Ans.}$$

EXCHANGE.

169. Although every problem coming under the head EXCHANGE can be worked by the rules already given, we consider it necessary to explain a few of the technical terms which are in common use.

The *course of Exchange* is the variable amount of money of one country which is given for a fixed sum of another. Thus £1 sterling is at some periods equivalent to 25·25 francs, at others 24·75 francs, and so on. This value depends upon the actual value of the coins in circulation, modified by political circumstances, and the state of trade between the two countries, that is, the quantity of imports compared with the exports.

The *Par of Exchange* is the sum of money of one place which is of the same intrinsic value as a certain sum of another: it depends upon the weight and purity of the coins in use. But with the United States the par is $109\frac{23}{40}$, or 4s. 6d. per dollar, which never represented the true value.

"A *Bill of Exchange* is a written order for the payment of a certain sum of money at an appointed time." These Bills are negotiable by the holder on endorsing them, provided they are properly drawn, and slightly fluctuate in the market, like other commodities, according to the supply and demand, and the current rate of exchange.

Bills drawn after *date* run from that day; those drawn after *sight* run from the date of acceptance, with allowance of three days' grace. A bill drawn at *sight* is the same as if drawn on demand.

170. If the London merchants, from any cause, owe the merchants of Paris £10000 at any particular time, and the Paris merchants owe them only £5000, it is evident the difference must be sent to Paris by some means; and as the transmission of bullion, the *universal medium* of exchange of all nations, is attended with trouble, and often with risk, the demand for Paris bills will be brisk, and there will be a natural tendency to export goods, thereby equalizing the balance of trade. If at this juncture the merchants of Lisbon owe money to the merchants of London, there will be a disposition by the latter to pay Paris debts by Lisbon bills. In the case we have considered the rate of exchange is said to be against London.

When money is transmitted through one or more countries the exchange is called *circular* or *indirect*, and sometimes *arbitration* of exchange.

Ex. 1. "If the exchange between London and Amsterdam be 35 shillings Dutch for £1, between Amsterdam and Lisbon 42 pence for 1 old crusade, and between Lisbon and Paris 480 rees for 3 francs; find the exchange between London and Paris. (1 old crusade = 400 rees, and 12d. = 1s. Dutch)."

By Chain Rule. ? = £1
£1 = 35s. Dutch.
42d., or 3½s. = 1 old crusade.
1 old crusade = 400 rees.
480 rees = 3 francs.

$$\therefore \textit{Ans.} = \frac{\overset{5}{\cancel{35}} \times \overset{5}{\cancel{400}} \times \cancel{3} \times \cancel{2}}{\cancel{7} \times \underset{\cancel{6}}{\cancel{480}}} = 25 \text{ francs.}$$

Sometimes ⅓ or ⅛ per cent. is charged on bills of exchange, which is calculated in the same manner as brokerage.

Ex. 2. "A person buys 1000 dollars United States bonds, 6 per cents, at 86¾, find what they cost him, brokerage ¼ per cent, and the nominal par of exchange 4s. 6d. per dollar; find also his income, the course of exchange being 4s. 0½d. per dollar."

Here adding the brokerage to the price we get (1000 × 87) ÷ 100 = 870, the number of dollars that will purchase 1000.

Now cost of dollars 870 × 4½s. = £199 15s.

$$\text{Again, his income} = \frac{1000 \times 6}{100} \times 4\text{s. } 0\tfrac{1}{2}\text{d.}$$

$$= 4\text{s. } 0\tfrac{1}{2}\text{d.} \times 60 = £12\ 2\text{s. } 6\text{d.}$$

XLVI.

Time at Competitions, 2 hours; at Tests, 3 hours.

1. Reduce 2435009 seconds to days, hours, &c.
2. If the cost of premium for insuring goods for £475 is £4 15s., what would it cost to insure for £1175?
3. What is the income tax on £1670 5s. at 1s. 4d. in the pound?
4. Find the Simple Interest on £5001 3s. 4d. for $12\frac{1}{2}$ years, at 8 per cent. per annum?
5. Add together $2\frac{23}{36}$, $\frac{3}{13}$, 25, and $\frac{5}{12}$.
6. Subtract $3\frac{1}{100}$ from $35\frac{1}{1000}$.
7. Multiply $7\frac{7}{11}$ by $8\frac{4}{9}$.
8. Divide $10\frac{2}{11}$ by $1\frac{1}{4}$.
9. Add together 100·005, ·1, 5, ·200857, and ·000839.
10. Subtract 257580·1 from 356010·08.
11. Multiply 50·607 by ·1207.
12. Divide 712·03 into 5·4 to four places of decimals
13. How many seconds are there in ·003125 of a day?
14. Reduce 3150479 inches to miles, &c.
15. If one clerk can copy a paper in $4\frac{1}{2}$ hours, in how many hours will another clerk copy it, who copies half as fast again as the former?
16. Find (by Practice) the cost of 20ac. 3r. 15p. at £675 per acre.
17. Find the amount of £5000 in 3 years at $4\frac{1}{2}$ per. cent. Compound Interest (neglecting fractions of 1d.).
18. Add together $\frac{4}{15}$, $\frac{101}{20}$, 15, and $7\frac{9}{30}$.
19. Subtract $\frac{103}{108}$ from $11\frac{24}{27}$.
20. Multiply $\frac{44}{21}$ by $\frac{429}{265}$.
21. Divide $\frac{22}{34}$ by $1\frac{11}{25}$.
22. Add together 10·070009, 856·39107, 3, ·000300, 9350049350·4.
23. Subtract 10700·3950 from 10701·4.
24. Multiply 3·43 by ·0785.
25. Divide ·5007 by ·08.
26. What decimal of £3 2s. 6d. is 5s. 3d.
27. Reduce 20 yards, 5 feet, 100 inches (cubic measure) to inches.
28. By a reduction of the income tax from 7d. to 5d. in the pound, a person saves £30 13s. 9d. in a year; what was his income?
29. Find (by Practice) the cost of a wire paling 220 miles 3 furlongs, 20 poles, 5 yards long at £11 per mile.
30. In what time will £191 0s. 5d. double itself at 8 per cent. per annum, simple interest.
31. Add together 1000, $\frac{5}{27}$, 20 $\frac{5}{6}$, and $\frac{1001}{9}$.
32. Subtract $31\frac{23}{36}$ from $101\frac{3}{13}$.
33. Multiply $4\frac{4}{7}$, $2\frac{3}{16}$, $4\frac{23}{35}$.
34. Divide $17\frac{1}{4}$ by $\frac{23}{6}$.
35. Add together 4678, ·3000, ·1033600, ·30056097, ·97006, and 500101·3.
36. Subtract 10·052003 from 19·05.
37. Multiply 3·03 by ·15601.
38. Divide ·012685 by 42·35 to 5 decimal places.
39. What decimal of half a guinea is 13s. $1\frac{1}{2}$d.?

XLVII.

1. In 4005201 grains troy, how many lbs., oz., &c.?
2. If two tons and a-half of coal cost £3 2*s.* 6*d.*, what would a hundred weight and a-half cost?
3. Find (by Practice) the price of 3 quarters, 2 bushels, 1 gallon of corn at £2 13*s.* 4*d.* per quarter.
4. Find the simple interest on £833 10s. for 20 years at $3\frac{3}{4}$ per cent. per annum.
5. Add together $\frac{1}{8}$, $\frac{7}{64}$, $\frac{3}{16}$, and $\frac{1}{2}$.
6. Subtract $\frac{25}{27}$ from $2\frac{1}{3}$.
7. Multiply $\frac{2}{15}$ by $3\frac{9}{17}$.
8. Divide $\frac{7}{25}$ by $5\frac{1}{5}$.
9. Add together 407·330719, ·000093, ·02, ·400, and ·005.
10. Subtract 3·070101 from 37·005.
11. Multiply 7840·6 by 20·471.
12. Divide 7·012 by 61·25, to four places of decimals.
13. Find the value of 2·003125 of £8.
14. In 4533206 inches how many miles, furlongs, poles, &c.?
15. If 9 men can build a wall 48 feet long and 24 feet high in 5 days, what would be the length of a wall built by them in the same time 8 feet in height?
16. Find (by Practice) the price of 7 ounces, 13 dwts., 15 grains of gold at £3 10s. per ounce.
17. Find the amount of £10 in 4 years at $4\frac{1}{2}$ per cent. compound interest (neglecting fractions of a penny.)
18. Add together $\frac{5}{3}$, $\frac{500}{33}$, and $\frac{13}{330}$.
19. Subtract $\frac{111}{40}$ from $3\frac{181}{360}$.
20. Multiply $\frac{63}{84}$ by $\frac{105}{1572}$.
21. Divide $3\frac{4}{7}$ by $\frac{25}{42}$.
22. Add together ·50145, ·00704, 4·00005, 8000·2, and ·000945.
23. Subtract ·44006 from 12·013.
24. Multiply ·17034 by 8572.
25. Divide 5·008 by ·049 to 3 places of decimals.
26. What decimal of a pound troy is $\frac{2}{3}$ of a dwt?
27. Reduce 1 acre 3 roods 5 poles to square feet.
28. How much land of the yearly value of £2 13*s.* 4*d.* per acre must be given in exchange for 188 acres of land of which the yearly value is £2 10s. per acre.
29. Find (by Practice) the wages of a man for 2 weeks, 4 days, 10 hours, at 36s. a week, reckoning 6 days to a week, and 12 hours to a day.
30. In what time will £540 amount to £712 16s. at 4 per cent?
31. Add together 9, $2\frac{3}{4}$, $\frac{211}{27}$, and $\frac{1}{54}$.
32. Subtract $2\frac{1}{13}$ from $10\frac{1}{39}$.
33. Multiply together $\frac{5}{156}$, $\frac{52}{15}$, and 9.
34. Divide $300\frac{3}{8}$ by 15.
35. Add together 20134, ·1992, ·0050434, ·061, and 1.
36. Subtract ·004301 from ·0102.
37. Multiply 8892 by ·002453.
38. Divide 15483·2 by ·001125.
39. Reduce $3\frac{3}{4}$ guineas to the decimal of £2 15s.

XLVIII.

1. Reduce 976,000 ounces to tons, hundred weights, &c.

2. If 69 lbs. of salt cost 7s. $2\frac{1}{4}$d., how much can be bought for £5 5s. 5d.

3. Find (by Practice) the dividend on £3173 15s. at 9s. 3d. in the pound.

4. Find the simple interest on £670 15s. for 8 months at 5 per cent. per annum.

5. $\frac{1}{3}+\frac{1}{2}+\frac{1}{4}+\frac{1}{5}+\frac{3}{8}$　　6. $10\frac{1}{8}-2\frac{3}{4}$.

7. $7\frac{1}{2}\times 1\frac{4}{15}$.　　8. $9\frac{1}{4}\div 2\frac{3}{17}$.

9. 6·7248 + ·003079 + 46·24 + 7·010709 + 10·643.

10. 141·72 − 93·608.　　11. 63·174 × 4·06045.

12. 46·732 ÷ ·007.

13. Express 6·105 lbs, Troy in grains.

14. Reduce 17 acres 14 perches to sq. inches.

15. If a 4 lb. loaf costs 6d. when wheat is 50s. a quarter, what ought a 6d. loaf to weigh when wheat is 66s. 8d. a quarter?

16. Find (by Practice) the cost of 2 qrs. 1 bushel 1 peck of oats at 4s. 4d. per bushel.

17. Find amount of £1485 in 3 years at 4 per cent. compound interest (neglecting fractions of a penny).

18. $\frac{2}{3}+1\frac{1}{2}+\frac{7}{24}+1\frac{3}{8}$.　　19. $2\frac{1}{13}-\frac{4}{9}$.

20. $4\frac{1}{8}\times\frac{2}{11}\times\frac{8}{27}\times 2\frac{1}{4}$.

21. $3\frac{4}{13}\div 7\frac{1}{6}$.

22. 7·04087 + 93·206 + ·00535 + 7·046 + 838·407.

23. 500·6203 − 471·3207.

24. 80·8054 × ·425.

25. 73·611 ÷ 743·1 to four places of decimals.

26. Find the value of 4·625 of 13s. 4d.

27. In 176325681 cubic inches how many cubic yards, feet, and inches?

28. If 15 men can build a wall 7 feet long and 15 feet high in 16 days, in how many days will 10 men build a wall 35 feet long and 2 feet high.

29. Find (by Practice) the cost of 3 silver candlesticks each weighing 4 oz. 7 dwts. at 7s. 6d. an ounce.

30. In what time will £2700 amount to £3219 15s. at $2\frac{3}{4}$ per cent.?

31. $\frac{3}{4}$ of $\frac{5}{6}$ + $\frac{4}{5}$ of $\frac{7}{8}$ + $\frac{1}{3}$ of $\frac{3}{5}$.

32. $72\frac{1}{3}-69\frac{1}{8}$.

33. $\frac{2}{3}$ of $\frac{4}{7}$ × $\frac{5}{6}$ of $\frac{7}{8}$ × $\frac{3}{5}$ of $\frac{5}{8}$.

34. $20\frac{1}{4}\div 2\frac{1}{4}$.

35. 471·0603 + 12·07043 + ·7131 + ·002083 + 16·97.

36. 20·103　17·897.

37. 863·2 × ·00763.

38. 63·611 ÷ 737·405.

39. Reduce 630 seconds to the decimal of a week.

PRELIMINARY PAPER.

XLIX.—Time, $2\frac{1}{2}$ hours.

1. Reduce 3 tons 5 cwts. 2 qrs. 7 lbs., to ounces

2. If 3 ac. 2 rds. of land cost £700, find the value of a plot containing $17\frac{1}{2}$ perches.

3. Find (by Practice) dividend on £476 15*s.*, at 4*s.* 10*d.* in the pound.

4. Find simple interest on £903 15*s.*, for 7 years, at $3\frac{1}{2}$ per cent. per annum.

5. $\frac{4}{17} + 2\frac{1}{3} + \frac{7}{34} + \frac{5}{11}$. 6. $52\frac{3}{7} - 25\frac{15}{28}$.

7. $5\frac{4}{9} \times 6\frac{3}{7}$. 8. $9 \div 11\frac{1}{27}$.

9. 876·543 + ·001324 + 4·1928374 + 257·62075.

10. 1005·20731 — 843·907262.

11. 65·2076 × 3·409. 12. 60·32414 ÷ 9·238.

13. Express 2*s.* $1\frac{1}{2}$*d.* as the decimal of £12.

14. In 61,397,258 in. how many miles, &c.?

15. A bankrupt's debts amount to £15,490, and his assets to £5615 2*s.* 6*d.* How much will a creditor receive on £237 10*s.*?

16. Find (by Practice) price of 2 qrs. 3 bush. 1 pk., at £5 16*s.* per quarter.

17. Find amount of £2762 for 3 years, at 4 per cent. per annum, compound interest.

18. $6\frac{1}{3} + \frac{5}{18} + 10\frac{1}{10} + \frac{8}{21}$. 19. $11\frac{3}{19} - 8\frac{5}{9}$.

20. $\frac{3}{8} \times 2\frac{5}{11} \times \frac{4}{9} \times \frac{11}{36}$. 21. $13\frac{12}{17} \div 2\frac{3}{4}$.

22. 1·010305 + 777·8989 + 6·235601 + 43·277652.

23. 2430·17258 — 84·2961. 24. 572·076 × ·00328.

25. 111·26 ÷ 0075.

26. Reduce ·0237 of £2 13*s.* 7*d.* to pence and the decimal of a penny.

27. Reduce 2 lbs. 3 oz. 2 dwts. to grs. troy.

28. If 3 horses plough 4980 sq. yards in $2\frac{1}{2}$ days, how long will it take 7 horses to plough 87,150 sq. yards?

29. Find (by Practice) price of 1230 articles, at £1 3*s.* 7*d.* per dozen.

30. At what rate per cent., simple interest, will £3125 amount to £3681 12*s.* 9$\frac{3}{4}$*d.* in 4 yrs. 9 months?

31. $2\frac{3}{4} + \frac{2}{11} + 3\frac{5}{6} + 9$. 32. $12\frac{1}{40} - 10\frac{2}{37}$.

33. $\frac{12}{13} \times 8\frac{2}{3} \times \frac{1}{4} \times 4\frac{5}{16}$. 34. $16\frac{5}{32} \div 12$.

35. 2·987463 + 9·00101 + 64·305699 + 459·9987.

36. 421·32079 — 289·643.

37. ·6200796 × ·10239. 38. ·3008096 ÷ 41·32.

39. What decimal of 2$\frac{1}{2}$ cwts. is 10$\frac{1}{2}$ lbs.?

L.—Time, 2$\frac{1}{2}$ hours.

1. Reduce 12 tons 3 qrs. 17 lbs. 4 oz. to ounces.

2. If silver be 5*s.* 6*d.* an ounce, find the value of 8 oz. 6 dwts. 16 grs.

3. Find (by Practice) the value of 13,840 articles, at 16*s.* 9*d.* a score.

4. Find simple interest on £656 6*s.* 8*d.*, for 2$\frac{1}{2}$ months, at 7$\frac{1}{2}$ per cent. per annum.

5. $\frac{3}{8} + 12\frac{1}{5} + 1\frac{2}{9} + 2\frac{7}{12}$. 6. $13\frac{2}{11} - 8\frac{1}{4}$.

7. $3\frac{3}{7} \times \frac{35}{48}$. 8. $\frac{1}{16} \div 1\frac{5}{8}$.

9. 993 + 2·0624 + ·0041 + 23·7421.

10. 623·51414 — 500·2107328.

11. 243·19 × ·0177. 12. 47·341 ÷ 329·7 to three places.

13. Reduce ·0253 of a mile to feet and the decimal of a foot.

14. In 6055 grs. how many lbs., oz., &c.?

15. A sack of potatoes weighs 178 lbs. If 6 such sacks cost £2 4*s.* 6*d.*, find the cost of 44 lbs.

16. Find (by Practice) the price of 3 pecks, 1 gal. 3 qts. 1 pint, at £4 12*s.* per gallon.

17. Find the amount of £1500 for 3 years, at 6$\frac{1}{2}$ per cent. per annum, compound interest (neglecting fractions of a penny).

18. $5\frac{1}{4} + \frac{13}{20} + 2 + \frac{413}{956}$. 19. $17\frac{2}{13} - 12\frac{6}{35}$.

20. $\frac{19}{48} \times 11\frac{7}{15}$. 21. $1\frac{35}{64} \div \frac{5}{32}$.

22. $57{\cdot}14 + 3842{\cdot}29731 + {\cdot}000246 + 467$.

23. $52{\cdot}412 - 37{\cdot}8467219$. 24. $6{\cdot}5862 \times 740$.

25. $3{\cdot}48223 \div {\cdot}0047$.

26. Express 42 lbs. as a decimal of a ton.

27. In 278,856,033 sq. inches, how many acres, &c. ?

28. The rates of a railway train and a coach are as 7 to 2. If the former goes 42 miles an hour, how long will it take the latter to go 342 miles ?

29. Find (by Practice) the price of 2 qrs. 14 lbs., at £5 16*s*. 8*d*. a ton.

30. At what rate per cent., simple interest, will £1250 amount to £1962 10*s*. in 6 years ?

31. $7\frac{1}{3} + \frac{5}{9} + \frac{14}{24} + 9\frac{11}{12}$. 32. $31\frac{9}{35} - 20\frac{3}{11}$.

33. $7\frac{3}{5} \times \frac{7}{11} \times \frac{8}{19} \times 12$. 34. $9\frac{1}{3} \div 3$.

35. ${\cdot}001234 + 1003{\cdot}5687 + 1012{\cdot}9763 + 99$.

36. $100 - 99{\cdot}0013$. 37. ${\cdot}001357 \times 1{\cdot}00661$.

38. Divide 99·765300 by 1·00134 to four places of decimals.

39. Give the value of 5·298 of 6*s*. 8*d*. in pence and decimal fraction of a penny.

LI.—Time, 2½ hours.

1. Reduce 257,691 ounces to tons, &c.

2. If 5 lbs. of butter cost 8*s*. 1½*d*., find cost of 213 lbs.

3. Find (by Prac.) the dividend on £12,135 16*s*., at 13*s*. 4*d*. in the pound.

4. Find simple interest on £960 for 12 yrs., at 2¼ per cent. per annum.

5. $3\frac{1}{42} + \frac{5}{21} + 1\frac{11}{14} + 2\frac{1}{8} + \frac{1}{3}$. 6. $9\frac{11}{12} - 4\frac{5}{8}$.

7. $5\frac{3}{6} \times 3\frac{3}{11}$. 8. $7\frac{1}{4} \div 12\frac{4}{7}$.

9. $8{\cdot}0012 + 3{\cdot}0546 + 789{\cdot}0005 + {\cdot}000601$.

10. $23{\cdot}4 - 2{\cdot}34$. 11. $83{\cdot}452 \times 36{\cdot}01$.

12. $4{\cdot}65 \div 812{\cdot}3$ to four places.

13. Express 4·238 lbs. in grains (troy).

14. In 1 m. 7 fur. 10 pls., how many feet?

15. If either 5 oxen or 7 horses eat up the grass of a field in 87 days, in what time will 2 oxen and 3 horses eat up the same?

16. Find (by Prac.) the value of 2 tons 21 lbs., at £12 10*s*. per cwt.

17. Find the amount of £6500 in 4 years, at 3 per cent., compound interest (neglecting fractions of a penny).

18. $\frac{4}{7} + \frac{3}{10} + 1\frac{5}{12} + 4\frac{17}{35} + 3\frac{15}{28}$.   19. $6\frac{3}{7} - 5\frac{19}{35}$.

20. $\frac{3}{8} \times 6\frac{1}{2} \times \frac{13}{14} \times \frac{7}{26}$.   21. $\frac{14}{45} \div 1\frac{17}{25}$.

22. $153{\cdot}5207 + {\cdot}00705 + {\cdot}2007063 + 63{\cdot}7 + {\cdot}194$.

23. $9{\cdot}112 - 7{\cdot}2045$.   24. $16{\cdot}012 \times 4{\cdot}0016$.

25. $17{\cdot}637 \div 84{\cdot}2$ to four places of decimals.

26. Reduce £4 9*s*. 3*d*. to guineas, and decimal fraction of a guinea.

27. Reduce 3 lbs. 3 oz. 3 dwts. to grs.

28. If 2 men, working 6 hours a day, finish a certain work in 10 days, in how many days will 10 men working 3 hours a day, get through the same work?

29. Find (by Prac.) cost of 1728 articles, at £3 17*s*. 6*d*. a dozen.

30. In what time will £369 17*s*. 10*d*. amount to £4972 10*s*. at $2\frac{1}{2}$ per cent. per annum?

31. At what rate per cent. will £3825 amount to £4972 10*s*. in 3 years?

32. $3\frac{1}{4} + 4\frac{1}{8} + 5\frac{1}{6} + \frac{3}{10}$.   33. $\frac{7}{11} \times 2\frac{1}{2} \times \frac{3}{7} \times 10\frac{1}{2}$.

34. $41\frac{5}{9} \div 11\frac{2}{3}$.

35. $16{\cdot}005 + 4{\cdot}0206 + 93{\cdot}707 + {\cdot}004203 + 700{\cdot}251$.

36. $120{\cdot}05 - 10{\cdot}697$.

37. $737{\cdot}2363 \times 0{\cdot}6013$.   38. $1{\cdot}006 \div {\cdot}061$ to four places.

39. Express 41,391 secs. as the decimal of 3 weeks.

LII.

1. Reduce 5 tons 3 qrs. 17 lbs. 13 oz. to ounces.

2. How much must be given for 29 articles at £3 16*s*. 6*d*. a dozen?

3. Find (by Practice) the value of 7128 articles at £4 5*s*. $10\frac{3}{4}$*d*. a piece.

4. Find the simple interest on £348 10*s*. for 7 years, at $4\frac{1}{2}$ per cent. per annum.

5. $3\frac{2}{5} + \frac{4}{9} + 1\frac{5}{12} + \frac{7}{10}$. 6. $8\frac{3}{5} - 5\frac{7}{8}$.

7. $14\frac{1}{9} \times 10\frac{1}{8}$. 8. $17\frac{5}{8} \div 19\frac{2}{5}$.

9. $381{\cdot}105 + 4{\cdot}692 + {\cdot}0001 + 54{\cdot}0096$.

10. $400{\cdot}0612 - 392{\cdot}68154$. 11. $129{\cdot}342 \times 5{\cdot}26$.

12. $79{\cdot}105 \div 12{\cdot}086$ to 4 places of decimals.

13. Reduce ·334875 of a £ to shillings and pence.

14. In 32,478,621 inches to miles, furlongs, &c.

15. If a baker gains $\frac{1}{2}$*d*. on a sixpenny loaf when wheat is 55*s*. per quarter, what should he gain by selling 400 similar loaves for 10 guineas when wheat is 60*s*. per quarter.

16. Find (by Practice) the price of 3 lbs. 5 oz. 14 dwts. 12 grs. at 17*s*. 6*d*. per ounce.

17. Find the amount of £412 10*s*. for 3 years, at 6 per cent., compound interest (neglecting fractions of a penny).

18. $\frac{13}{14} + 1\frac{17}{18} + \frac{3}{4} + 2\frac{11}{12}$. 19. $16\frac{5}{21} - 10\frac{11}{13}$.

20. $4\frac{2}{3} \times \frac{3}{7}$. 21. $10\frac{8}{9} \div \frac{42}{53}$.

22. $39{\cdot}0016 + 4215{\cdot}15 + {\cdot}07364 + 1{\cdot}009$.

23. $100 - 38{\cdot}09652$. 24. $30{\cdot}561 \times {\cdot}0016$.

25. ${\cdot}030734 \div 20{\cdot}56$ to 4 places of decimals.

26. Reduce 121121 in. to the decimal of a mile.

27. Reduce 10 acres 3 poles $10\frac{1}{4}$ square yards to square inches.

28. If 7 men working 16 days can mow a field of corn 1320 yards long and 880 wide, what will be the length of the side of a field 1320 yards broad which 4 men can mow in 42 days?

29. Find (by Practice) the cost of a telegram consisting of 425 words, at £1 12*s*. 6*d*. per twenty words.

30. In what time will £225 amount to £256 10*s*. at 4 per cent., simple interest?

31. $3\frac{1}{7} + 10\frac{13}{112} + \frac{15}{28} + 1\frac{1}{4}$. 32. $2\frac{1}{4} - \frac{14}{15}$.

33. $\frac{2}{9} \times \frac{14}{15} \times 1\frac{5}{28} \times \frac{81}{82}$. 34. $3\frac{2}{3} \div 4\frac{1}{9}$.

35. $374{\cdot}826 + {\cdot}19241 + {\cdot}09059 + {\cdot}0006$.

36. $510{\cdot}81 - 426{\cdot}5793$.

37. ${\cdot}10065 \times 428$. 38. ${\cdot}00329875 \div {\cdot}0754$

39. Reduce $10\frac{3}{4}$*d*. to the decimal of 1*s* 6*d*.

LIII.

1. In 1350173 seconds, how many days, hours, &c.?

2. If 14 lbs. of tea cost £1 10*s*. 4*d*., what would 129 lbs. cost?

3. Find (by Prac.) the dividend on £1867 4*s*. 6*d*. at 9*s*. 8*d*. in the £.

4. Find the simple interest on £1560 10*s*. for 12 yrs. at $3\frac{3}{4}$ per cent.

5. Add together $1\frac{1}{3}$, $2\frac{5}{6}$, $3\frac{7}{9}$, $\frac{11}{15}$. 6. Subtract $\frac{5}{6}$ from $4\frac{2}{3}$.

7. Multiply $\frac{14}{15}$ by $\frac{84}{49}$. 8. Divide $\frac{7}{22}$ by $\frac{27}{8}$.

9. Add together 329·01, 13·319, 78·4349, ·061.

10. Subtract 5·973 from 31·95. 11. Multiply 17 371 by 4·06.

12. 74·60 ÷ 53·25 to 3 decimals. 13. Find value of ·178125 of a ton.

14. In 5078352 inches, how many miles, furlongs, &c.?

15. If a railway train goes at the rate of $27\frac{3}{4}$ miles in 2 hrs. and 45 min., how far will it go in an hour?

16. Find (by Practice) the price of 4 oz. 10 dwts. 18 grs. at £2 16*s*. 8*d*. per oz.

17. Find the compound interest on £1540 for $4\frac{1}{4}$ years at 3 per cent.

18. Add together $3\frac{1}{4}$, $5\frac{2}{5}$, $\frac{17}{100}$, and $10\frac{21}{25}$.

19. Subtract $2\frac{9}{10}$ from $16\frac{7}{12}$. 20. Multiply $\frac{53}{42}$ by $\frac{105}{318}$.

21. Divide $\frac{25}{84}$ by $1\frac{11}{14}$.

22. Add together 6003, ·072, ·46, and 21304·56.

23. Subtract 703·500 from 9000. 24. Multiply 5·032 by ·09301.

25. Divide ·4007 by ·09.

26. Reduce $7\frac{1}{2}$ guineas to the decimal of £100.

27. Reduce 6 acrs. 12 pls. to feet.

28. In what time will £2275 amount to £2673 2*s*. 6*d*. at 5 per cent. simple interest.

29. Find (by Practice) the rent of 26 ac. 3 rds. 25 pls. at £2 13*s*. 4*d*. per acre.

30. Find the compound interest on £5000 for 2 yrs. at 4 per cent.

31. Add together $\frac{5}{8}$, $6\frac{3}{40}$, $\frac{17}{5}$, $\frac{9}{80}$. 32. Subtract $6\frac{13}{16}$ from $60\frac{3}{8}$

33. Multiply together $\frac{12}{13}$, $\frac{52}{7}$, $\frac{7}{2}$. 34. Divide $\frac{59}{9}$ by $\frac{21}{237}$.

35. Add together 154·235409, 907·0001, and ·00002.

36. Subtract 2·10809 from 84302.

37. Multiply 2·43 by ·0586. 38. Divide 1·9854 by ·11250.

39. Reduce £3 2*s*. 6*d*. to the decimal of half a guinea.

LIV.

The following paper is used in preliminary test examinations and others.
(*Time* $2\frac{1}{2}$ *hours.*)

1. How many lbs. oz., &c., are there in 721572 grains of gold?

2. A ton of potatoes cost £7 : how many pounds might have been bought for 3*s.*?

3. Find (by Practice) the dividend on £1430 12*s.* at 13*s.* 4*d.* in the pound.

4. Find the simple interest on £435 15*s.* for $2\frac{1}{4}$ years, at $2\frac{1}{2}$ per cent. per annum.

5. $8\frac{1}{7} + \frac{9}{14} + 4\frac{5}{21} + 2\frac{17}{42}$.

6. $8\frac{1}{30} - 3\frac{4}{45}$. 7. $9\frac{5}{11} \times 4\frac{1}{8}$

8. $72\frac{1}{2} \div 7\frac{1}{4}$.

9. ·813 + 420·91 + ·00093 + 7·043 + 12560.

10. 506·222 — 59·87. 11. 757·04 × 15·8.

12. Divide 22·097 by 543·14 to 4 places of decimals.

13. Find the value of 2·625 of a pound sterling.

14. In 5864642 inches, how many miles, furlongs, &c.?

15. If a horse trots $23\frac{3}{4}$ miles in 2 hours and a half, what is his rate per hour?

16. Find (by Practice) the price of 7 ozs. 14 dwts. 21 grs. of gold, at £2 13*s.* 4*d.* per ounce.

17. Find the amount of £2700 for 4 years, at $3\frac{1}{2}$ per cent., compound interest (neglecting fractions of a penny).

18. $12\frac{1}{3} + \frac{7}{15} + \frac{47}{20} + \frac{7}{45}$. 19. $4\frac{3}{26} - \frac{19}{52}$.

20. $18\frac{3}{4} \times 1\frac{13}{15} \times \frac{1}{7}$ of 5. 21. $1\frac{7}{22} \div 8\frac{2}{7}$.

22. 50[illegible]04 + ·46 + ·579 + 1201·043.

23. 75 — 19·9876. 24. 45·267 × 7·045.

25. 15·96 ÷ ·065.

26. Reduce 18*s.* $8\frac{1}{4}$*d.* to a decimal of 2*s.* 2*d.*

27. Reduce 5 acres 13 poles to square feet.

28. If $4\frac{1}{8}$ cwts. of sugar cost 21 guineas, what is the price of $195\frac{1}{4}$ lbs.?

29. Find (by Practice) the price of 34 cubic yards 3 feet 288 inches of earth, at £3 6*s.* 9*d.* per yard.

30. In what time will £2220 be doubled at 6 per cent. simple interest?

31. $\frac{7}{15} + 6\frac{5}{48} + \frac{17}{8} + \frac{13}{24}$. 32. $11\frac{3}{44} - 7\frac{15}{22}$.

33. $\frac{19}{20} \times 11\frac{1}{5} \times 3\frac{21}{38}$ of $3\frac{4}{5}$. 34. $7\frac{7}{11} \div 3\frac{3}{55}$.

35. ·2764 + 1824 + 176·09032 + ·470.

36. 872·70032 — 7·20147. 37. 7·045 × 974·01.

38. Divide 1·765 by 2470 to five places of decimals.

39. Reduce 1*s.* 2*d.* to the decimal of 12*s.* $8\frac{1}{4}$*d.* to four places.

LIV.*

Given 1877.—Time, $2\frac{1}{2}$ hours.

1. In 234563 grs. troy, how many lbs. &c.?

2. If 21 cwt. of sugar cost £96 12*s.* what is the price of 70 lbs.?

3. Find (by Practice) dividend on £418 16*s.* at 5*s.* 10*d.* in the £.

4. Find simple interest on £1715 1*s.* 8*d.* for 5 years at 3 per cent. per annum.

5. $\frac{15}{19} + 1\frac{1}{2} + 6\frac{7}{114} + \frac{13}{57}$. 6. $15\frac{1}{9} - 14\frac{3}{8}$.

7. $\frac{14}{15} \times 2\frac{1}{22}$. 8. $\frac{8}{17} \div \frac{11}{51}$.

9. 34·9362 + ·00063 + 43·2875 + 5432·92543.

10. 101·207359 — 69·74352. 11. 763·15 × ·0129.

12. 765·123 ÷ 12·56 to three places of decimals.

13. Reduce 13*s.* $6\frac{3}{4}$*d.* to the decimal of £4.

14. Reduce 3 qrs. 2 bus. 1 peck 3 qts. to pints.

15. If a fourpenny loaf weighs 2 lbs. when wheat is 60*s.* a quarter, what will be the price of 30 lbs. of bread when wheat is 40*s.* a quarter?

16. Find (by Practice) the cost of 12 cwt. 3 qrs. at $3\frac{1}{2}$ lbs. at 48*s.* a cwt.

17. Find compound interest on £3695 in 3 years at $4\frac{1}{2}$ per cent. (neglecting fractions of a penny).

18. $2\frac{3}{4} + \frac{9}{16} + 3\frac{5}{12} + \frac{17}{24}$. 19. $1\frac{1}{12} - \frac{18}{25}$.

20. $4\frac{5}{8} \times \frac{3}{4} \times 1\frac{1}{14} \times 1\frac{1}{2}$. 21. $22\frac{1}{7} \div 16\frac{6}{19}$.

22. 3·50621 + ·00403 + 177·42 + 26·325 + 5·0735.

23. 40·002 — 38·5601. 24. 34·065 × 10·0068.

25. 56·277 ÷ ·507. 26. Reduce ·0465 of a mile to yards.

27. Reduce 40 acres 3 roods 16 poles to square feet.

28. If 12 men can build a wall 6 feet high and 3 feet thick in 9 days, how many men should build one 5 feet high and 4 feet thick in 24 days?

29. Find (by Practice) value of 90072 articles at £2 17*s.* 6*d.* a dozen.

30. In what time will £3745 amount to £3932 5*s.* at $2\frac{1}{2}$ per cent. per annum simple interest?

31. $3\frac{7}{11} + \frac{11}{12} + 4\frac{2}{9} + \frac{13}{66}$. 32. $\frac{13}{18} - \frac{29}{45}$.

33. $10\frac{1}{2} \times 2\frac{5}{8} \times \frac{4}{21} \times 3\frac{3}{25}$. 34. $\frac{8}{35} \div 9\frac{4}{11}$.

35. 2·453 of a day + ·055 of an hour, giving answer in minutes, and decimal of a minute.

36. From 6 furlongs take ·036 of a mile, giving result in yards and decimal fraction of a yard.

37. ·006392 × ·0015. 38. ·01404 ÷ ·195.

39. Express ·015 of a cubic yard in cubic inches, and decimal fraction of a cubic inch.

LV.—Time, $2\frac{1}{2}$ hours.

1. $5\frac{2}{7} + \frac{2}{35} + 4\frac{9}{10} + \frac{11}{12}$. 2. $5\frac{13}{18} - 3\frac{14}{15}$.
3. $3\frac{29}{30} \times 2\frac{14}{23} \times \frac{9}{14} \times 2\frac{1}{34}$. 4. $7\frac{2}{3} \div \frac{15}{16}$.
5. 896·51 + ·00867 + 1·92 + ·806.
6. 1046·201 — 345·0629.
7. 762·81 × ·0047. 8. 8224·6385 ÷ 170·3.
9. Express $\frac{3}{5}$ of 12*s.* 6*d.* as the decimal of £40.
10. $1\frac{7}{15} + \frac{27}{35} + 4\frac{5}{7} + \frac{29}{70}$. 11. $6\frac{11}{40} - 5\frac{7}{9}$.
12. $2\frac{6}{7} \times \frac{14}{15} \times 5\frac{1}{3} \times 7\frac{7}{8}$. 13. $5\frac{7}{8} \div 6\frac{13}{16}$.
14. Add ·527 of a bushel and 1·836 of a gallon, and give the answer in pints and decimal fraction of a pint.
15. From 1·0045 of a pound troy take 9·75 of an oz.
16. 409·7 × 65·02. 17. ·0004743 ÷ ·144.
18. Reduce ·50612 of 2*s.* $8\frac{3}{4}$*d.* to farthings and the decimal of a farthing.
19. Reduce 5 tons 3 cwts. 2 qrs. 13 lbs. to ounces.
20. A field of 8 acres produces 220 bushels of wheat; how much is that upon 22 square yards?
21. Find (by Practice) the cost of 455,436 articles, at £1 17*s.* $11\frac{3}{4}$*d.* per dozen.
22. In what time will £1637 16*s.* 8*d.* amount to £2129 3*s.* 8*d.*, at $3\frac{3}{4}$ per cent. per annum, simple interest?
23. In 946,345 inches how many miles, furlongs, &c.?
24. If 20 men dig 30 yards of earth in 4 days, how many yards can be dug by 40 men in 6 days?
25. Find (by Practice) the value of 7 qrs. 3 bus. 3 pks. of wheat, at £5 4*s.* a quarter.
26. Find amount of compound interest of £3670 in 3 years, at 6 per cent. per annum (neglecting fractions of a penny).
27. $\sqrt{8264{\cdot}446281}$; $\sqrt[3]{{\cdot}300,405,224}$.
28. A river 30 feet deep and 200 yards wide flows 4 miles an hour; find the number of cubic feet of water that falls into the sea in a minute; also its weight in tons (1 cubic foot = 1000 ozs.)
29. Two clocks are at 12 the same instant; one loses 7″ and the other gains 8″ in 24 hours; when will one be half-an-hour before the other, and what time will each clock then show?
30. A cistern has two two pipes, one of which can fill it in 2, the other in 3 hours. A third pipe can empty it in 5 hours. If while the cistern is empty all these are opened, in what time will it be filled?

LVI.

Cavalry and Infantry ($2\frac{1}{2}$ *hours*).—*December* 1874.

1. $6\frac{3}{5}+2\frac{1}{4}+\frac{3}{10}+2\frac{1}{2}$. 2. $30\frac{7}{30}-22\frac{5}{13}$.
3. $21\frac{7}{15}\times4\frac{2}{7}$. 4. $5\frac{1}{4}\div19\frac{3}{8}$.
5. 42·307+·05869+57·933+439·2865.
6. 150·293104—47·3825.
7. 56·443×·0285. 8. 11·83÷26.
9. Reduce ·275 of £1 13*s*. 4*d*. to pence.
10. Reduce 1 ton 3 cwt. 2 qrs. 9 lbs. to oz.
11. If 3 acres of land cost £465 18*s*., how much can be bought for £6716 14*s*. 6*d*.?
12. Find the simple interest on £635 for 9 years at $3\frac{1}{4}$ per cent. per annum.
13. $2\frac{1}{16}+5\frac{4}{9}+\frac{2}{3}+6\frac{3}{8}$. 14. $45\frac{1}{60}-19\frac{2}{39}$.
15. $6\frac{5}{18}\times1\frac{2}{25}\times16\frac{2}{3}\times\frac{3}{4}$. 16. $3\frac{9}{20}\div2\frac{13}{10}$.
17. Add together ·0625 of 1*s*. and ·0375 of £1.
18. From ·315 of a gallon take 2·25 pints.
19. ·087654×·385. 20. 688·2÷·555.
21. Express 4 hours as decimal of 1 week.
22. In 234247 grains (troy), how many lbs., ozs., &c.?
23. If 14 grains of gold cost 2*s*. $2\frac{1}{4}$*d*.; find price per oz.?
24. What is the income of a man who pays £19 13*s*. 9*d*. per annum income tax at rate of 5*d*. in the pound?
25. In what time will £445 amount to £486 14*s*. $4\frac{1}{2}$*d*. at $2\frac{1}{2}$ per cent. per annum simple interest?
26. Find (by Practice) value of 5 cwt. 3 qrs. 21 lbs. at £15 per ton.
27. If unwrought silver be 4*s*. 11*d*. per oz., and the workmanship equals $\frac{1}{5}$ the value of the metal; find the whole cost of a salver weighing 45 ozs.

LVII.

Royal Marines.

1. $5\frac{1}{8}+3\frac{1}{15}+2\frac{8}{9}$.
2. Simplify $2\frac{1}{6}$ of $3\frac{1}{10}$ of $\frac{5}{13}$ of $\frac{3}{62}$.
3. Simplify $\dfrac{3\frac{1}{16}-1\frac{5}{12}.}{16\frac{2}{3}\text{ of }\frac{7}{200}.}$
4. Reduce $\frac{1}{315}$ to a decimal fraction.
5. Multiply 2705·6 by ·987 and divide product by ·00375.
6. Simplify $\dfrac{\cdot\dot{3}\div\cdot\dot{0}\dot{9}}{\cdot\dot{3}-\cdot\dot{0}\dot{9}}$ giving result as a decimal.

7. Find value of £$3\frac{5}{8}+9\frac{3}{16}$*s.*$+5\frac{3}{4}$*d.*

8. Bring $7\frac{1}{5}$ of £2 3*s.* $6\frac{1}{2}$*d.* to the fraction of 7*s.* 6*d.*

9. Express 5 sq. yds. 2 ft. 72 in. as the fraction of a square mile.

10. Find value of ·176 of 1 fur. 36 pls. 2 yds. 5 in.

11. Find value of $\cdot 39791\dot{6}$ of £1 $+\cdot 4097\dot{2}$ of a guinea.

12. If an oz. of gold be worth £$4\cdot\dot{0}09\dot{9}$, find the value of a bar weighing $1\cdot 68\dot{3}$ lbs.

13. If a cubic foot of wood weighs $11\frac{10}{11}$ lbs., what is the weight of a beam 24 feet long $2\frac{3}{4}$ feet wide and $2\frac{1}{2}$ feet thick? Find its value at $3\frac{4}{45}$*s.* per cubic foot.

14. If 5 per cent. be lost by selling an article at 10*s.*, find gain or loss per cent. by selling at 12*s.* 6*d.*

15. $\sqrt{53107\cdot 2025}$.

16. A, B, and C rent a farm at £270. A puts 200 sheep on it, B 150, and C 100. After 6 months A sells $\frac{3}{4}$ of his flock to C, and 3 months later B sells $\frac{2}{3}$ of his to A. How much of the rent should each pay at the end of the year?

LVIII.

1. The difference between the simple interest and the discount on a certain sum for 3 months is $4\frac{1}{2}$*d.*; find the sum, the rate being 5 per cent.

2. Find the difference between the simple and compound interest on £1800 for 4 years at 4 per cent.

3. The difference between the simple and compound interest on a sum for 3 years at 5 per cent. is £4 12*s.* 6*d.*; find the sum.

4. Find value of $\dfrac{\frac{2}{35}+\frac{1}{63}}{\frac{5}{11}+\frac{1}{4}} \div \dfrac{\frac{4}{7}+\frac{2}{9}}{\frac{5}{3}+\frac{2}{5}}$ of 13 tons 7 cwt. 3 qrs. 12 lbs., and express result as the decimal of a ton.

5. A person can discharge a debt by paying at the end of one, two, and three years equal instalments of £1,771; what is the amount of the debt, interest being reckoned at 5 per cent.?

6. If ·0359375 of a ton cost £1·203125, how much can be bought for $1\cdot 3541\dot{6}$ of a guinea?

7. A person, by selling an article which cost £14 per cwt. at 2*s.* $9\frac{3}{4}$*d.* per lb. makes 5 per cent. more profit than he would do if he sold the whole for £55 15*s.* $3\frac{3}{4}$*d.*; how much was sold?

8. A man pays rates and taxes amounting to £22 15*s.* 7*d.* a year, made up of:—(*a*) Queen's taxes, 11*d.* in the pound on his rent; (*b*) Parochial rates, average 1*s.* 1*d.* per quarter on a rating of $\frac{4}{5}$ of the rent; (*c*) Water-rate, at 4 per cent. on the rent, together with 15*s.* additional for extra service; find his rent.

LIX.

1. Find the cost of carpeting a room 18 ft. 6 in. long, and 12 ft. 6 in. broad, with carpet $\frac{3}{4}$ of a yard wide at 3*s*. 3*d*. per yard.

2. Find the cost of painting the walls of a room 20 ft. 10$\frac{1}{4}$ in. long, 15 ft. 1$\frac{3}{4}$ in. broad, and 10 ft. 3 in. high, at 9$\frac{1}{2}$*d*. per square yard.

3. A room is 25 ft. 3 in. long, 14 ft. 10 in. wide, and 11 ft. 2 in. high: find the cost of a carpet for it 2 ft. wide at 3*s*. 9*d*. per yard; and find the cost of papering it with paper 2 ft. 4 in. wide, at 1*s*. 2*d*. per yard.

4. A room is 22 ft. 9 in. long, 20 ft. 10 in. broad, and 18 ft. 5 in. high; it has a door 7 ft. 6 in. high and 3 ft. 4 in. wide, and two windows each 5 ft. 2 in. by 3 ft. 6 in.; find the cost of papering it at 3*s*. 4$\frac{1}{2}$*d*. per square yard.

5. A room is 39 ft. 7 in. long, 21 ft. 5 in. broad, and 12 ft. 3 in. high; find the cost of papering it with paper 1 ft. 10 in. wide at 6*d*. per yard; also the cost of carpeting it with carpet 5 quarters wide at 2*s*. 4$\frac{1}{2}$*d*. per yard.

6. If 20 men can perform a piece of work in 12 days, how many will perform another piece of work 6 times as great in one-tenth of the time?

7. If 3 men mow 20 acres in 11 days, working 11 hours a day, how many men will it take to mow a rectangular field 384 yards long, and 300 broad, working 12 hours a day, in 4 days?

8. A steamer going 12 miles 3 fur. 9 pls. 4 yds. per hour, goes 7 times over a certain distance in 12 hours; how long would a sailing vessel, whose speed is 8 miles per hour, be going 8 times over the same distance?

9. In a manufactory a certain number of persons receive 50*s*. per week, thrice as many 37*s*. 6*d*., and 11 times as many 14*s*. each. If £94 19*s*. be the weekly payment to all, find the number of workmen.

10. A merchant buys 128 galls. of wine at 8*s*. 6*d*. per gall.; how many galls. of water must he add to it that he may gain 8 guineas on his outlay and retail the wine at 5*s*. per gallon?

11. A clock is set at 12 o'clock on Monday, at noon on Tuesday it is 3 minutes too fast; what will be the true time when the clock strikes 4 on Thursday afternoon?

12. Find (by Practice) the cost of repairing a road of 87 yds. 1 ft. 10 in. at £3 6*s*. 8*d*. per chain; and show how square links can be reduced to acres by moving the decimal point five places to the right.

LX.

1. Bring 167948604 square inches to acres.

2. A man contracts to perform a piece of work in 30 days, upon which he employs 15 men: and in 24 days it is only half finished: how many additional men must he employ to finish the work in time?

3. A cistern can be filled by a spout in 2 hrs., but on account of a leak it takes $2\frac{1}{2}$ hours to fill it: how long would the leak be in emptying it?

4. A country, the population of which is 10 millions, has births yearly of 1 in 20 and deaths 1 in 30: what will its population be 5 years hence?

5. A person paid £45 2*s.*, including a duty of 10 per cent. for a gold watch; how much was the duty?

6. If the alloy in a shilling be $\frac{1}{13}$ of its mass, and the coin be worth $\frac{1}{4}$*d.* if all alloy; what would be its exact value if it were all pure silver?

7. A cask whose weight is 2 cwts. 12 lbs. 4 oz. is floating in a square cistern whose edge is 2 ft. 6 in.; if the cask be removed, how far will the water sink in the cistern, a cubic foot of water being = 1000 oz.?

8. The telegraph posts on a railway are 77 yards apart; find the smallest number of miles that correspond to an exact number of posts.

9. A trader uses a light weight of $13\frac{1}{2}$ lbs. instead of a stone; find how much per cent. he gains fraudulently.

10. If the officers of a regiment be ·042 of the whole number, but after 50 privates are added the officers are only ·04 of the whole; find the number of men in the regiment.

11. The discount on a bill of £743 3*s.* 6*d.* due 5 years hence is £192 13*s.* 6*d.*; find the rate per cent. per annum calculated.

12. If a merchant sells goods to a retail dealer at a profit of forty per cent. and the retail dealer becoming bankrupt only pays 12*s.* 6*d.* in the £.; what per cent. does the merchant lose?

LXI.

1. Assuming that the circumference of a circle is to the diameter as 22 : 7, and that the circumference of the earth is to its diameter as 160 mètres are to 167 feet, find the ratio of a mètre to a foot.

2. Find the fraction of a pound which is equivalent to the excess of $\frac{2}{3}$ of a guinea above the sum of $\frac{3}{4}$ of a shilling and $\frac{4}{9}$ of 7*s.* 6*d.*

3. A gallon contains 277·274 cubic inches, and a cubic foot of water weighs 1000 oz.; find the weight of a quart of water.

4. The price of a book is 17*s.* and 25 are sold as 24; if 531 copies are sold for £434 7*s.*, how many volumes were sold in lots of 25?

5. Add together:—¾ of $\frac{5}{9}$ of £2 5*s.*, $\frac{3}{7}$ of 3 guineas, $\cdot\dot{8}1$ of £1 18*s.* 6*d.* and 2·154 of £2 15*s.* and reduce the result to the decimal of 25 guineas.

6. If 5763 bushels of corn are imported for £2377 4*s.* 9*d.* and a duty is then paid of 10¼ per cent. on the money expended: find the duty per bushel.

7. If a postage stamp be an inch long and ⅘ of an inch wide, how many stamps will be required to cover the walls of a room 18 ft. 9 in. long, 16 ft. 9 in. wide, and 10 ft. 6 in. high?

8. Standard gold is coined at the rate of £3 17*s.* 10½*d.* per ounce; find the least number of ounces that can be coined into an exact number of half-sovereigns.

9. A crew pulls 36 strokes in a minute, another 40 strokes per minute: 11 strokes of the former are equal to 12 of the latter: if the latter pull a certain distance in 20 minutes, in what time should the former pull the same distance?

10. A piece of cloth, when measured with a yard measure which is ⅔ of an inch too short, appears to be 10½ yards long: find its true length.

11. A person after paying a poor-rate, and also an income-tax of 7*d.* in the pound, has £486 remaining; the poor-rate amounts to £22 10*s.* more than the income-tax: find his gross income and the number of pence per £. in the poor-rate.

12. A person sells 72 quarters of wheat at a profit of 8 per cent. and 37 quarters at a profit of 12 per cent.; had he sold the whole at a profit of 10 per cent. he would have received £2 13*s.* 1*d.* more than he actually did. What price per quarter did he pay for the wheat, and what sum did he gain by the transaction?

13. Show if the following rule for the calculation of interest on any number of pounds at 5 per cent. per annum for any number of days be correct:—

"Multiply the number of pounds by one-third of the number of days, or the number of days by one-third the number of pounds; reject the unit figure, and the result will be the answer in pence, if one penny be deducted for every six shillings."

14. To find the daily wage corresponding to any given number of pounds per annum: Take two-thirds of the number of pounds and the

result will be the daily wage in pence, provided that one half-penny be deducted from every three shillings in the sum so found. Show that the error in this rule cannot exceed one half-penny if the sum be under £3000 per annum.

15. A wine which contains $7\frac{1}{2}$ per cent. of spirit is frozen, and the ice which contains no spirit being removed, the proportion of spirit in the wine is increased to $8\frac{3}{4}$ per cent. How much water in the shape of ice was removed from 504 gallons of the original wine?

16. If 70 acres of wheat yielding 4 quarters per acre give a profit of £238 when wheat is 57 shillings per quarter, what profit will be made from 60 acres of wheat yielding 5 quarters per acre when wheat is at 55 shillings per quarter, the expense of cultivation being the same *per acre* in both cases?

17. £1 per ton being the average price of coals during nine months exclusive of summer months, when the average price is only 18*s.* per ton: what is the average price during two months of mid-winter, that of the other ten months being 19*s.* 8*d.* per ton?

18. If 2 men and 1 boy can do in 5 days twice as much work as 1 man and 2 boys can do in 3 days, how many boys will be needed to do the work of 8 men?

19. A man's wages are £1 7*s.* 6*d.* per week, and on Saturday, January 4, 1868, he puts by one-fourth of that week's wages, and if he does so every fourth Saturday till the end of 1875, find how much he will then have saved.

20. A party of 27 persons set out to travel, and the fare for each of them is £1 16*s.* 5*d.*, but in consequence of some of the party having no money, the others pay their fares in equal proportions, paying £2 2*s.* 9*d.* each: how many of the party did not pay?

21. Reduce to its lowest terms the product of

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\frac{1}{36}, \text{ and } \frac{3}{13}-\frac{1}{7}+\frac{8}{59}.$$

22. If 400 labourers make an embankment 5 miles long in 25 days, how much overtime must 120 men work in order to complete a similar embankment 2 miles long in 32 days, the day's work being 12 hours?

23. Resolve the following numbers into their prime factors: 5940, 19008, and 269568; and write down the greatest number which will divide them without a remainder, and the smallest number which they will all divide without a remainder.

24. Find, to four places of decimals, the square root of ·001 and of ·5; and find what numbers have ·1, and ·03 for their square roots.

25. A can copy 6 pages while B is copying 5, B copies 15 while C copies 12, and C can copy 4 while D is copying 3. A, who can write 20 pages a day, receives a paper of 240 pages to copy, and, after doing

one-fourth of it, calls in B, C, and D to assist him: in what time will they have it finished?

26. From the known diameter (4 feet) of a carriage-wheel, two persons calculate the number of its turns in a given distance, one estimating the circumference as 3 times the diameter, the other using the ratio as 22 : 7, their results differ by 100 turns. Find the distance.

LXII.

Temporary Writers.

1. Write down in figures—Three hundred million thirty thousand and eighteen.

2. Write down in figures—Seventeen hundred and ninety-eight million one hundred and twenty thousand and seventy.

3. Write down in words 30600101.

4. Add together—

9426712
901
569815
75643
16374128

5.

£	s.	d.
12	14	$10\frac{3}{4}$
809	10	$11\frac{1}{2}$
365	4	9
37	17	$3\frac{1}{4}$
194	9	7
36	2	$11\frac{1}{2}$

6. From 710520605
Take 691730908

7.

£	s.	d.
3125	17	$3\frac{1}{2}$
1982	17	$6\frac{3}{4}$

8. 893612 × 40106.
9. 4327986210 ÷ 18.
10. 216938 × 5123.
11. 3798462 ÷ 937.
12. £243695 12s. $2\frac{1}{2}$d. × 7.
13. 846305217 ÷ 2846.
14. £36152 17s. $3\frac{1}{4}$ × 305.
15. £105574 9s. $5\frac{3}{4}$d. ÷ 35.
16. £691 3s. $11\frac{3}{4}$d. × 780.
17. £11937948 11s. $7\frac{1}{2}$d. ÷ 486.

18. In 53618 oz. how many tons, cwts., qrs., &c.?

19. How much oats at £21 0s. 9d. for 51 cwts. may be bought for £64 7s.?

20. Find (by Practice) the dividend on £4586 13s. 4d. at 7s. 6d. in the pound.

21. $3\frac{5}{6} + \frac{5}{8} + 4\frac{2}{9} + 1\frac{13}{24}$.
22. $24\frac{1}{24} - 21\frac{1}{21}$.
23. $\frac{7}{11} \times 2\frac{1}{2} \times \frac{3}{7} \times 10\frac{1}{2}$.
24. $\frac{11}{12} \div 7\frac{17}{18}$.
25. 734·60125 + 5·90123 + ·8466721 + ·0043.
26. 2160 − 812·63159.
27. 35·716 × ·016.
28. 181·3 ÷ ·00037.
29. Give the value of ·625 of a pound troy in grains.

Note.—The candidates for writerships also get twelve columns of tots to be done in half an hour.

Given at various Examinations.

30. Find true discount on £501 10*s.* due six mo. hence, at 4 p. ct.

31. A bankrupt has good debts to the amount of £456 18*s.* 1*d.*, and the following bad debts, £360 7*s.* 10*d.*, £120 13*s.*, and £19 18*s.*, for which he receives respectively 4, 5, and 9 shillings in the pound; his own liabilities amount to £3408 12*s.*; how much can he pay in the £?

32. The shares of a certain company are £3 15*s*: what is the gain of a person who buys 77 of them when they are at 4 per cent. below par, and sells again at 1 per cent. premium.

33. A man skates 6 miles at the rate of 10 miles an hour with the wind, and returns against it in 50 minutes. How much does the wind accelerate or retard him?

34. An estate with a rental of £437 is sold for £17480. How much per cent. must the purchaser raise the rental to have 2¾ per cent. for his money?

35. A coach goes 9 miles in an hour, and a railway train goes 23 miles while the coach goes 11. How much time will be saved in a journey of 138 miles by taking the train instead of the coach?

36. Two persons start from London and Brighton (50 miles apart) at same time; the first goes 4½ miles an hour, and rests 20 minutes at the end of every ten miles: the other walks steadily 3½ miles an hour; where will they meet?

37. An event occurred 40,000 days preceding January 1st, 1871; find the year, the day of the month, and the day of the week on which it happened.

38. Assuming that the value of 2 Suacan scudi is greater than 10⅓d. and less than 11d.: and the value of 1 Austrian Swanziger is greater than 8d., and less than 8⅓: prove that the value of 544 Suacan scudi differs from 355 Austrian Swanziger by less than that of 19 Austrian Swanziger.

39. A ditch is being dug at the rate of 81 feet per day by 54 men: after 13 days' work 8 of them are replaced by boys, and the work goes on for 11 days more, at the end of which the whole length dug is 1889 feet; how much work per day is done by the boys?

40. If 16 men and 12 boys earn £23 4*s.* in 8 days, how much should 20 men and 15 boys earn in 10 days, if a man's wages be 2*s.* a day?

41. Extract the square root of ·00005476, and the cube root of $2\frac{1371}{2744}$.

42. A tenant holds a farm of 350 acres, subject to a tax of 3*s.* 6*d.* per acre, and a corn rent of 100 qrs. of wheat, barley, oats, and beans, respectively. Find the amount of his rent, when the average price of wheat, barley, oats, and beans per qr. are 34*s.* 9*d.*, 26*s.* 4*d.*, 17*s.* 2*d.*, and 33*s.* 10*d.* respectively.

43. The sum of £1001 was laid out in the 3 per cents. at $89\frac{3}{8}$, and a half year's dividend having been received upon it, it was sold out, the whole increase of capital being 72 guineas. Find at what price it was sold out.*

44. A sum is laid out in the 3 per cents. at $89\frac{3}{8}$ and a half-year's dividend received upon it; the stock being then sold at $94\frac{5}{8}$ and the whole increase of capital being £54, find the original sum laid out.†

45. A dealer buys 80 tons of coals, and after selling them at 1s. 6d. per sack finds he has gained £4. Had he sold them at 1s. 4d. per sack, he would have lost £6. Find the cost price per ton, and the weight of a sack of coals. ‡

MISCELLANEOUS QUESTIONS.

Ex. 1. 'At what time between 3 and 4 o'clock will the hands of a clock be together?'

At 3, the minute hand points to 12 and the hour hand to 3. The minute hand travels 12 times as fast as the hour hand, and the question becomes 'In what time will the minute hand, travelling 12 minutes while the hour hand is going 1, overtake the other which has 15 minutes of a start?'

It is evident 55 minutes will be gained in 60 by the minute hand.

∴ 11 minutes will be gained in 12 „

and 1 minute will be gained in $\frac{12}{11}=1\frac{1}{11}$ „

∴ 15 minutes will be gained in $1\frac{1}{11}\times 15=16\frac{4}{11}$. The time between 3 and 4 when the hands will be together is $16\frac{4}{11}$ minutes past 3.

Ex. 2. John can do a certain work in 12 hrs., James in 10 hrs., and Richard in 8 hrs.; in what time should they do it, all working together?

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* $89\frac{3}{8} : 100 :: £1001 : £1120$: then interest of £1120 at 3 per cent.=£33 12s., and £33 12s.÷2=£16 16s. the half-year's dividend; then £1001+72 guineas−£16 16s.= £1059 16s.; therefore $\frac{£1059\ 16s.\times 100}{1120}=94\frac{5}{8}$. *Ans.*

† Here, gain on every $89\frac{3}{8}$ invested=$34\frac{5}{8}-89\frac{3}{8}=5\frac{1}{4}$, to which add a half year's dividend, and $5\frac{1}{4}+1\frac{1}{2}=6\frac{3}{4}$ whole gain on £$89\frac{3}{8}$: whence $(54\div 6\frac{3}{4})\ 89\frac{3}{8}=£715$ *Ans.*

‡ Here 2*d.* a sack makes a difference of £10.

Here, John does $\frac{1}{6}$ in two hours.
James $\frac{1}{5}$ in two hours.
Richard $\frac{1}{4}$ in two hours.

All working together will do $\frac{1}{6}+\frac{1}{5}+\frac{1}{4}=\frac{37}{60}$ in two hours; therefore $\frac{1}{60}$ will be done in $\frac{1}{37}$ of 2 hrs. or $\frac{2}{37}$ of an hr,

And $\frac{60}{60}$, the whole, in $\frac{2\times 60}{37}=\frac{120}{37}=3\frac{9}{37}$ hours. *Ans.*

Ex. 3. If oranges be bought at 10 for a penny, how many should be sold for 2*s.* 2*d.* to gain 30 per cent.?

Without gain or loss he would sell,

$$26\times 10=260.$$

In order to gain 30 per cent. he must sell less, in the ratio of 130:100.

$$\therefore 130:100::260:200. \textit{ Ans.}$$

Ex. 4. The annual losses of a merchant during 4 years average $3\frac{3}{4}$ per cent. upon his original capital; at the end of the four years his capital is £2783 15*s.*; what was it at first?

An average loss of $3\frac{3}{4}$ per cent. for 4 yrs. $=3\frac{3}{4}\times 4=15$. Whence an original capital of £100 would (in this time) be reduced to £85; whence, 85 : 100 :: £2783 15*s.* : £3275 *Ans.*

Ex. 5. If A do $\frac{3}{4}$ of a certain work in 2 hrs. and B $\frac{2}{3}$ of the remainder in 1 hour, and C finish it in 10 minutes: in what time will they do it all working together?

A will do $\frac{1}{2}$ of $\frac{3}{4}=\frac{3}{8}$ of it in an hour.
B will do $\frac{2}{3}$ of $\frac{1}{4}$ $=\frac{1}{6}$ of it in an hour.
C will do $\frac{1}{3}$ of $\frac{1}{4}=\frac{1}{12}$ of it in 10 minutes;
hence, $\frac{1}{12}\times 6=\frac{1}{2}$ of it in an hour.

Therefore they will all do $\frac{3}{8}+\frac{1}{6}+\frac{1}{2}=\frac{25}{24}$ of it in an hour working together, and the whole work in $\frac{24}{25}$ hr. $=57\frac{3}{5}$ minutes. *Ans.*

Ex. 6. If a man rows 14 miles in $3\frac{1}{2}$ hrs. against the stream, the rate of which is 6 miles an hour: in what time should he row 10 miles with the stream?

Against the stream he rows 14 miles in $3\frac{1}{2}$ hrs., that is, $14\div 3\frac{1}{2}=4$ miles an hour; but the stream brings him back or resists him 6 miles an hour: hence his rate of rowing is 10 miles an hour. With the stream he will row $10+6=16$ miles an hour; therefore he will row 10 miles in $\frac{10 \times 1 \text{ hr.}}{16} = \frac{10}{16} = \frac{5}{8}$ of an hour $=37$ m. 30 secs.

Ex. 7. The rate of a clock is ·04 *per cent.* too fast; how much will it gain in a day?

It is evident that a day by the clock will be $\cdot 000\dot{4} = \cdot 0\dot{4} \div 100$, too much; and $\cdot 000\dot{4} = \frac{4}{9000} = \frac{1}{2250}$ of a day is the time it will gain $= \frac{1 \times \cancel{24} \times \cancel{60} \times \cancel{60}}{\cancel{2250}} = \frac{192}{5} = 38\frac{2}{5}$ secs. *Ans.*

Ex. 8. Simplify the expression

$$\frac{\frac{1}{3} \text{ of } \frac{4}{5} \div 2\frac{2}{5} + 2\frac{1}{2}}{(\frac{1}{7} \text{ of } \frac{2}{5} + \frac{1}{2}) \div \frac{4}{7} - \frac{3}{4}}$$

Taking the numerator we have $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$, which $\div 2\frac{2}{5}$ gives $\frac{1}{9}$, to which add $2\frac{1}{2}$ and we have $2\frac{11}{18}$, the value of numerator. Again, $(\frac{1}{7} \times \frac{2}{5} + \frac{1}{2}) \div \frac{4}{7} - \frac{3}{4} = (\frac{2}{35} + \frac{1}{2}) \div \frac{4}{7} - \frac{3}{4} = \frac{39}{70} \div \frac{4}{7} - \frac{3}{4} = \frac{39}{40} - \frac{3}{4} = \frac{9}{40}$, the value of denominator. Now, $2\frac{11}{18} \div \frac{9}{40} = \frac{47}{18} \times \frac{40}{9} = 11\frac{49}{81}$. *Ans.*

LXIII.

Miscellaneous Questions.

1. If 110 dollars are worth £23, what are 15 dollars 2 dimes 7 cents worth. (1 dol. = 10 dimes = 100 cents.)

2. Find (by Practice) the value of 5 acres 2 roods 11 perches, at £95 per acre; and also 95 acres at £5 2*s.* 11½*d.* each.

3. Find the simple interest on £3728 19*s.* 7½*d.* for 4¾ years, at 5 per cent.; and also £760 for 65 days, at 2½*d.* per cent. per day.

4. Find the compound interest on £5750 for 3 years at 4¾ per cent.

5. Add together $9\frac{1}{18}$, $3\frac{5}{6}$, $2\frac{11}{12}$, and $\frac{5}{7}$; add $125\frac{7}{12}$, $2\frac{1}{6}$, $\frac{3}{5}$, $\frac{7}{9}$.

6. Add together 39·51, 425·76, ·237, 4·0032; and 52·11, 328·01, 64, ·024, expressing the difference of the two results as the decimal of 22·5.

7. Reduce $\frac{5}{8}$ of 13*s.* 6*d.* to the decimal of a guinea.

8. Find the value of ·38246 of 10*s.* 6*d.*, and express the result as the decimal of a guinea.

9. Extract the cube root of 589365742·983688.

10. Find the difference between $\frac{3}{8}$ of $\frac{2}{9}$ of a mile, and $\frac{2}{3}$ of $\frac{1}{20}$ of $2\frac{6}{7}$ of a furlong.

11. Find the cost of carpeting a room that is 25 feet long and 20 feet wide, the price of a yard of carpet 2 feet 6 inches wide being

4*s*. 3*d*. If the room be 10 feet 7½ inches high, find the cost of painting the walls at 4*s*. 1½*d*. the square yard.

12. Convert 22 lbs. 5 oz. 15 dwts. 20 grs. troy into avoirdupois weight; one pound avoirdupois being to one pound troy as 175 to 144.

13. If 56 cubic feet 1044 cubic inches of timber are required to floor a room 29 feet 3 inches by 25 feet 4 inches, what is the thickness of the boards?

14. A person sells out of the 3 per cent. consols at 99 and invests in Exchequer bills, bearing interest at the rate of 2½*d*. a day per cent. when the bills are at a premium of 7*s*. 6*d*. What effect has this on his income?

15. In the month of December last the number of paupers in a union was 336, the number of women being double that of the men, and the children being as many as the men and women together. If a man cost one-third more than a woman, and three children as much as a man and a woman together, and the whole cost for the month be £83 6*s*.; how much is the daily cost of each man, woman, and child?

16. A bankrupt owes £25,962 10*s*., what must his assets be worth in order that he may pay 7*s*. 11½*d*. in the pound; and how much will a creditor lose whom he owes £980 10*s*.?

17. A tradesman's annual losses during 5 years average 1¼ per cent. on the capital with which he began, and at the end of the 5 years his effects are worth £2531 5*s*., what capital did he begin with?

18. If 90*s*. will pay 5 men for 12 days' work, how much will pay 32 men for 24 days' work?

19. What should the result be if the efficiency of the second set were half that of the first, and their day's work five-fourths as long?

20. Find the amount of £875 at the end of 15 months, at 4 per cent. per annum, compound interest, the interest accruing quarterly.

21. What would be the amount of a salary of £95 per annum, subject to a deduction of £2 7*s*. 6*d*. per cent. per annum, for 1 year and 25 days?

22. Suppose 6948 4/7 dollars were purchased at 48⅞*d*. each, and that they were issued at ⅝ of a penny more than that rate; what would be the total cost of the dollars, the total amount for which they were issued, and the total gain upon the transaction?

23. What will £100 6*s*. 8*d*. amount to with compound interest for five years at 5 per cent. per annum, the interest being paid annually?

24. If 939 men consume 351 quarters of wheat in 168 days, how many will consume 1404 quarters in 56 days?

25. Silk sold at 12*s.* 6*d.* per yard gained £7 10*s.* per cent., what did it cost per yard?

26. Reduce 2 fur. 11 yds. 1 ft. 9 in. to the decimal of a mile.

27. A and B earn £1 18*s.* 6*d.* in 7 days; A and C earn £2 18*s.* 4*d.* in 10 days; B and C earn £3 9*s.* 8*d.* in 11 days: find the separate daily wages of each.

28. A owns $\cdot56\dot{7}$ of an estate, and B the remainder; express B's share as a vulgar fraction, and find its value if the whole estate be worth £13,500.

29. Find the value of $1\cdot9\dot{5}$ tons, less 17·09375 cwts. at 6*s.* $3\frac{1}{4}$*d.* an ounce.

30. Find the difference between $\frac{5}{8}$ of £·7875 and $1\cdot\dot{2}$ of 11*s.* $5\frac{1}{4}$*d.*

31. If 1185 parts copper be added to 715 parts tin and 100 parts zinc, find the per centage of each metal in the compound.

32. A can complete a job in 6 days, and B in 8 days; A works at it 2 days, how long will both be in finishing it if they work together?

33. Find the difference in 7 years at simple interest between £925 at 5·825 per cent. and £2500 at $2\frac{1}{8}$.

34. A rectangular cistern, 9 feet long, 5 feet 4 inches wide, and 2 feet 3 inches deep is filled with a liquid that weighs 2520 pounds; how deep must a rectangular cistern be which will hold 3850 pounds of the same liquid, its length being 8 feet, and its width 5 feet 6 inches?

35. Reduce to prime factors and thence find G. C. M. 35035, 41580, 24255.

36. A man engages to ride 500 miles in 60 hours; he rides 2 miles in $11\frac{1}{4}$ minutes, and stops an hour at the end of every 40 miles: how much time has he to spare?

37. Divide an estate so that A may get $\frac{5}{8}$ of the whole, and B twice as much of the remainder as C. If in this way B gets 27 acres more than C: find the acreage of the whole estate.

38. $\sqrt{70599\frac{178}{289}}$; and $\sqrt[3]{2326203\cdot125}$.

39. If an article be sold at the rate of 17 lbs. for 1*s.* 3*d.*, of which 2 cwt. 1 qr. 3 lbs. was bought for 15*s.*; find the gain per cent.

40. Express 300 yards as the decimal of 1 furlong. If a path 3 feet broad be put round a house 57 feet long and 37 feet wide, express the area of path as the decimal of an acre.

41. A merchant has teas worth 4*s.* 6*d.* and 3*s.* 6*d.* per lb. respectively, which he mixes in the proportion of 3 lbs. of the former to 2

of the latter, and sells the mixture at 4*s.* 4*d.* per lb.; what does he gain or lose per cent.?

42. The populations of 3 towns in the year 1841 were 15875, 64575, and 29185; and in the year 1851 the two former had increased respectively 4 and 8 per cent., and the latter had decreased 20 per cent.; find the increase or decrease per cent. of the total population of the three towns.

43. The breadth of a room is 14 feet; the cost of papering it at 1s. per square yard is £4, and the cost of covering the floor with a carpet at 4s. 6d. per square yard is £5 12s. Find the length and height of the room.

44. What must be the market value of 3 per cent. stock in order that, after deducting the income-tax of 9*d.* in the £, it may yield $3\frac{1}{2}$ per cent. interest?

45. The cost price of a book is 12*s.* 6*d.* If the expense of sale be 9 per cent. upon this, and the profit 21 per cent. upon the same, what is the retail price?

46. From the following table:—

Railway Passengers	Great Britain	France	Prussia
1st Class	4743213	2124917	307492
2nd Class	10291749	6172429	1292476
3rd Class	21409217	15473551	2374237

find the proportion per cent. which the whole number of 1st class passengers bears to the whole number of 2nd and 3rd class passengers.

47. The rate of a clock is ·075 per cent. too fast. How much will the clock gain in the week?

48 A man rents a house at £142 a-year, payable half yearly; if he sub-lets at £160 a-year, payable quarterly, and money be worth 4 per cent., find his gain in 3 years.

49. A tradesman marks his goods at a gain of 12 per cent., but he allows a discount of £1 6*s.* 3*d.* on payment of an account for 20 guineas; find his actual gain per cent.

50. Add together $\cdot0\dot{6} + \sqrt{\cdot09} + \sqrt[3]{\cdot125}$ and express the sum as a vulgar fraction of $\cdot3\dot{6}$.

51. Extract the square root of 531441 and of $32\frac{1}{9}$.

52. If the $3\frac{1}{2}$ per cents., paying half-yearly dividends on 1st October and 1st April, are 105 on 1st August, what should be the price of the 3 per cents. same day which pay dividends on 1st July and 1st January?

53. Find (by Practice) the premium on £126,315 at 8*s.* 8*d.* per cent.

54. A person's annual expenditure from the year 1833 to the year 1853 inclusive is £293 17*s.* 4*d.*; he finds that in 1833 he spent £283 17*s.* 10*d.*, and in 1854, £365 14*s.* 1*d.* What was the average annual expenditure from 1834 to 1854 inclusive?

55. Find the discount on £126 13*s.* 4*d.* due 8 months hence at 4 per cent. simple interest. What would be the amount of the error in this case if interest were taken instead of discount, and in whose favour would the error be?

56. Find the amount of £13333 6*s.* 8*d.* for 4 years at 5 per cent. compound interest.

57. Find the present worth of £2157 12*s.* due 9 months hence at $3\frac{1}{2}$ per cent.; also of same sum at same rate due 292 days hence.

58. The population of Great Britain in the year 1851 was 21121967 and the increase during the previous half century had been 93·5 per cent. What was the population in 1801?

59. A train starts with its full complement of passengers. At the first station it drops $\frac{1}{3}$ of these and takes in 96 more; at the next it drops $\frac{1}{2}$ of the new total and takes in 12 more; on reaching the third station there are 248 left. What number started?

60. Reduce $\frac{\cdot 4\dot{3}4\dot{9}}{3\cdot 42\dot{5}5\dot{9}}$ to a vulgar fraction.

61. In what proportion must wines worth 15*s.*, 20*s.*, 26*s.*, and 30*s.* per gallon be mixed so as to give a profit of $14\frac{2}{7}$ per cent. on the cost when sold for 24s. a gallon?

62. Find the side of a square garden which contains 4 roods 1 pole 29 yards $6\frac{3}{4}$ feet.

63. Find the value of $\cdot 0\dot{5}$ of $\cdot\dot{0}\dot{6}$ of a mile.

64. A merchant buys 15 dozen of port at 82*s.* per dozen, and 60 dozen more at 50*s.* per dozen; he mixes them and sells the mixture at 70*s.* per dozen; what profit per cent. does he realize on his original outlay?

65. Extract the cube root of 946966168.

66. There are 58 insolvents, 3 of whom paid 20*s.* in the pound; 11, 12*s.* 6*d.*; 19, 7*s.* $1\frac{1}{2}$*d.*; 17, 3*s.* $4\frac{1}{2}$*d.*; 2, 1*s.* 6*d.*; the rest nothing. What was the average dividend?

67. If coffee worth 2*s.* 9*d.*, 3*s.* 2½*d.*, and 2*s.* 4½*d.* per lb., be mixed in equal quantities, and the mixture sold at £16 9*s.* per cwt., what is the gain or loss per cent. ?

68. If oranges be bought at the rate of 20 for a shilling, how many should be sold for £1 8*s.* to gain 40 per cent. ?

69. The number of persons employed in the railways are, respectively :—

Year	England		Scotland		Ireland	
		Miles		Miles		Miles
1852	55331	5433·9	8271	961·4	3999	680·4
1853	66267	7544·2	8979	977·9	5163	786·9

Hence find the average number of men per mile employed on railways in the whole United Kingdom, (1) in 1852, (2) 1853.

70. The price of the 3 per cents. and the market value of railway stock paying 5¼ per cent. being respectively 92⅝ and 99¼. compare the rates of interest which investments in those stocks would give.

71. The rate of a clock is ·0625 too fast ; how much should it gain in a week ?

72. The populations of 3 towns in 1841 were 20565, 53415 and 30125, and in 1851 the two former increased respectively, 8 and 12 per cent., and the latter decreased 40 per cent.; find the increase or decrease per cent. of the total population of the 3 towns.

73. Find what will be the rate of interest in order that the discount on £1936 18*s.* at the end of 3 years may be £207 10*s.* 6*d.*

74. If 48 pioneers in 5 days of 12½ hours dig a trench 139·75 yards long, 4½ yards wide, and 2½ yards deep ; how many hours per day must 90 pioneers work, during 84 days, to dig a trench 4910$\frac{1}{16}$ yards long, 4⅞ yards wide, and 3⅕ yards deep ?

75. If by selling goods for £272 I lose 15 per cent., how much per cent. should I have lost or gained had I sold for 320 guineas ?

76. If a man, his wife, and child consume 36 pounds of bread in 9 days, when bread is at 5*d.* the four pound loaf, how much bread a day would a family consisting of a man and wife and three children have to restrict each member to so as not to exceed the same rate of expenditure when the four pound loaf cost 6*d.* ? In each case assume that the rate of consumption of a man, woman, and child are as 4, 3, and 2.

77. Find the cube root of 84·9.

78. If £533 6*s.* 8*d.* be taken as payment of £560 due 2 years hence calculate the rate of discount.

79. It appeared from a daily paper that the annual expenses of the Suez Canal might be estimated at about 17,800,000 francs, exclusive of 10,000,000 of francs needed for the payment of the interest on the shares. Assuming that all the receipts over and above the sum of these amounts are to be divided as profits among the shareholders, the founders and the Egyptian Government, the holder of shares taking 70 per cent. and each of the latter parties 15 per cent., what must the yearly receipts be in order that an English holder of 20 shares out of the 400,000 representing the whole stock, may receive £73 10*s.* as his proportion of the profits? (£1 = 25 francs).

80. The specific gravities or relative weight of proof-spirit and water may be taken as 923 and 1000, disregarding temperature. From a gallon jar filled with proof-spirit a certain quantity is drawn off and water added till the vessel is again full. The mixture is then found to have a specific gravity of 992. How much spirit has been abstracted?

81. What is the length of the side of a square whose area is equal to a rectangle of 513 yards 1 foot 11 inches × 1628 yards 0 feet 11 inches?

82. Reduce to a simple fraction $\frac{3}{7} \times 1\cdot1\dot{6} + 2\cdot15\dot{6} \times \frac{11}{14}$.

83. If A. do $\frac{3}{8}$ of a piece of work in 4 hours, and B. do $\frac{3}{4}$ of the remainder in 1 hour, and C. finish it in 20 minutes: in what time will they do it all working together?

84. If two clocks, one of which gains 2 minutes and the other loses 4 minutes a day, be set at noon, what hour will the second indicate when the first indicates noon 12 days afterwards?

85. The imperial gallon contains 277·27 cubic inches, and a cubic foot of water weighs 62·42 lbs.: find the weight of a pint of water correctly to two places of decimals.

86. Extract the cube root of 731189187729.

87. Multiply by the method of duodecimals 2 feet 10 inches 11 parts by 3 feet 7 inches 5 parts; and the product by 1 foot 10 inches.

88. Express the result of the last question in cubic feet, cubic inches, and fraction of a cubic inch.

89. A sum of £3000 is to be divided between 20 men, 64 women, and 96 children: if each man's share is to be equal to the shares of two women, and if the 64 women are to have twice as much as the 96 children, how much will the several individuals receive?

90. When a vulgar fraction is reduced to a decimal, in what cases will the decimal terminate? 91. In what cases will it recur?

92. What is the limit in the number of figures in the period?

93. Divide £177 7*s.* 0¾*d.* among four persons, in the proportion of the fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$.

94. Divide $4\cdot0\dot{3}$ by $\cdot1\dot{4}0\dot{7}$.

95. What is discount? If the discount on £567 be £34 14*s.* 3¾*d.*, simple interest being reckoned at 4½ per cent. per annum, when is the sum due?

96. A. barters sugar with B. for flour which is worth 2*s.* 3*d.* per stone, but uses a false stone weight of 13½ lbs. (instead of 14 lbs.), what value should B. set on his flour that the exchange may be fair?

97. A grocer sold 1 lb. of tea and 3 lbs., of sugar for 6*s.*, but if the tea rose 10 per cent. and the sugar 50 per cent. they would have cost 7*s.* What was the price per lb. of each? *

98. The populations of three towns, in the year 1841, were 20325, 42405, and 1423; and in the year 1851 they had increased respectively 9, 10, and 12 per cent.; find the average population of the three towns in 1851.

99. If goods which were bought at £2 5*s.* 10*d.* per cwt. be sold at £2 14*s.* 1*d.*, what is the gain per cent.?

100. A number of men were arranged into a hollow square 4 deep. In the outer rank there were 50 in the face of the square, and in the inner rank 44. How many men were there in all?

101. A grocer buys 3 cwts. of sugar at 5*d.* per lb., and 7 cwts. at 6½*d.*: he sells 5½ cwts. at 5½*d.* per lb.; at what rate per lb. must he sell the remainder in order to make 15 per cent. on his outlay?

102. A person sells out of the 3 per cents. at 98, and invests his money in railway 5 per cent. stock at par; find how much per cent. his income is increased.

103. The present prices of the 3 per cent. consols and Midland Railway stock, paying 5¼ per cent., are respectively 95⅜ and 108¾. Compare the rates of interest which investments in these stocks would give.

104. A person buys coffee at £5 12*s.* 6*d.* per cwt. and chicory at £2 5*s.* 5*d.*, and mixes them in the proportion of two of chicory to five of coffee. He retails the mixture at 1*s.* 3*d.* per lb. What is his gain per cent.?

105. By selling tea at 5*s.* 4*d.* per lb. a grocer clears ⅛ of his outlay: he raises the price to 6*s.* 2*d.* per lb.: what does he clear per cent. upon his outlay at the latter price?

105. (*a*). Three silver candlesticks, weighing in all 36 oz., cost £18, the silver being valued at £12 and the workmanship at £6; what should 5 candlesticks cost, each of which weighs 24 oz., and the workmanship charged for at twice the rate per ounce?

106. A and B enter into partnership. A puts in £2100 stock, B puts in £1500 stock; four months after C enters the partnership with a capital of £2700. At the year's end the profits are found to be 10 per cent. on the whole capital invested: how much of the profits should each receive?

107. What is the difference between interest and discount? If a person buys 325 cows for £5000, payable 12 months hence, and sells them immediately at £15 a head: find how much he gains by the transaction, reckoning money value for 5 per cent. per annum.

108. Two clocks strike 9 together on Tuesday morning; and on Wednesday morning one is 10 minutes to 11 when the other strikes 11: how many minutes must the slow one be put on so that they may both strike nine together on the same evening?

109. A person invests £5460 in the 3 per cents. at 91; he sells out £200 stock when they have risen to $93\frac{1}{2}$, and the remainder when they have fallen to 85. How much does he gain, or lose, by the transaction? If he invests the produce in the $4\frac{1}{2}$ per cents. at 102, what will be the difference in his income?

110. A person bought goods on the continent; the cost of freight and insurance was 15 per cent., and that of duty 10 per cent. on the original outlay; he was obliged to sell them at a loss of 5 per cent.; but if he had made £3 more of them he would have gained 1 per cent.: what was the original outlay?

111. The exchange at Paris upon London is at the rate of 25 francs 70 centimes for £1 sterling, and the exchange at Milan upon Paris is at the rate of 42 Austrian lire for 20 francs: find how many Austrian lire should be paid at Milan for a £20 note.

112. A ship carries 5 chronometers, and estimates Greenwich time by the mean time of the five. Two of them lose, respectively, 39″ in 35 days and 4·3″ in 6 days; the others gain, respectively, 5″ a week, 1·3″ a day, and 23″ in 14 days. By how much will the estimated time be in error after a cruise of 176 days?

113. After a certain number of men had been employed on a piece of work 24 days, and had half finished it, 16 men more were put on, by which the remaining half was completed in 16 days. How many were employed at first?

Ex. A person who has £3300 in the 3 per cents. sells out at 88, and invests in Exchequer Bills, which pay $2\frac{1}{2}$*d.* per cent. per day when the premium is 7*s.* 6*d.*: find the alteration in his income.

£3300 × 3 ÷ 100 = £99 first income.
Then $100\frac{3}{8}$: 88 : : £99 first income : £110 second income.
£3 × 240 : $2\frac{1}{2}$*d.* × 865.
∴ £110 − £99 = £11 increase. *Ans.*

114. Find the interest on £1376 11*s.* 3*d.* at $4\frac{3}{4}$ per cent., lent on the 6th April, and repaid on the 29th of August, the former day being excluded and the latter included.

115. A society subscribes £45 18*s.* 9*d.* to a charity; each member pays as many pence ar there are members in the society; find the number of members.

116. If 10 men can mow 126 acres in 12 days of $9\frac{1}{2}$ hours each, how many can be mown by 45 men in 11 days of $8\frac{4}{5}$ hours each?

117. What is the difference between the interest and discount on £350 for 3 years at 4 per cent.?

118. Find the amount of £7000 in 4 years, at 5 per cent. per annum, compound interest.

119. Two persons have each a capital of £12,000: the one invests it in the 3 per cent. consols at $90\frac{5}{8}$, the other in railway shares paying 5 per cent. at $103\frac{3}{4}$: find by how much the income of the one exceeds that of the other.

120. The expense attending the production of a book the retail price of which is 7*s.* 6*d.* is 2*s.* $4\frac{1}{2}$*d.* per copy. The publisher allows the bookseller 25 per cent. on the retail price, and gives 13 copies to the dozen; 3900 copies are printed and sold; the author is to have half the profits; how much will he receive?

121. In what time will the interest on £75 12*s.* 6*d.* equal the interest on £15 2*s.* 6*d.* for 105 days at any rate of interest?

122. Supposing that in England gunpowder is made of 75 parts of nitre, 10 of sulphur, and 15 of charcoal; in France of 77 of nitre, 9 of sulphur, and 14 of charcoal: if a ton of each be mixed, what weight of nitre, sulphur, and charcoal will there be in the compound?

123. By a reduction of the interest on Exchequer Bills from $2\frac{1}{2}$*d.* to $1\frac{1}{2}$*d.* per cent. per day a person loses at the rate of £152 7*s.* 9*d.* per annum, what amount of Exchequer Bills does he hold?

124. How many yards of matting, $4\cdot\dot{8}$ feet broad, will cover a floor that is $27\cdot\dot{3}$ feet long and $20\cdot1\dot{6}$ feet broad?

125. If $\frac{2}{57}$ of $3\frac{1}{2}$ of $7\frac{3}{5}$ of $\frac{4}{7}$ of 36 lbs. of sugar cost 12*s.* $6\frac{1}{2}$*d.*, how much will 17 tons 17 cwt. cost?

126. Convert 44 lbs. 11 oz. 11 dwts. 16 grs. troy into avoirdupois weight, one pound avoirdupois being to one pound troy as 175 to 144.

127. If a coach travelling 9 miles an hour occupies 4 hours more than a railway train in a journey of 69 miles, what is the proportion between the rates of the coach and the train?

128. If A invests £33,075 in the purchase of 3 per cents. at $87\frac{1}{2}$, and B invests the same sum in the purchase of 4 per cents. at $94\frac{1}{2}$, what

will be the difference in their net incomes after deducting income-tax at 16*d.* in the pound?

129. What is the value of one pound avoirdupois in troy weight?

130. Find the fractions equivalent to $\cdot 2\dot{6}$, $\cdot 83\dot{2}\dot{6}$, and $\cdot \dot{2}602739\dot{7}$.

131. If oranges be bought at the rate of 20 for a shilling, how many should be sold for £2 8*s.* to gain 40 per cent. on the outlay?

132. A can reap a field in 5 days and B in 6 days working 11 hours a day. In what time could A and B together reap it working 10 hours a day?

133. Railway stock is sold for 108, and with the proceeds the government stock bought at $91\frac{1}{8}$; after a time the government stock is sold at $95\frac{3}{8}$ and the original stock repurchased at 109, leaving a profit of £109 on the transaction: find the amount of stock sold.

134. What will be the cost of painting the walls of a room at 1*s.* 7*d.* per square yard, the length being 19 feet $10\frac{1}{4}$ inches, the breadth 16 feet $1\frac{3}{4}$ inches, and the height 10 feet 3 inches? Solve by decimals.

135. How much will £3500 amount to in 4 years at $4\frac{1}{2}$ per cent. compound interest?

136. At what rate per cent. will £1303 6*s.* 8*d.* amount to £1884 18s. 11*d.* in 7 years at simple interest.

137. If I buy 14 oxen for £157 5*s.* 10*d.* and sell 6 of them at £7 4*s.* each, for what must the remainder be sold that I may gain 4 per cent. on the whole?

138. Explain the difference between interest and discount, and find the present worth of £1215 due 4 years hence at $5\frac{3}{8}$ per cent.

139. If $1\frac{3}{7}$ of $\frac{3}{40}$ of $1\frac{3}{4}$ of a ton is worth £1 10*s.*, what is the value of $\frac{3}{5}$ of it?

140. $\sqrt{4\frac{213}{289}}$. 141. $\sqrt[3]{\cdot 51478848}$.

142. One horse gives 5 strides while another is giving 4, but the latter horse's is to the former's as 6 to 7; which is the faster, and how much would he gain in a distance of 4 miles?

143. The net rental of an estate, after deducting 7*d.* in the pound for income-tax and 5 per cent. on the remainder for expenses of collecting, is £959 3*s.* 8*d.*; find the gross rental?

144. Find the compound interest on £364 for 3 years at $3\frac{1}{4}$ per cent.

145. A person sells £7500 3 per cent. consols at $90\frac{3}{8}$ and invests it in railway shares paying 5 per cent. at $104\frac{3}{8}$; find how his income is affected, and the gain or loss per cent.

146. If by selling an article at 5*s.* 6*d.* I gain $\frac{3}{8}$ of my outlay, what would I have gained per cent. if I sold it at 6*s.* 6*d.*?

147. What sum of money will produce as much interest in $3\frac{1}{4}$ years as £210 3*s.* in 5 years 5 months?

148. The price of consols is 88⅜, how many railway shares, the market price of which is £8 15*s.*, can be purchased with £1000 consols?

149. The rates of the express and mail trains on a railway are 40 and 28 miles per hour respectively; what time is saved by taking the faster train for a journey of 192 miles?

150. Define discount. If the discount on a bill of £226 2*s.* 8*d.*, due at the end of a year and a half, be £12 16*s.*, what is the rate of interest?

151. If I lay out £1911 in 3 per cent. consols at 79½, at what price must I sell out, to gain £150, paying ⅛ per cent. commission on each transaction?

152. Extract the square root of 13104400.

153. Extract the cube root of 586376253.

154. Reduce ⅜ of 17*s.* 6*d.* to the decimal of a guinea.

155. A person invests £5617 10*s.* in the 3 per cents. at 93½. He sells out when they have fallen to 91¾, and invests the proceeds in Russian 5 per cents. at 98. What is the difference in his income under the two investments? Brokerage is to be reckoned at ⅛ per cent. on all the stock sold or bought.

156. A railway cutting is 38 feet wide at bottom, and 74 feet at top, 35 feet deep, and ¾ of a mile long: how many solid yards of earth were removed?

157. A traveller sets out to walk from A to B, at the rate of 4 miles an hour. One hour later another traveller sets out from B towards A at 3½ miles an hour: when they meet the first has walked 6 miles more than the second. Find the distance from A to B.

158. How much will £500 amount to in 3 years, at 4½ per cent., compound interest?

159. It is required to divide the number 34 into two parts such that the difference between the greater and 18 shall be the difference between 18 and the less in the ratio of 2 to 3.

160. A man rows 10 miles in 2½ hours against the stream, the rate of which is 3 miles an hour: how long will he be in rowing 5 miles with the stream?

161. A ship 40 miles from shore springs a leak which admits of 3¾ tons of water in 12 minutes; 60 tons would be sufficient to sink her, but the ship's pumps can throw out 12 tons of water in an hour. Find her average rate of sailing that she may reach the shore just as she is beginning to sink.

162. A tradesman's stock is valued at £8975 at the beginning of a year, during which he sells goods to the amount of £5023; 7½ per cent. of his receipts go for wages, and he expends £3782 in the

purchace of goods, and pays £75 shop-rent; at the end of the year his stock is value for £8763; how much does he gain or lose?

163. A solar day being 24 solar hours, a sidereal day is 23 hours, 56 minutes, 4 seconds $\frac{9}{100}$. A sidereal day being now divided into 24 sidereal hours, find the number of sidereal hours, minutes, and seconds in a solar day.

164. What must be the market value of 4 per cent. stock, in order that, after deducting the income-tax of 10*d.* in the pound, it may yield $4\frac{1}{2}$ per cent. interest?

165. Find the cost of laying with gold a table $6\cdot\dot{4}28571\dot{4}$ yards long and $1\cdot7\dot{8}$ yards wide, at 25*s.* the square inch.

166. If 24 pioneers in $2\frac{1}{2}$ days of $12\frac{1}{2}$ hours long can dig a trench 139·75 yards long, $4\frac{1}{2}$ yards wide, and $2\frac{1}{2}$ yards deep: how many hours per day must 180 pioneers work during 21 days in order to dig a trench $4910\frac{1}{16}$ yards long, $4\frac{7}{8}$ yards wide, and $3\frac{1}{8}$ yards deep?

167. Borrowed £220 at 5 per cent., and 12 months after £340 at 4 per cent.: when paid it took £607 18*s.* to discharge them both: how long had these sums been kept?

168. A had 12 pipes of wine which he sold to B at $4\frac{1}{2}$ per cent. profit, B sold them to C for £40 12*s.* profit, C sold them to D for £605 18*s.*, who cleared 6 per cent.: what did A give a gallon for the wine?

169. Bought a quantity of goods for £375, and 3 months after sold them for £412 10*s.*, how much per cent. per annum was the gain?

170. The content of a cistern is the sum of two cubes, whose edges are 10 inches and 2 inches, and the area of its base is the difference of two squares whose sides are $1\frac{1}{4}$ and $1\frac{3}{4}$ feet. Find its depth.

171. How much paper, $\frac{3}{4}$ yard wide, will be required for a room that is 22 feet long, 14 feet wide, and 9 feet high, if there be three windows and two doors, each 6 feet by 3 feet?

172. A stone 18 inches long, 17 broad, and 7 deep, weighs 278 lbs.: how many cubic feet of this kind of stone will freight a vessel of 230 tons burthen?

173. Express a degree, $69\frac{1}{12}$ miles in French metres, given that 32 metres are equal to 35 yards.

174. What is the present value of a bill of £151 17*s.* 6*d.* due at the end of 4 years, at $5\frac{3}{8}$ per cent. per annum?

175. If the 8*d.* loaf weighs 48 oz. when wheat is 52*s.* per quarter, what would be the price of wheat, when the 6*d.* loaf weighs 32 oz. 8 drs.?

176. Find exactly the square root of $1040520\frac{1}{289}$.

177. Find the cube root of 352045367·981.

178. Multiply by the method of duodecimals 3 feet 2 inches 5 parts by 1 foot 4 inches 7 parts, and the product by 4 feet 8 inches.

179. Transform the result of the above into cubic feet, cubic inches, and the fraction of a cubic inch.

180. A cistern has two pipes, one of which fills it in 30 minutes, the other in 40 minutes: it also has a discharge pipe, which empties it in 20 minutes. In what time will the cistern be filled, if all the pipes are set going?

181. Reduce to a decimal correct to 7 places.

$$\frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \text{\&c., \&c.}$$

182. In Reaumer's thermometer the freezing point is zero, and the boiling point 80°. In the common thermometer (Fahrenheit's), the freezing point is 32°, and the boiling point 212°. What does Reaumer's mark, when Fahrenheit's marks 62°, 10°, and — 18° respectively?

183. A person converts £27,851 17*s.* 3*d.* into 3 per cent stock at $78\frac{3}{8}$, and invests the same sum in a $3\frac{1}{2}$ per cent. stock at $95\frac{3}{16}$: he sells out when the former has risen $\frac{1}{16}$ per cent. and the latter has fallen $\frac{1}{16}$ per cent. How much has he gained?

184. A cistern contains 600 gallons. Its external dimensions are 7 ft. in length, 2 ft. 11 in. width, and 2 ft. 6 in. depth: if the thickness of the sides be $1\frac{1}{2}$ in., find the thickness of the bottom. ($6\frac{1}{4}$ gallons = 1 c. foot.)

185. Two trains start from opposite stations. The faster from A would reach B in 2 hours; had the slow one kept time they would have met at $\frac{3}{8}$ the whole distance from B, but owing to an accident to it they met at $\frac{1}{4}$ the whole distance from B; how much was the slow train late on reaching A?

186. If 25 thalers are equal to 3 francs, 5 scudi equal to 27 francs, 135 gulden equal to 62 scudi, 10 gulden equal to £1, how many thalers in £3?

Passengers.	Great Britain.	France.	Prussia.
1st Class	4,743,213	2,124,917	307,492
2nd Class	10,291,749	6,172,429	1,292,476
3rd Class	21,409,217	15,473,555	2,374,237

187. From the above table, find the proportion per cent. that the whole number of passengers in Prussia bears to the whole number of second-class passengers.

188. Multiply by the method of duodecimals 8 feet 5 inches by 6 feet 7 inches 11 parts, and express the result in square inches and the fraction of a square inch.

189. If 24 pioneers in 2 days of 12 hrs. each, can dig a trench 140 yds. long, 4 yds. broad, and 2 yds. deep, in how many days of 9 hrs. each, will 180 pioneers dig a trench 480 yds. long, 4 broad, and 3 deep?

190. In a class of 21 children, there are 7 boys and 14 girls. The ages of the boys are: 3 boys are 8 yrs., 2 are 11, and 2 are 12. The girls' ages are: 4 of them 9 yrs., 2 are 10, 4 are 13, and 4 are the average age of the boys. Find the average age of the whole class.

191. A person transfers £300 stock, 3 per cent consols at 99, to Exchequer bills, paying 2½*d.* per cent. per day at a premium of 7*s.* 4*d.* Find the alteration in his income.

192. A person goes to France with 33 sovereigns, ½ a sovereign, and 7 florins: he spends 577 francs 50 centimes: how many florins has he remaining, supposing the exchange to be 26 francs 25 centimes for £1?

193. The cost of carpeting a room, twice as long as it is broad, at 5*s.* a square yard, amounted to £6 2*s.* 6*d.*: and the painting of the walls at 9*d.* a square yard amounted £2 12*s.* 6*d.* Find the height of the room.

194. What would be the cost of papering a room 18 feet, 8¼ inches long, 16 ft. 3¾ in. broad, and 10 ft. 3½ inches high, at 4¾*d.* per square yard? Solve by decimals.

195. Extract the cube root of 1194389981; and of $35\frac{162}{343}$.

196. A person invests in the 4 per cents., and receives 5 per cent. interest for his money: required the price of the stock.

197. If 25 thalers = 93 francs: 135 gulden = 62 scudi: 10 gulden = £1: £3 = 20 thalers: find the number of francs in 5 scudi.

198. Subtract 5 florins 3 cents. 5 mils from 9 florins 6 cents. What fraction of £4 13*s.* 6*d.* is the remainder?

199. Multiply by the method of duodecimals 3 feet 7 inches by 4 feet 9 inches 11 parts; and show what the product will become when reduced to square inches and the fraction of a square inch.

200. The average number of people passing a certain point between the hours of 8 A.M. and 10 P.M. is 105 per minute: and between 10 P.M. and 8 A.M. the number is 14 per minute. Find how many pass in a year.

201. From the table given in preceding page, find the proportion per cent. which the whole number of 1st and 2nd class passengers bears to the whole number of 3rd class passengers.

202. Required the length of the edge of a cube, whose content is 22,214 yards 10 feet 64 inches.

203. At what price must the 3½ per cents. be in order that they may give the same income as an investment in the 3 per cents. at 72?

204. A certain sum, if invested in the 3 per cents., would give an annual income of £105, and if invested in railway shares at par,

paying 5¼ per cent., would yield £177.6*s*. 4½*d*. Find the sum and the price of consols.

205. The distance between two towns, A and B, is 57½ miles, and the fare by railway 11*s*.: the distance between two other towns, C and D, is 20½ miles, and the fare 2*s*. How much must be added to the fare per mile between A and B to make it exactly double the fare between C and D?

206. The capital of a railway company is £11,400,000. Besides this there is £3,420,000 preference shares paying 5¼ per cent., and £2,800,000 debentures, paying on an average 4¼ per cent. per annum. The half-yearly receipts are £725,000, and the current expenses are 59 per cent. of the receipts: find the rate per cent. for the shareholders half-yearly, laying aside £1200.

207. How many yards of carpet ¾ of a yard wide will cover a room 26 ft. long and 16 ft. broad? find its cost at 4*s*. 6*d*. per yard.

208. Find the value of the series $1 + \frac{1}{1} + \frac{1}{1 \times 1} + \frac{1}{1 \times 2 \times 3} +$ &c., as far as six places of decimals.

209. Reduce 1·875 of a day to seconds; bring 3″ to decimal of a week; and 3 pints to the decimal of a quarter.

210. Required the present worth of £2369 5*s*. for ¼ of a year at 5 per cent. per annum, and find value of $\frac{\sqrt{8}+\sqrt{5}}{\sqrt{8}-\sqrt{5}} + \frac{\sqrt{8}-\sqrt{5}}{\sqrt{8}+\sqrt{5}}$.

211. Find the price of 10740 cubic feet of gas, at 4*s*. 2*d*. per 1000 cubic feet; also of 5 tons 3 qrs. 17 lbs. at £7 3*s*. 4*d*. per ton.

212. A person has £3000 in the 3 per cents.: he sells out at 99 per cent. and invests in Exchequer bills at 2¼*d*. per cent. per day when the premium is 7*s*. 6*d*.; what is the effect on his income?

213. Extract the cube root of 428661064.

214. Multiply by the method of duodecimals 4 feet 5 inches by 2 feet 7 inches 11 parts; express the result in square inches and fraction of a square inch.

215. A person goes to France with 33 sovereigns, 1½ sovereign, 7 florins. He spends 577 francs 20 centimes: how many florins does he bring home, the exchange being 26 francs 25 centimes to the pound?

216. If 240 pioneers in 2 days of 4 hrs. each, dig a trench 140 yds. long, 4 yds. deep, and 2 yds. wide, in how many days of 12 hrs. each should 980 pioneers dig a trench 1000 yds. long, 4 yds. deep, and 3 yds. wide.

217. What is meant by a circulating decimal; and how is it known whether a vulgar fraction will give a terminate decimal?

218. Find the value of: $(\cdot 5\dot{3} \div \cdot 0261\dot{6}) \div \cdot 3\dot{5}67\dot{8}$.

219. What interest per cent. per annum has a person for his money who buys in the 4 per cents. at 95⅜?

220. Find the amount, at compound interest, of £2500 payable quarterly for 2 yrs. at 4 per cent.

221. Pure gold is £3 17*s.* 10$\frac{1}{2}$*d.* per ounce—gold coin is compounded of 22 parts pure gold and 2 parts alloy; the alloy being worth 1$\frac{1}{2}$*d.* per oz., what is the value of alloy in £100?

222. A person sold at 104$\frac{5}{8}$; he invests in 3 per cent. consols at 93; after getting a half year's dividend he sells at 93$\frac{3}{8}$, his whole gain being £168 15*s.*; how much railway stock did he sell?

223. A bankrupt whose assets were £70 10*s.* paid 3$\frac{1}{2}$*d.* in the pound to his creditors; by how much were they losers?

224. If 1200 passengers travelling on an average of 30 miles each give a weekly return to a railway company of £1600, how many will give a yearly return of £286,000, travelling 25 miles each, if the fares be reduced by one-fourth?

225. A can do a piece of work in 6 days of 8 hours each, B in 5 days of 10 hours each, and C. in 12 days of 6 hours; how many hours can they do twice as much work all working together?

226. Two trains 92 and 84 feet long respectively, when moving with uniform velocities on parallel rails in opposite directions pass each other in 1$\frac{1}{2}$ seconds, but when they move in the same direction the faster passes the other in 5 seconds; find their rates of speed.

227. A person who owes £100 pays £25 both for principal and interest yearly; how much is due at the end of 3$\frac{1}{2}$ years at 5 per cent.?

LXIV.

Averages and Per-centages. (*Time,* 2 *hours.*)

1. Find the average of $13\frac{1}{2}\frac{2}{5}$, 21, $7\frac{3}{4}$, ·0023, $3\frac{1}{8}$, 0, $106\frac{1}{2}$, and $57\frac{7}{20}$: express the fractional part decimally.

2. If by selling wine at 15*s.* per gallon I lose 6 per cent., at what price must I sell it to gain 17$\frac{1}{2}$ per cent.?

3. Of 32 selected candidates for the East Indian Civil Service in 1859, 3 were above 20 years of age when they went to India, 4 above 21, 12 above 22 and 23 respectively, and 1 above 24. From these data, find what is the average age at which the men went to India.

4. A merchant has teas worth 4*s.* 6*d.* and 3*s.* 6*d.* per lb. respectively, which he mixes in the proportion of 3 lbs. of the former to 2 of the latter, and sells the mixture at 4*s.* 4*d.* per lb.: what does he gain or lose per cent.?

5. Between the years 1841 and 1851 the population of England increased 14·2 per cent. In the latter year it was 21121290. What was it in the former year?

6. A person invests £5460 in the 3 per cents. at 91, he sells out 2000 stock when they have risen to 93$\frac{1}{2}$, and the remainder when they have fallen to 85: he then invests the produce in the 4$\frac{1}{2}$ per cents. at 102. What is the difference in his income?

7. What must be the market value of 6 per cent. stock, in order that after deducting the income-tax of 10*d.* in the pound, it my yield 6$\frac{1}{4}$ per cent. interest.

8. Reduce $\frac{5{\cdot}1183}{{\cdot}00705}$ of $11{\cdot}\dot{1}$ of ${\cdot}2\dot{9}$ of ${\cdot}1\dot{1}\dot{7}$ to a vulgar fraction.

These Questions were also given at Competitive Examinations.
(*Time 2 hours.*)

LXV.

1. When a 3¼ per cent. stock is at 93, find what price a 4½ per cent. stock must bear, that an investment may be made with equal advantage in either stock.

2. A person sells Midland Stock paying 6½ per cent. at 128½, and invests in Great Western stock paying 3 per cent. at 72½. By how much per cent. will the interest of his investment be altered?

3. A person invests £5000 in the new 6 per cent. Turkish loan, issued at 68 per cent. at 2½ premium. How much stock will he have, and what rate of interest will the investment give?

4. What must be the market value of 3 per cent. stock, in order that, after deducting an income-tax of 10*d.* in the pound, it may yield 3½ per cent. interest?

5. What is meant by the *par of exchange* between two countries? When is the exchange said to be against a country? Explain briefly why the course of exchange between two countries varies.

6. If £3 = 20 thalers; 25 thalers = 93 francs; 27 francs = 5 scudi; and 62 scudi = 135 gulden: how many gulden = £1?

7. A trader in London owes a debt of 1000 pistoles to one in Cadiz: find what he gains by sending it to him through France, the exchanges being £1 = 25·4 francs; 19 francs = 1 Spanish pistole; 4 Spanish pistoles = £3.

8. A person in London owes another in St. Petersburg 920 roubles, which must be remitted through Paris. He pays the requisite sum to his broker, at a time when the exchange between London and Paris is 25·15 francs for £1, and between Paris and St. Petersburg 1·2 francs for 1 rouble. The remittance is delayed until the rates are 25·35 francs for £1, and 1·15 francs for 1 rouble. What does the broker gain or lose by the delay?

9. An event occurred on 30th January, in the last century, within 14 years of the end of it; find the year, the day of the week having been Sunday?

10. A and B set out together from the same place and travel in the same direction; after A, who goes uniformly 18 miles a day, travels 9 days, he turns back and goes as far as B has gone in those 9 days; he turns again and pursuing his journey overtakes B in 22½ days from they first set out, at what rate did B travel?

The following 15 Questions were proposed to Candidates who had already passed a Test Examination. (Time 3 hours.)

LXVI.

1. If a steamer makes the passage from New York to Liverpool (say 2,760 miles) in 9 days 14 hours, and a train goes from London to Edinburgh (say 405 miles) in 18 hours: compare the rates of the steamer and the train.

2. Find the square root exactly of $2515\frac{294}{361}$.

3. Extract the cube root of 5·78 to three places of decimals.

4. Multiply by the method of duodecimals 3 feet 1 inch 11 parts by 2 feet 6 inches 7 parts, and the product by 1 feet 7 inches.

5. Express the result of the last question in cubic feet, cubic inches, and a fraction of a cubic inch.

6. Divide $4\cdot0\dot{3}$ by $\cdot1\dot{4}0\dot{7}$.

7. Find the average of $21\frac{2}{3}$, $73\frac{4}{5}$, 0, 3·065, 82, $17\frac{3}{20}$, $5\frac{1}{4}$, $9\frac{5}{12}$: express the fractional part decimally.

8. A person sells as many 3 per cent. consols at $98\frac{5}{8}$ as produce £2000, and invests this sum in railway stock, paying $4\frac{1}{4}$ per cent. at $93\frac{3}{4}$. How is his income affected?

9. A person buys coffee at £5 12*s.* 6*d.* per cwt., and chicory at £2 5*s.* 5*d.* per cwt. and mixes them in the proportion of 2 of chicory to 5 of coffee. He retails the mixture at 1*s.* 3*d.* per lb. What is his gain per cent.?

10. Find the true discount on £512 15*s.* 3*d.*, due 52 days hence, at $2\frac{1}{2}$*d.* per cent. a day.

11. If 5 men can perform a piece of work in 12 days of 10 hours each, how many men will perform a piece of work four times as large in a fifth part of the time, if they work the same number of hours in the day, supposing that 2 of the second set can do as much work in an hour as three of the first set.

12. A canal 10 miles long is 8 yards wide at the top, 6 yards wide at the bottom, and 5 feet deep. How soon would the excavation of it be completed by 800 men, each removing on an average 15 cubic yards per day?

13. The rate of a clock is ·0375 *per cent.* too fast. How much will the clock gain in a week?

14. A vessel whose speed was $9\frac{1}{4}$ miles per hour started at 8 o'clock to go a distance of 74 miles. A second vessel, whose speed was to that of the first as 8 to 5, starting from the same place, arrived 5 minutes before the first. When did the second vessel start?

15. At a siege it was found that a certain length of trench could be dug by the soldiers and navvies in 4 days, but that when only half the navvies were present, it required 7 days to dig the same length of trench. What proportion of the work was done by the soldiers?

LXVII.

London University.

1. Find the least number divisible by all the numbers from 1 to 12.

2. Simplify the expression:

$$\frac{4\frac{1}{2}-3\frac{1}{3}+5\frac{1}{12}}{7\frac{1}{2}-4\frac{1}{3}+11\frac{1}{12}}-\frac{11\frac{3}{4}-5\frac{7}{15}}{11\frac{3}{4}+5\frac{7}{15}}.$$

3. Extract the square roots of 443556; ·0000004489; and ·4.

4. Divide ·000356 by ·0000057, and calculate to 5 places of decimals the value of $\frac{1}{\sqrt{2}}$

5. Define Interest, Discount, and Present Value, and find the present value of £911 13*s.* 3*d.* due 5 years hence at 3 per cent. simple interest.

6. If the $3\frac{1}{2}$ per cents. be at £$95\frac{5}{8}$ and the 3 per cents. at 82, which is the better investment?

7. A person walking along the road in a fog meets one waggon and overtakes another which is travelling at the same rate as the former, and he observes that between the time of his first seeing and passing the waggons, he walks 20 yds. and 60 yds. respectively; find how far he can see in the fog, and compare his rate of walking with the rate at which each waggon is moving?

8. The weights of pure gold and of alloy in a sovereign are very nearly in the ratio of 113 : 10 and the price of pure gold is £3 17*s.* $10\frac{1}{2}$*d.* per oz.; find the weight of a sovereign, supposing its value to be that of the pure gold it contains.

9. Find the square root of $\cdot\dot{0}12345678\dot{9}$.

10. How many yards of carpet, three quarters of a yard wide, will cover a room whose width is 20 ft. and length 22 ft.?

11. By selling tea at 4*s.* 6*d.* per lb. a grocer cleared $\frac{1}{11}$ of his outlay, what would he have gained if he had sold the tea at 5*s.*?

12. Express $\frac{2}{9}$ of 7*s.* 6*d.* + ·625 of 16*s.* as a decimal of £2 1*s.* 8*d.* Divide 2·88 by ·00016.

13. Find the square of $\cdot\dot{2}$ in the form of a recurring decimal; and find $\sqrt{\cdot 69\dot{4}}$.

14. An electric cable is laid between two ports and it is found that the gross annual receipts amount to 20 per cent. on the original outlay;

the annual cost of repairs is 2 per cent. on the original cost, and the expense of working is found to be £600 a year, in addition to 20 per cent. on the *net* receipts. After a year a second cable is laid down, and it is found that the gross receipts and the expense of working are doubled, while the cost of repairs for the new cable is only 1 per cent., and that the *net* receipts now amount to £14500. Find the cost of a cable and the *net* receipts for the first year.

15. A person invests in £10 railway shares when they are at a premium of 10*s*. At the end of a year he receives a guinea per share: what interest does he get?

16. If 81 bushels of wheat are consumed by 56 men in 5 days, how long will 16 men take to consume 28 bushels?

Miscellaneous.

17. Reduce 875 seconds to the decimal of a day.

18. A merchant bought a fifty gallon cask of wine for £37 4s. It lost $3\frac{1}{2}$ gallons by leakage. At what price per dozen must he sell the remainder, to clear 15 per cent. on his outlay, if 9 bottles = 1 gallon.

19. A legacy of £897 1*s*. is to be divided among 3 persons A, B, C. A is to receive $\frac{1}{7}$; B is to receive $\frac{2}{3}$, and C the remainder. Find the sum to be received by each, and the fraction of the whole paid to C.

20. In what number of years will £1876 10*s*. amount to £2439 9*s*. at 4 per cent. simple interest?

21. A wine merchant mixes two kinds of wine and sells the mixture so as to gain 8 per cent. on what the wine cost him. Had he sold each kind of wine at the same price per gallon as he sells the mixture, he would have gained 10 per cent. and 6 per cent. respectively on the cost price of each. In what proportion were the two kinds of wine mixed?

22. The stuff out of a lead mine contains at first 15·9 per cent. of lead. After washing, by which process the amount of lead ore is not diminished, the stuff contains 87·45 per cent. of lead. How much rock was washed away out of 216 tons 5 cwts. of the original stuff?

23. The income derived by a legatee from money invested in his behalf in the 3 per cents. at $93\frac{1}{2}$ is £68 3*s*. 6*d*. What was the amount of the legacy?

Direct Commissions—January 1863.

24. In standard gold 11 parts of 12 are pure gold. What weight of alloy is there in 3 oz. 5 dwts. of standard gold?

25. Add $\frac{5}{7}$, $\frac{1}{2}$, $\frac{1}{8}$, and $\frac{3}{28}$ of $3\frac{1}{3}$.

26. Find the value of ·36875 of a sovereign.

27. Divide 27·5264 by ·0374.

28. In how many days will the interest on £4600 at 1½*d.* per cent. daily amount to £18 8*s.*?

29. If the corn of 13 horses for 63 days cost £17 6*s.* 8*d.* when corn is 4*s.* per bushel, how many horses will cost £10 13*s.* 4*d.* for corn in 56 days when corn is 4*s.* 6*d.* per bushel?

30. Extract the square roots of 8 and ·0002 each to three places of decimals, and multiply the square roots together.

Woolwich, 1861.

31. A sovereign standard gold weighs 5·136 dwts.; a shilling standard silver weighs $\frac{1}{66}$ of a lb. Troy; what weight in standard silver is equal to 4 oz. standard gold?

32. If a merchant with a capital of £20000 gain £500 in 3 months, what sum will he gain with a capital of £30000 in 7 months?

33. Add together $\frac{5}{18}$ of a guinea, $\frac{3}{32}$ of a pound, $\frac{7}{20}$ of a crown, and $\frac{5}{16}$ of a shilling, and reduce the result to the decimal of a pound.

Queen's University, Woolwich, etc.

34. Reduce to simple fractions:

$$\frac{2}{3}+\frac{3}{4}+\frac{1}{12},\ \frac{3}{5}+\frac{1}{2}-\frac{7}{10},\ \text{and}\ \frac{2}{3}\times\frac{9}{10}\times4\frac{1}{4}.$$

35. Divide $\frac{3}{4}$ of $2\frac{1}{3}$ by $\frac{5}{8}$ of $7\frac{1}{2}$.

36. Find the simple interest of £275 10*s.* for 2 years 146 days, at 5 per cent.

37. Extract the square root of 1562500 and of ·015625.

38. Multiply $3\frac{1}{2}$ by $4\frac{1}{4}$ and from the result subtract $7\frac{5}{8}$.

39. At what per cent. simple interest will 3½d. produce 3½s. interest in 33⅓ years?

40. In the 3 per cents. what fraction of a given amount of stock is paid for annual interest—(1) without any deduction; (2) after 9d. in the pound has been deducted for income tax?

41. If an oz. of standard gold, of which the weight of the alloy is represented by two parts out of 12, be worth £3 17s. 6d., find the value of 10 lbs. of jewellers' gold in which the weight of alloy is represented by the decimal ·41̇6̇, the value of the alloy being neglected?

42. A and B can do a piece of work in 4 days, B and C in 6 days, and A and C in 8 days; in what time could they all do it working together?

43. Ten excavators dig 12 loads of earth in 16 hrs., whilst 12 others can only dig 9 in 15 hrs.; in what time will they jointly dig 100 loads, and in what time 1 man of each set separately?

purchace of goods, and pays £75 shop-rent; at the end of the year his stock is value for £8763; how much does he gain or lose?

163. A solar day being 24 solar hours, a sidereal day is 23 hours, 56 minutes, 4 seconds$\frac{9}{100}$. A sidereal day being now divided into 24 sidereal hours, find the number of sidereal hours, minutes, and seconds in a solar day.

164. What must be the market value of 4 per cent. stock, in order that, after deducting the income-tax of 10*d.* in the pound, it may yield $4\frac{1}{2}$ per cent. interest?

165. Find the cost of laying with gold a table $6\cdot4\dot{2}8571\dot{4}$ yards long and $1\cdot7\dot{8}$ yards wide, at 25*s.* the square inch.

166. If 24 pioneers in $2\frac{1}{2}$ days of $12\frac{1}{2}$ hours long can dig a trench 139·75 yards long, $4\frac{1}{2}$ yards wide, and $2\frac{1}{2}$ yards deep: how many hours per day must 180 pioneers work during 21 days in order to dig a trench $4910\frac{1}{16}$ yards long, $4\frac{7}{8}$ yards wide, and $3\frac{1}{5}$ yards deep?

167. Borrowed £220 at 5 per cent., and 12 months after £340 at 4 per cent.: when paid it took £607 18*s.* to discharge them both: how long had these sums been kept?

168. A had 12 pipes of wine which he sold to B at $4\frac{1}{2}$ per cent. profit, B sold them to C for £40 12*s.* profit, C sold them to D for £605 18*s.*, who cleared 6 per cent.: what did A give a gallon for the wine?

169. Bought a quantity of goods for £375, and 3 months after sold them for £412 10*s.*, how much per cent. per annum was the gain?

170. The content of a cistern is the sum of two cubes, whose edges are 10 inches and 2 inches, and the area of its base is the difference of two squares whose sides are $1\frac{1}{3}$ and $1\frac{2}{3}$ feet. Find its depth.

171. How much paper, $\frac{3}{4}$ yard wide, will be required for a room that is 22 feet long, 14 feet wide, and 9 feet high, if there be three windows and two doors, each 6 feet by 3 feet?

172. A stone 18 inches long, 17 broad, and 7 deep, weighs 278 lbs.: how many cubic feet of this kind of stone will freight a vessel of 230 tons burthen?

173. Express a degree, $69\frac{1}{22}$ miles in French metres, given that 32 metres are equal to 35 yards.

174. What is the present value of a bill of £151 17*s.* 6*d.* due at the end of 4 years, at $5\frac{3}{8}$ per cent. per annum?

175. If the 8*d.* loaf weighs 48 oz. when wheat is 52*s.* per quarter, what would be the price of wheat, when the 6*d.* loaf weighs 32 oz. 8 drs.?

176. Find exactly the square root of $1040520\frac{1}{289}$.

APPENDIX I.

GREATEST COMMON MEASURE.

177. The G. C. M. of two or more numbers is the product of their common prime factors; thus, if it be required to find the G. C. M. of 120 and 168; the prime factors of 120 are 2 × 2 × 2 × 3 × 5, and those of 168 are 2 × 2 × 2 × 3 × 7: therefore the factors common to both numbers are 2 × 2 × 2 × 3, and their product is 24, which is the G. C. M. required. In finding the G. C. M. any one of the numbers may be divided by a measure by which the other numbers are not divisible.* For instance, 168 might have been divided by 7, and the quotient 24 used instead. The following contracted method of finding the G. C. M., founded upon these principles, will be understood from the examples below.

Ex. 1. Find the greatest common measure of 161 and 253.

After writing the numbers, we see by inspection 7 will measure 161, and not 253: we divide by 7, and find for quotient 23, a *prime* number, which we find by trial measures 253.

$$\begin{array}{rrl} & \cancel{161} & \quad 253 \\ \text{G. C. M.} & 23) & 253(11 \\ & & \underline{253} \end{array}$$

Ex. 2. Find the G. C. M. of 1216, 832, 424.

Here we find by inspection that 8 will measure the three numbers. We divide 152 and 104 by 2, a factor which cannot enter into the G. C. M. of the quotients, because it is not a factor of 53. Now 52 and 53 being *prime to each other*, and neither of them a measure of 76, we conclude that the G. C. M. of the given numbers is 8.

$$\begin{array}{rrrr} 8 & \cancel{1216} & 832 & 424 \\ & \cancel{152} & \cancel{104} & 53 \\ & 76 & 52 & \end{array}$$

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* The number used as divisor must not contain a factor which is common to any of the numbers it does not divide. It sometimes, but very rarely, happens that no common factor can be found.

DISCOUNT.

180. The following formulæ, the reason of which is evident from the nature of proportion, will be found useful.

Let P = Present worth, D = Discount, X = the value of the bill at some future time, A = Amount, and I = Interest of £100 for the same time and at the same rate, and R = rate per cent.

$$\text{I.}\quad P=\frac{£100\times X}{A}.$$

$$\text{II.}\quad D=\frac{X\times I}{I+£100}.$$

$$\text{III.}\quad X=\frac{A\times P}{100},\ \text{or}\ \frac{A\times D}{I}.$$

$$\text{IV.}\quad I=\frac{A\times D}{X}.$$

$$\text{V.}\quad R=\frac{100\times D}{P\times T}.$$

In working No. 11, given in page 144, we apply the second of the two formulæ, III., and we have $\frac{£105\times\cancel{15}}{\cancel{5}}=£105\times 3=£315$. *Ans.*

181. Square Root (Formula for).

$$(x+y)^2=x^2+2xy+y^2.\quad \text{(Euclid, Prop. 4, B. II.)}$$

$$34=30+4\ \therefore 34^2=30^2+2(30\times 4)+4^2=30^2+(2\times 30+4)4=1156$$

$$\sqrt{1156}=\sqrt{900+256}$$

$$\begin{array}{r|l} & 900\quad (30+4\ \textit{Ans.} \\ \hline 2\times 30+4=60+4 & 256 \\ & 256 \\ \hline \end{array}$$

$$\sqrt{1156}$$

$$\begin{array}{r|l} & 9\quad (34\ \textit{Ans.} \\ \hline 64 & 256 \\ & 256 \\ \hline \end{array}$$

182. Cube Root (Formula for).

$$(x+y)^3=x^3+3x^2y+3xy^2+y^3=x^3+(3x^2+3xy+y^2)y$$

$$25=20+5\ \therefore 25^3=20^3+(3\times 20^2+3\times 20\times 5+5^2)5=15625$$

$$\sqrt[3]{15625}$$

$$\sqrt[3]{15625}=\sqrt[3]{8000+7625}$$

$$\begin{array}{r|l} 20^3= \quad 8000 & (20+5\ \textit{Ans.} \\ \hline 3\times 20^2=1200 & 7625 \\ 3\times 20\times 5=\ \ 300 & \\ 5^2=\ \ \ 25 & \\ \hline 5\times 1525 & 7625 \\ \hline \end{array}$$

$$\sqrt[3]{15625}$$

$$\begin{array}{r|l} 8 & (25\ \textit{Ans.} \\ \hline 3\times 20^2=1200 & 7625 \\ 3\times 20\times 5=\ \ 300 & \\ 5^2=\ \ \ 25 & \\ \hline 1525 & 7625 \\ \hline \end{array}$$

DECIMAL COINAGE.

183. The following system of coinage, based upon the pound sterling as its standard, has elicited a large amount of approval for its admirable simplicity and its little interference with our present currency.

'Pound and Mil System.'

10 mils = 1 cent.
10 cents = 1 florin.
10 florins = £1.

It is obvious from the above table that the cent would be value for $24d. \div 10 = 2\frac{2}{5}$, or $2\frac{1}{2}d.$ very nearly, and that the mil would be value for $2\frac{2}{5}d. \div 10 = \frac{12}{50}d.$, or very nearly one farthing.

This system, when introduced, will have the effect of rendering Compound Arithmetic entirely unnecessary; an equivalent to adding nearly twenty minutes to each school day in our primary schools.

Ex. 1. Reduce £16 7 florins 6 cents 5 mils to mils.

Here we have 16765 m. at once, without performing any reduction. The complete reduction would be

$$\begin{array}{llll}
\pounds & \text{fl.} & \text{c.} & \text{m.} \\
16 & 7 & 6 & 5 \\
\underline{10} & & & \\
\multicolumn{4}{l}{160 + 7 = 167 \text{ florins}} \\
\multicolumn{4}{l}{\qquad\quad \underline{10}} \\
\multicolumn{4}{l}{\qquad\quad 1670 + 6 = 1676 \text{ cents}} \\
\multicolumn{4}{l}{\qquad\qquad\qquad\qquad \underline{10}} \\
\multicolumn{4}{l}{\qquad\qquad\qquad\qquad 16760 + 5 = 16765 \text{ m.}}
\end{array}$$

Ex. 2. Reduce 74689 mils to pounds, &c.

If we place the decimal point three places to the left, we have the required pounds, thus £74·689; if two places, in florins—thus 746·89 fl.; and if only one place, in cents—thus, 7468·9 c.

The following example in Addition will show more fully the advantages of a decimal coinage.

Ex. 3. Add together

£	fl.	c.	m.		mils.
24	7	6	3	=	24763
19	6	5	2	=	19652
46	9	4	7	=	46947
49	7	8	5	=	49785

141147m = £141 1fl. 4c. 7m. *Ans.*

Ex. 4. Reduce £78 17*s.* 10½*d.* into the new coinage.

We have given a simple and accurate rule for reducing shillings and pence to the decimal of a pound (**97**), of which we now avail ourselves.

17*s.* = ·85
10½*d.* in farthings = ·042
(⅛ of 10·5) = ·00175
17*s.* 10½*d.* = ·89375

Consequently in the new coinage it is written 78893·75m. = £78 8fl 9c. 3·75m.

Although a more simple method might be given, yet this has the great advantage of being rigorously accurate. The reverse operation—reducing any sum in the new to its equivalent value in the present currency—may be performed by expressing it in pounds, and then finding the value of the decimals of a pound by the rules already given.

Ex. 5. Express £78 8fl. 9c. 3·75m. in £ *s.* *d.*

£78·89375
20
17·875~~00~~
12
10·5~~00~~
4
2·~~0~~
£78 17*s.* 10½*d.* *Ans.*

We can work this and similar questions very expeditiously by (**99**).

THE METRIC SYSTEM.

184. This system, at present in use on the Continent, was recommended by a committee of the House of Commons in 1862; and 'for the promotion and extension of our internal as well as our foreign trade, and for the advancement of science' is legalised in this country by an Act of Parliament, dated July 29th, 1864. The *metric system* was formed in France in 1789, and derives its name from the *mètre* which is its base, which was obtained by actual measurement, being one-ten-millionth part of the quarter of a meridian—the distance on the arc from the pole to the equator. All other measures are either multiples or sub-multiples of the mètre in a tenfold ratio, and consequently are subject to the same laws of calculation as the numbers in our common decimal notation. The mètre is a little longer than our yard, being 39·371 inches: *its square* is the standard for surfaces, *its cube* for solids, and a cube described upon the one-hundredth part of its length the unit of weight.

When greater lengths are expressed, such as distance between towns, it is usual to express them in kilometres; minute distances, on the contrary, are expressed in millimetres.

			British yds.	feet	inches.
1 mètre			1	0	3·3708
10 mètres	= 1 décamètre	=	10	2	9·7079
100 mètres	= 1 hectomètre	=	109	1	1·079
1000 mètres	= 1 kilomètre	=	1093	1	10·79
10,000 mètres	= 1 myriamètre	=	10,936	0	11·79
($\frac{1}{10}$) ·1 of a mètre	= 1 décimètre	=	.	.	3·9371
($\frac{1}{100}$) ·01 „ mètre	= 1 céntimètre	=	.	.	·3937
($\frac{1}{1000}$) ·001 „ mètre	= 1 millimètre	=	.	.	·0394

The prefixes *deca*, &c. which are used in the multiples, are taken from the *Greek: deci*, &c. forming the subdivisions, are taken from the *Latin.* It is usual to place the decimal point after the mètre, and then the number is expressed in mètres, the figures to the right being tenths, hundreds, &c. of a mètre, just as in the common notation numbers are expressed in units. We can express a number in any other denomination by shifting the point: thus 1347·3 metres when written 1·3473 become kilometres, and when written 13473 become decimetres.

SURFACE.

The principal unit is the *square metre;* land measure is usually expressed in square decametres, which are called ares.

	Square metres.	British square yds.
Centiare, i.e. $\frac{1}{100}$ are . . .	1	1·1960
Are	100	119·6033
Dekare, i.e. 10 ares . . .	1,000	1196·0333
Hectare, i.e. 100 ares . . .	10,000	11,960·3326

CAPACITY.

The principal unit is the *cubic metre*; the standard for liquids and grain is the *litre.*

	Litres.	Cubic metres.	British pints.
Kilolitre . . .	1000	1	1760·77
Hectclitre . .	100	$\frac{1}{10}$	176·077
Dekalitre . .	10	$\frac{1}{100}$	17·6077
Litre . . .	1	$\frac{1}{1000}$	1·76077
Decilitre . . .	·1	$\frac{1}{10000}$	·176077
Centilitre. . .	·01	$\frac{1}{100000}$	·0176077

WEIGHT.

The fundamental unit of weight is the *gram*. The kilogramme is the weight *in vacuo* of a decimetre cube of distilled water, the thermometer being at 4° centigrade.

	Grams.	Stones.	Lbs.	Oz.	Drs.
Gram . . .	1	...	...	...	·56438
Dekagram . .	10	...	...	...	5·6438
Hectogram . .	100	...	...	3	8·438
Kilogram[*] . .	1,000	...	2	3	4·3831
Myriagram . .	10,000	1	8	0	11·8304
Quintal . . .	100,000	15	10	7	6·304
Millier . . .	1,000,000	157	6	9	15·04

The above tables have been taken from the Act of Parliament referred to.

LXVIII.

1. If a metre be 39·371 inches, express a kilometre as a decimal of an English mile; also a yard as a decimal of a metre.

2. Reduce 150 hectars to acres, roods, and perches.

3. Find the weight in kilograms of a block of stone 12·37 by 7·14 by 0·83 metres, supposing the stone to be 2½ times heavier than water, bulk for bulk.

4. Find the price of paving a court-yard 25·34 metres by 18·36 metres at 4*s*. 6*d*. per square yard.

5. Show that 32 metres = 35 yards very nearly. If the circumference of the earth be 40 millions of metres, express it in English miles?

6. How much cotton, 4 feet wide, at 3*d*. per square foot, is equal in value to 34·45 metres of silk, ¾ of a yard wide, at 4 francs per square metre (£1 = 25·15 frs., and 39·37 in. = 1 metre)?

7. It is computed that every individual in France consumes yearly 1160 grams of coffee and 9 of tea, and that each person in England consumes 473 grams of coffee, and 1679 of tea; if tea be 3*s*., and coffee 1*s*. 8*d*. per lb., and a franc worth 10½*d*., find in francs the value of the Englishman's consumption above the Frenchman's.

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[*] Ordinary merchandise is always referred to this weight, which is = 15432·3478 grains British.

LXIX. *Given May* 15, 1872.—Time, 3 Hours.

1. A person has £642 6d. 8d. remaining after paying an income-tax of 4d. in the pound. Find his gross income.

2. Find the cost of 5A. 3R. 7P. 88YDS. at £161 6s. 8d. per acre.

3. Find the difference between 30 chains 15 links, and 3 furlongs 6 rods. Express the answer in yards, and the fraction of a yard.

4. Find the present worth of £455 12s. 6d. due in 4 years hence at 5⅝ per cent. per annum.

5. By selling sugar at 4¾d. per lb. a person loses 10 per cent. Find what it cost him per cwt.

6. Find the cost of papering the four walls of a room 27 ft. 8 in. long, 21 ft. 4 in. broad, and 12 feet high, when a piece of paper 12 yards long and 3 quarters wide costs 4s. 6d.

7. Find within an inch the diagonal of a square field containing 3½ acres.

8. Multiply duodecimally 13 ft. 5 in. 7 pts. by 9 ft. 3 in. 11 pts. giving answer in sq. ft., sq. in., and fraction of a sq. in.

9. The price of £100 bank stock, paying 6½ per cent. per annum, is 185⅝; £6,250 stock is transferred from this to the 3 per cents. at 92⅞. Find the decrease in income, neglecting brokerage.

10. Find by duodecimals the height of a parallelopiped containing 659 ft., 1,248 in., the area of whose base is 26 ft. 6 in.

11. Find the number of lineal feet in the edge of a cube containing $18{\cdot}\dot{9}6\dot{2}$ cubic yards. What is the length of the diagonal, and the area of one of the faces?

12. A watch is 10 min. too fast at 12 o'clock on Monday, and loses 3 m. 10 sec. per day. Find the time indicated by the watch at a quarter past 10 on the Sunday morning following.

13. On May 15th in the present year a person had lived 30,000 days. Find the year and day of the month of his birth.

14. What vulgar fraction$=10{\cdot}5624\dot{9}$? show that its sq. root is $3{\cdot}24\dot{9}$.

15. The French metre$=39{\cdot}371$ English inches. How many metres, &c., in a mile, and how many English yards in a kilometer? What is the unit of weight and of capacity in the French system of measurement?

LXX. *Given October* 22, 1873.—Time, 3 Hours.

1. Add together ·05735 of a mile and 46·25 yards; and reduce the result to the decimal of 7 furlongs.

2. A person invests £3,395 in the 3 per cent. Consols, when the

price of the stock is at 84⅞; find the income he derives therefrom, and also the rate of interest he receives for his money.

3. If the price of coal be 42s. per ton, and the value of coal to the same weight of coke be in the ratio of 5 : 4, calculate the value of 10 chaldrons of coke, if 3 chaldrons=2 loads, and each load weighs 21 cwt.

4. A detachment of cavalry consisted of 520 men after engaging in two battles, in the first of which it lost 1 man in every 25, and in the second $\frac{5}{18}$ of the remainder. Of how many men did the troop originally consist?

5. From two places, which are 154 miles apart, two persons set out to meet each other, one walking at the rate of 3 miles in two hours, the other 5 miles in 4 hours. When will they meet?

6. Find by duodecimals the cubical content of a tank 36 ft. 8 in. long, 7 ft. 11 in. broad, and 6 ft. 5 in. high; and express the result in cubic yards, feet and inches.

7. Distinguish between interest and discount. Find the difference between the interest and discount on £908 5s. 9d., for 3 years, at 4¾ per cent.

8. Find the square root of $\cdot 0069\dot{4}$. Of what number is ·2 the square root?

9. Find the cube root of 11, to 4 places of decimals.

10. Two circular gold plates, each 1 inch thick, of which the diameters are respectively 6 and 8 inches, are melted into one plate an inch thick. Find its diameter, having given that the area of a circle is proportional to the square of its diameter.

11. The height of a solid six-inch cube is diminished by pressure to 5⅜ inches; supposing the lateral expansion to be uniform throughout the mass, find accurately, to 3 places of decimals, the length of one of the edges of the new case of the solid, whose volume is supposed to remain unaltered.

12. Find the exact time between 7 and 8 o'clock when the hands of a watch coincide.

13. A rectangular grass plot, the lengths of whose sides are in the ratio of 5 : 6, costs £16 11s. 6⅜d. for turfing, at 5d. per square yard. Find the length of its sides.

14. What principal put out to compound interest for 3 years at 5 per cent. would amount to £144 14s. 0¾d.?

15. If 60 cubic inches of lead, together with 54 cubic inches of cork, are equal in weight to 1,538⅞ cubic inches of fir, and the weights of equal quantities of lead and fir are represented by the numbers 11·324 and ·45; what number represents the weight of an equal quantity of cork?

LXXI. —CLASS II.—MARCH 1875 (3 *hours*).

1. Find $\sqrt{\cdot 0003418801}$, and $\sqrt[3]{\frac{1}{4}}$ correct to 7 places of decimals.

2. Find the least number which is divisible by all the numbers up to 15 inclusive.

3. If 7 ozs. of gold, 22 carats fine, 12½ ozs., 21 carats fine, and 17 ozs., 19 carats fine are melted together, what is the fineness of the compound?

4. A garrison of 4500 men is provisioned for 15 weeks at the rate of 13 ounces per diem per man, how many men must leave, that the same provisions may last those who remain 27 weeks at 10 ounces per diem each man?

5. Reduce to their simplest forms,

$$\frac{3\frac{2}{3}-9}{\frac{5}{6}-\frac{7}{8}} \div \frac{\frac{1}{2}+\frac{1}{3}+\frac{9}{10}}{\frac{4}{9}-\frac{7}{8}}; \text{ and } \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\frac{1}{2}}}}$$

6. What is the present worth of a bill for £694 10*s.* due 3½ years hence, at 4½ per cent., simple interest?

7. If 1⅔ yards of cloth are worth $\frac{5}{12}$ of a bushel of corn, and 12 yards of cloth will fetch 4½ dollars, what is the value of 5 quarters of corn, a dollar being equal to 4*s.* 2*d.*?

8. If a company of 180 men, in 6 days of 11 hours each, can dig a trench 230 yards long, 5½ wide, 1½ deep, in how many days of 8 hours each would 96 men dig a trench 220 yards long, 3½ wide, and 1 deep, supposing the hardness of the ground in the former case is to that of the latter as 5 to 7, and that 4 men of the latter company can do as much work as 5 of the former in the same time?

9. Two persons, A and B, hold ground in a common at a rental of £54, A puts in 23 horses for 27 days, and B 21 horses for 39 days, how much of the rent should be paid by each?

10. Find by duodecimals, the capacity of a tank 36 ft. 7 in. long, 7 ft. 11 in. deep, 6 ft. 10 in. wide, giving result in cubic measure.

11. A and B working together do a piece of work in 7 days, B working alone can do it in 13 days. Suppose B works 3 days, how long will A take to finish the work?

12. A contractor agrees to sink an artesian well on the following ms: £5 per fathom for the first 30 fathoms (consisting of gravel only), £35 per fathom through the sand, £20 per fathom through clay, and £25 through chalk. He found the thickness of the chalk to be equal to half the depth of the well, and that the sand beds were three times as thick as the clay. At the end of the work he got £2700 for his trouble. What was the depth of the well?

1 . A and B starting at opposite corners of a square, whose side is 100 yards long, walk round it in the same direction, A walks 19 yards, while B walks 16. At what part of the square will they be together, and what number of times will each have passed the corner from whence they started?

14. A speculator bought Spanish bonds (yielding no interest) at 18, 18 Honduras £100 bonds paying 10 per cent. interest at 35, and 27 Turkish £100 bonds paying 5 per cent. at 45. At the end of two years he sells the Honduras stock at 15, and the following year he sold

the Spanish at 25, and the Turkish at 46. Including the interest, he gained £1146. What number of Spanish bonds did he buy?

15. A and B enter into business together, A gives $\frac{3}{5}$ of the capital; at the end of the first year they have made a profit of $3\frac{1}{2}$ per cent., and at the end of the second a profit of 5 per cent., at the end of the third year they are bankrupts, and can only pay 10*s*. in the pound. The remaining money is £16,663 10*s*. How much did each contribute?

LXXII.—*Excise—now Customs.*

1. $3\frac{7}{10}+\frac{11}{15}+1\frac{13}{18}+\frac{17}{20}$. 2. $8\frac{2}{27}-5\frac{5}{6}$.

3. $15\times\frac{7}{11}\times\frac{22}{5}\times\frac{9}{14}$. 4. $13\frac{1}{3}\div2\frac{26}{27}$.

5. 34·25 + 285·632 + ·00685 + 1400·06.

6. 1000·423 — 836·0982.

7. 652·394 × ·0476. 8. 219·46232 ÷ ·03542.

9. Express 5*s*. $1\frac{3}{4}$*d*. as a decimal of £5 2*s*. 11*d*.

10. $9\frac{1}{5}+\frac{4}{21}+\frac{3}{35}+\frac{11}{12}$. 11. $6\frac{11}{20}-3\frac{11}{18}$.

12. $\frac{2}{3}\times3\frac{1}{4}\times15\times\frac{9}{20}$. 13. $14\frac{124}{231}\div4\frac{14}{33}$.

14. Add together ·542 of an hour and 10·864 of a minute, giving result in seconds.

15. From 3·0565 of an oz. take 9·15 of a grain.

16. 30·96 × 43·8. 17. ·03796 ÷ 365.

18. Reduce ·0162 of 4*s*. 6*d*. to farthings and decimal of a farthing.

19. Reduce 11 qrs. 7 bus. 3 qrts. to pints.

20. What weight of sugar can be bought for £187 4*s*., if 13 cwt. cost £55 9*s*. 4*d*.?

21. Find (by Prac.) dividend on £5069 10*s*. at 14*s*. $11\frac{1}{2}$*d*. in the £.

22. Find simple interest on £4000 for 3 years at $5\frac{1}{2}$ per cent. per annum.

23. In 1468935 inches how many miles, &c.?

24. If I get 1 lb. of bread for 1*s*. when wheat is 6*s*. a bushel, what should be the price of wheat per quarter when I get 12 ozs. of bread for 4*d*.

25. Find (by Practice) the value of 2 rds. 19 pls. 19 sq. yds. of land at £40 6*s*. 8*d*. an acre.

26. Find amount of £3750 for 3 years at 6 per cent. per annum compound interest.

27. A can dig a certain length of ditch in 3 days, B can do the same in 4 days, and C in 5 days. How long will it take A, B, and C together to dig the same length of ditch? Also what proportion of the work will be done by each?

28. A prize of £2770 was divided amongst three persons in the proportion of the fractions $\frac{3}{4}$, $\frac{14}{15}$, and $\frac{5}{8}$; what were their respective shares of the prize?

29. A pond whose area is 3 acres is frozen over with ice to the uniform thickness of 6 inches; if a cubic foot of ice weigh 896 ounces avoirdupois, find the weight of ice on the pond in tons.

30. Find the square root of ·0043046721 and $\sqrt[3]{40\cdot353607}$.

31. A merchant sells tea to a tradesman at a profit of 60 per cent. but the tradesman becoming bankrupt pays only 2*s*. 6*d*. in the pound. How much per cent. does the merchant gain or lose by his sale?

LXXIII.

1. If 16 men and 12 boys earn £23 4s. in 8 days, how much should 20 men and 15 boys earn in 10 days, a man's daily wage being 2s.?

2. $\sqrt{39148797{\cdot}61}$; and $\sqrt[3]{28934443}$.

3. Find the sum which must be invested in the three per cents. at 90 at simple interest payable half-yearly, to amount in 23½ years to £2,317 sterling, the price of the funds remaining unchanged. If the funds rose to 96, in how many years sooner could the required amount be realized?

4. In a school the numbers for the week were:—Monday morning 67, Tuesday morning 60, Wednesday morning 65, Thursday morning 68, Friday morning 62, Monday afternoon ·5 more than the average of Monday and Tuesday mornings, Tuesday afternoon 59, Wednesday afternoon ·5 less than the average of Tuesday, Thursday the average of Monday morning and Tuesday afternoon, Friday afternoon 60. Find the average attendance for the week.

5. A tradesman professes to retail his goods at a profit of 10 per cent. He mixes with them $\frac{1}{4}$ of their weight of an inferior article which cost him only $\frac{4}{5}$ of the price of the better. How much per cent. profit does he make? Also, in what proportion must he mix them in order to gain 20 per cent?

6. A can perform a work in 10 days by getting 2 days' assistance from B, but B can do the same in 8 days by getting 2 days' assistance from A. In what time could they do it, both working together?

7. By investing $\frac{1}{8}$ of a sum in the 3 per cents. at 90, $\frac{1}{5}$ of it in 4 per cent. preference shares at par, $\frac{1}{4}$ of it on mortgage at 4¼ per cent., and the remainder in New Zealand 5 per cent. stock at 104, a person has an income of £481 16s. 8d. Find the sum invested.

8. A man travels 60 miles in 3 hours, partly by coach and partly by rail; had he gone all the way by rail he would have ended the journey an hour sooner and saved $\frac{2}{5}$ of the time he was on the coach. How far did he go by coach?

9. In a British regiment ·4 were Irish, ·3 Scotch, and the remainder English; after 200 Irish and the same number of Scotch recruits are added ·225 are English. How many did it at first contain?

10. If the 3 per cents. are at 95, and Government offer to receive a loan of £5,000,000, the lender to receive five millions in the 3 per cents., together with a certain sum in the 3¼ per cent. stock, what sum in the 3¼ per cents. ought the lender to accept?

11. A passenger and a goods train start from the ends of a line 120 miles long, and meet in the middle of it; the goods train started an hour earlier than the other, and goes at $\frac{2}{3}$ its speed; find the rate of each.

12. Multiply $(1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\frac{1}{36})$ by $(\frac{3}{13}-\frac{1}{7}+\frac{8}{59})$.

13. If a halfpenny stamp be $\frac{1}{2}$ an inch long and $\frac{2}{5}$ of an inch wide, how many will cover the walls of a room 18 ft. 9 in. long, 16 ft. 10 in. broad, and 10 ft. 6 in. high?

LXXIV. (Class II.) 3 Hours.—*May* 1875.

1. Multiply 4,327 by 814, and explain the method. Multiply 32,856 by 121,711 in three lines of multiplication only.

2. Define G. C. M., and find the G. C. M. and L. C. M. of 157 days, 7 hours, 4 minutes, 7 seconds, and 243 days, 2 hours, 11 minutes, 49 seconds.

3. $$1\tfrac{5}{6} \text{ of } \frac{\frac{1}{2} + 1\frac{1}{12} + \frac{3}{4}}{2\frac{1}{3} - 3\frac{1}{8} + 4\frac{1}{4}} \times \frac{\dfrac{2\frac{1}{2}}{3\frac{1}{7}} + \dfrac{\frac{3}{16}}{1\frac{3}{8}}}{\dfrac{3}{4\frac{1}{2}} + \dfrac{4\frac{1}{3}}{3}}$$

4. Gold is worth £3 17s. 10½d. per ounce, and silver 5s. 2½d. per ounce; how many sovereigns weigh as much as 5000 shillings?

5. A person shooting at a target distant 550 yards, hears the bullet strike the target four seconds after he fires. A spectator, equally distant from the target and the shooter, hears the shot strike the target two and a half seconds after he heard the report. Find the velocity of sound.

6. Divide $8{\cdot}0\dot{6}$ by $\cdot 1\dot{8}0952\dot{3}$.

7. What is the number of yards in the side of a square field containing 3,952 acres, 3 roods, 18 perches, 21½ square yards.

8. Multiply by duodecimals 9 feet 7 inches 3 parts by 2 feet 7 inches 5 parts, and the result by 5′ 7″; what does the product become when expressed in cubic feet, cubic inches, and the fraction of a cubic inch?

9. Find the length of the edge of a cube containing 41,063 cubic feet 1,080 cubic inches.

10. A person buys five shares in a concern at 40, next year the price is 45, and every succeeding year there is a fall of 4. Each year he sells out 1 share, and finds, at the end, he has neither gained nor lost. What interest did the concern pay?

11. A person sells an article at 5 per cent. profit. If he had bought it at 5 per cent. less and sold it for 1s. less, he would have gained 10 per cent. What was the cost price?

12. A person has £5,000 stock in the 3 per cents., which he invests in the 3½ per cents. at 87½, thus increasing his income by £5. Find the price of the 3 per cents.

13. On a piece of work 2 men and 5 boys are employed who do half of it in 6 days. After this, 1 man and 1 boy more are put on, and ¼ more is done in three days. How many more men must be put on that the whole may be completed in one day more?

14. In a mile race, A starts at 270 paces of 48 inches per minute, and B 300 paces of 44 inches per minute. After 4 minutes, B quickens his pace to 320. Which wins, by how much, and in what time?

APPENDIX II.

EXAMINATION PAPERS.

(Time, 2½ hours.)

A.

1. The population of 3 towns in 1841 were 4235 2553 and 6432, respectively; and in 1851, their average population was 4402·6, the first and second having increased 12 and 20 per cent. respectively. Find the increase or decrease per cent. in the third town.

2. At what rate of simple interest will £1439 12*s*. 6*d*. become £1691 11*s*. 2½*d*. in 5 years?

3. A father and son contract to do a work for £5 8*s*., their united earnings being 9*s*. a day: after 6 days the father leaves. In what time will the son do the remainder, the father's rate of working being ½ better than his?

4. A man buys articles at £12 15*s*. per 100, and sells them at 2½ guineas per dozen: find his per centage profit.

5. Find in yards the side of a square field, containing 15ac. 2rds. 29pls. 3yds.

6. A, B, and C met: A had 5 loaves; B, 3 loaves; and C had 8d. They divided the loaves fairly among them, and C paid A and B proportionately. How much did each receive?

7. What must be the value of 5 per cent. stock that after paying an income tax of 10*d*. in the pound, it may yield 5¼ per cent. interest?

8. The Brighton Railway Company carries volunteers from London to Brighton (52 miles), and back for 1*s*. 6*d*.: if the working expenses of the train are 3*s*. 6*d*. per mile, what profit does the company make on 600 men?

9. Find the exact cube root of $47307\frac{1}{216}$.

10. A man starts on a journey and walks 10½ miles the first day, 16½, 18⅙, 27⅓, 21¼ on the four following days, respectively. He then returns home by the same road in 4 days. What was his average daily walk on his way home, and on his whole journey?

11. By selling a horse for £64 15*s* 0*d*., I lost 7½ per cent. How much per cent. would I have gained by selling him for 70 guineas?

12. Multiply by the method of duodecimals 2ft. 7in. by 3ft. 5in. 7pts., and the product by 2ft. 11in. 5pts., and express the result in cubic feet, cubic inches and the fraction of an inch.

B.

1. Show that $\frac{4}{11}$ of [illegible] of $8\frac{1}{4}$ of a guinea is to $4\cdot\dot{1}\dot{2}$ of 16*s.* 6*d.*, as 35 to 34.

2. A person has £4000 worth of property: he insures half of it at 2*s.* 6*d.* per cent., and the other half at 3*s.* 6*d.* per cent. What does the insurance cost him?

3. By selling an article for 15*s.*, I gain $\frac{1}{8}$ of my outlay. What would I have gained per cent. had I sold it for 16*s.* 6*d.*?

4. If 12 men do a piece of work in 25 days, in what time will 10 men do a piece of work $1\frac{3}{4}$ times as great, if 3 of the first set do as much in an hour as 4 of the second set can do in an hour and a-half and the second set work half as long again as the first set?

5. Three farms of 125, 400, and 150 acres are bought for £3500, £11200, and £22750, respectively. Find the average price of each farm, and the average price per acre.

6. The produce of the income tax at 5*d.* in the one pound was equal to £5250000; how much would it be if it was at 5 per cent.?

7. A ship has 3 chronometers—one of which loses $\frac{1}{2}$ a minute in the week, and the others gain 11″ and 8″, respectively, per day. The captain computes by the average of the 3: after a voyage of 150 days, by how much is he wrong?

8. A wall whose height is $\frac{1}{8}$ of its length, and whose breadth is $\frac{1}{5}$ of its height, contains 54925 cubic inches. Find the breadth.

9. One horse goes $1\frac{1}{4}$ mile in 2′ 26″, another $1\frac{3}{4}$ mile in 3′ 36.″ How much per cent. does the first one go more than the second?

10. A man starts at 6 o'clock, a.m., on 1st of May, to walk 1000 consecutive hours. When will he have accomplished the task?

11. A cubic foot of water weighs 1000 oz., find the side of a cubical vessel which contains 4 tons 12 cwts. 3 qrs. 10lbs. 7 oz.

12. A baker's outlay for flour is 70 per cent. of his gross receipts, and other trade expenses 20 per cent. The price of flour falls 50 per cent., and other trade expenses are thereby reduced 25 per cent. What reduction should he make in the price of a 5*d.* loaf, allowing him still to realize the same amount of profit from it?

C.

1. Reduce 29454 oz. troy to the decimal of 1 lb. avoirdupois.

2. Find the difference between the discount and interest on £866 5*s*. for 3 years, at 5 per cent.

3. Find the value of $10{\cdot}\dot{0}\dot{9}$, of $\cdot 0\dot{6}9\dot{3}$, of $\cdot\dot{2}8571\dot{4}$, of 15*s*.

4. The annual consumption of spirits in a country is 4500000 gallons, and the duty is 8s. 4d. per gallon: the duty is reduced 4 per cent. and the consumption then increased 6 per cent.: how is the revenue affected?

5. Divide £658 17*s*. among A, B, C, and D, giving A $\frac{1}{5}$ of the sum; B $\frac{1}{2}$ of that sum, C $\frac{1}{3}$ of it, and D the remainder,

6. Find the cost of papering a room 24ft. 3in. long, 15ft. 8in. broad, and 11ft. 6in. high, at 4s. per square yard.

7. Multiply by the method of duodecimals 4ft. 5in. by 6ft. 2in. 11pts., expressing the answer in square inches.

8. By selling goods for £116 17*s*., I lose 5 per cent.; for what should I have sold them to gain $3\frac{3}{4}$ per cent?

9. Three vessels containing 5267 gallons of wine are turned into a vat of which 3123 galls. are fortified by whiskey to the extent of 7·4 per cent., 1017 to 9·3 per cent., and the remainder contains no spirits: how much per cent. is the whole fortified?

10. What is a man's gross income, if he has £1728 after paying 6d. in the pound income tax?

11. A boat falls down a river with the stream in 40′, and is pulled back in 70′: compare the rate of rowing and the rate of the stream.

12. A train 160 yards long is overtaking another train 150 yards long: the first is going at the rate of 40 miles per hour, and the second at the rate of 18. Find how long a person sitting in the *long* train will see the other while passing it.

D.

1. Find $\sqrt{459684}$; and find the length of the side of a square field whose area is equal to that of a rectangle, the sides of which are 29·16 and 18·69 yards.

2. Find $\sqrt[3]{4134520125}$.

3. A person sells out of the 3½ per cents. at 92¾, and realizes £18550; if he invests ⅕ in the 4 per cents. at 96, and the remainder in the 3 per cents. at 90, find the alteration in his income.

4. If the Hamburg foot = 126·9667 Paris lines, and 10 lines = ⅜ of an English foot; find in English measure the length of the Hamburg foot.

5. Multiply duodecimally 3ft. 4in. 7pts. by 1ft. 8in. 5pts., and the result by 4ft. 11in.

6. Express the result of the previous question in cubic inches and the fraction of an inch.

7. One pound of silver is coined into 66s, find the profit per cent. on the silver coinage when the price of silver is 5*s.* 2*d.* per oz.

8. A grocer buys two sorts of tea at the rate of 5*s.* 6*d.* and 3*s.* 6*d.* per lb.; he mixes them in the proportion of 5 parts of the cheaper to 7 parts of the dearer. At what rate per lb. must he sell the mixture, so as to gain 25 per cent.?

9. The price of gold is £3 17*s.* 10½*d.* per oz., what ought 539 sovereigns to weigh, supposing $\frac{11}{12}$ of each to be pure gold, and that the value of a sovereign is = the gold it contains?

10. Find by decimals the interest on £358 3 fl. 1 ct. 2½ mils. for 2¾ yrs., at 2½ per cent.

11. Find the value of 37 kils. 8 decs. 4 grams, at £13 5 fl. 7 cts. per kilogram, and express the result in English money.

12. In 1861 there were saved by life-boats 13½ per cent., by rocket and mortar apparatus 8 per cent., by ships, boats, &c., 62 per cent., and by individual exertion ½ per cent.: lost 16 per cent. Find the number of lives lost corresponding to 729 saved by life-boats, and also the corresponding numbers saved by the other means enumerated.

E.

1. The profits of a company for one year amount to £3296 13*s.* 5¼*d.*, and a person holding 70 shares received for his dividend the sum of £1025 12*s.* 7½*d.* How many shares were there in all?

2. Reduce $\cdot 7\dot{8}46153\dot{3}$ to a vulgar fraction.

3. Multiply by the method of duodecimals 4ft. 3in. 7pts. by 5ft. 7in. 3pts., and the result by 1ft. 11in.; and give the answer in cubic feet, cubic inches, and the fraction of a cubic inch.

4. Find the present value of £10 due 2 years hence at 5 per cent. per annum; and prove that the present value will pay the sum in 2 years.

5. Find the square root of

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3.$$

6. If 7 cwts. 2 qrs. 15 lbs. cost £25 19*s.* 10*d.*, what must it be sold at per lb., to gain 15 per cent.?

7. On what day of the week will the 1st of January fall in 1880, in 1890, and in 1900, respectively?

8. A sheet of paper 5½ feet long, and 2½ feet wide is cut in stripes 1 inch wide. How many such sheets of paper will it take to go round the earth, a distance of 25000 miles?

9. What must be the market value of 4 per cent. stock in order that after deducting an income tax of 4*d.* in the pound, it may yield 5 per cent. interest?

10. If a 2*d.* loaf weighs 22oz., when wheat is 54*s.* per quarter, what is the price of wheat per quarter, when the 3*d.* loaf weighs 24oz.?

11. If 2 turkeys and 9 fowls cost £3, or 5 turkeys and 3 fowls £5; find the cost of 1 turkey, and 1 fowl.

12. The expense of carpeting a room was £11 5*s.*; but if the breadth had been 3 feet less than it was, the expense would have been £9. What was the breadth?

F.

1. If by selling articles at 3*s*. 7*d*. each, I gain 7½ per cent., what will I gain or lose by selling them at 3*s*. 4*d*. each?

2. Divide £459 among 5 persons in the proportion of ⅓, ¼, ⅕, ⅙, and ⅐.

3. A cube contains 37c. ft. 64c. in. Find its diagonal.

4. What is the difference between interest and discount? and find the difference between the amount of £123 15*s*., and its present worth in 2 years, at 5 per cent.

5. If in a race A beats B by 100 yards, C beats D by 40 yards, and D beats B by 60 yards: how much faster does A go than C, and how much will he gain in a race of ¾ of a mile?

6. In a school of 19 children, 7 are boys and 12 girls: of the boys 3 are 8 years old, 2 are 11, and 2, 12: of the girls 3 are 9 years old, 2 are 10; 3 are 13, and 4 the average age of the boys. Find the average age of the class.

7. Multiply duodecimally 3ft. 1in. 5pts. by 4ft. 7in. 2pts., and the result by 2ft. 5in., giving answer in cubic feet and inches.

8. Multiply $2{\cdot}\dot{0}3174\dot{6}$ by $8{\cdot}818\dot{9}5238\dot{0}$, and divide product by $\cdot\dot{1}5873\dot{0}$.

9. The 3 per cents. are at 87½, and the 5 per cents., at 131¼; which is the better investment? and what amount invested would produce a difference of £50 in a man's income?

10. A can do a piece of work in 8 days, but with the help of B he can do it 3⅓ days: how long would B alone take to do it?

11. Find what time between 6 and 7 o'clock are the hands of a watch together, and 23 minutes apart.

12. A train is travelling from one station to another; after travelling an hour it breaks down, and is delayed for an hour; it then proceeds at ⅗ its former speed, and arrives 3 hours too late: had it gone on for 50 miles further before the accident, it would have arrived 1 hour 20m. sooner Find the rate of travelling per hour, and length of the line.

G

1. A gentleman, after paying 8*d*. in the £ income tax, finds he has a net income of £759 18*s*. 8*d*.: what is his nominal income?

2. A person sells £3000 consols at 95⅞; and on their falling, he sells £3000 more at 92⅜; but on their again rising, he buys back the whole at 94¼. What does he gain or lose by the transaction?

3. Multiply 5ft. 6in. by 8ft. 9in. 7pts., expressing the result in feet, inches, &c., and also in square inches and fraction of a square inch.

4. A wholesale dealer sells to a retailer at 30 per cent profit, and the retailer sells to the consumer at 50 per cent. profit; what percentage of the price paid by the consumer is profit?

5. What should be the value of £135 in the 4½ per cents., when the 3 per cents. are at 97⅛?

6. A person sells £3000 Midland Stock, paying 5¼ per cent., at 105½; and with the product purchases South Eastern Stock, paying 4 per cent., at 78¾ How will his income be affected?

7. Three men go round a circle 180 miles in circumference, the first at 10 miles per day, the second at 6 miles, and the third at 4 miles per day; in what time will they all be together, and how many miles will each have travelled?

8. A person buys 50, 60, and 70 quarters of wheat, at 50*s*., 60*s*. and 70*s*., respectively, and mixes them. At what price per quarter should he sell the whole to gain 20 per cent.?

9. If two clocks, one of which gains 2 minutes a day, and the other loses 4 minutes be set right at noon. What will be the time by the first clock when the second shows noon 5 days after?

10. Extract the square root of $\cdot\dot{0}4761\dot{9} \div \cdot06\dot{8}5714\dot{2}$.

11. Find amount of £2800 in 4 years, at 4½ per cent. per annum compound interest.

12. If A does ⅔ of a piece of work in 4hrs., and B ¾ of the remainder in 1hr., and C finish it in 20 minutes What time would all together require to do it?

H.

1. At what rate of interest will the discount on £387 7s. $7\frac{1}{5}$d., payable in 8 yrs., amount to £41 10s. $1\frac{1}{5}$d.?

2. Multiply 7ft. 9in. 10pts. by 6ft. 11in. 11pts., and the result by 5 ft. 4in., expressing the result in cubic feet, inches and fraction of an inch.

3. A map of a plot, which is drawn on a scale of an inch to mile, contains 6 feet 6 in.: find its acreage.

4. Find exactly $\sqrt[3]{47558\frac{2185}{2197}}$.

5. A person sells £8000 stock out of the 3 per cents., at $87\frac{1}{2}$ and invests in $3\frac{1}{2}$ per cent., at 91. Find the alteration in his income.

6. A and B set out from London and Brighton at the same time, a distance of 50mls. A walks 5 miles an hour, but after 10 miles rests 40 minutes. B walks constantly 4 miles an hour. Where will they meet?

7. A ship carries 3 chronometers: one gains 7″ per diem, another 10″ per diem, and the 3rd. loses 1′ per week. How much will the average time be in error after a cruise of 270 days?

8. If 24 pioneers, in 10 days of $6\frac{1}{4}$ hrs, dig a trench $628\frac{3}{8}$ yds. long, 9 yds. wide, and 5 yds. deep. How many hours a day must 60 pioneers work to dig a trench $9820\frac{1}{3}$ yds. long, $4\frac{7}{8}$ yds. wide, and $1\frac{3}{5}$ yds. deep, in 63 days?

9. Multiply and divide $1\cdot\dot{4}2857\dot{1}$ by $1\cdot\dot{6}\dot{3}$, and find the difference in the results.

10. A man buys £2000, £4000, and £5000, 3 per cent. stock, at $87\frac{1}{2}$, 95, and 85, respectively. Find the average price paid, and average rate of interest he receives.

11. Find the number of yards of matting required to cover a room $29\cdot\dot{5}$ft. long, $17\cdot1\dot{9}$ft. wide, with matting $2\cdot\dot{4}$ft. wide.

12. Simplify $\dfrac{4\frac{1}{9} \div 6\frac{1}{6}}{\frac{11}{23} + \frac{35}{69}} + \dfrac{7\frac{5}{11} \times 3\frac{3}{41}}{\frac{5}{9} - \frac{3}{7}} \div 6\frac{11}{18}$.

I.

1. At a sale 8 horses were sold for £24 each, 7 at £32 10s. each, 5 at 45 guineas each, 3 at £50 each: find the average price of each horse.

2. From the first of 2 mortars in a battery, 36 shells are thrown before the second is ready for firing. Shells are thrown from both in the proportion of 8 from the first to 7 from the second. The second requires as much powder for 3 charges, as the first does for 4. After how many shots will the quantity of powder consumed by the second equal that consumed by the first?

3. The average number of persons passing a certain point between the hours of 9 a.m. and 11 p.m. is 16 per minute, and between 11 p.m. and 9 a.m. is 5 per minute. How many pass in a year?

4. A certain sum invested in the 3 per cents. gives an income annually of £75; the same sum in a 5¼ per cent. stock at par would produce an income of £126 13s. 1½d. What was the sum and the price of the 3 per cents.?

5. The specific gravity of hammered gold and silver being 19·35 and 10·15, respectively: find the ratio of the quantities of matter contained in a bar of gold, measuring 4·18 inches long, ·64 inches broad, and ·31 inches deep, and in a bar of silver 13·02 inches long, 1·14 in. broad, and ·65 in. deep.

6. A person enters into a building contract for £75000. On the money he expends he has a gain of 15 per cent. The cost of labour and materials is as 2 : 1 : when half completed, the price of materials falls 20 per cent., and labour rises 10 per cent. What sum will the contractor gain?

7. I invest in the 4 per cents and obtain 4⅜ per cent. interest for my money: find the price of the stock.

8. Find the cost of painting the outside of a cubical box which contains 42·875 cubic feet, at $1\cdot\dot{3}$s. per square yard.

9. A person buys 360 shares at £2$\frac{3}{16}$ each, his broker charging him ⅛ per cent. commission on the purchase money. He sells them at £2⅜ each, his broker again charging ½ per cent. What is his loss?

10. If a train travelling 36 miles an hour, performs a distance in 3½ hrs. How long will another train going 48 miles an hour be in completing half the distance?

11. An army of 24000 men lost by desertion to the enemy 6 per cent.; find the increase per cent. to the enemy, their number being 28000.

12. The number of 1st, 2nd, and 3rd class passengers on the railways in England was 4174213, 10291749, 21409217; in Scotland 2124917, 6172429, 15473551; in Ireland, 307492, 1292476, 2374237 respectively. Find the proportion per cent. which the whole number of 3rd class bears to the whole number of passengers.

J.

1. After paying 4 per cent. income tax, a person had £1250; find his gross income.

2 Find the average of $13\frac{1}{2}\frac{2}{5}$, 21, $71\frac{3}{8}$, ·0023, $3\frac{1}{8}$, 0, ·95., $67\frac{7}{20}$.

3. By reduction in the interest on Exchequer Bills from $2\frac{1}{4}$*d.* to $1\frac{1}{2}$*d.* per cent. per day, a person loses at the rate of £76 3*s.* $10\frac{1}{2}$*d.* per annum. What amount in bills does he hold?

4. How much is the income of a person increased who sells £61000 3 per cent. consols, at $75\frac{1}{4}$, and invests in railway shares, paying 5 per cent., at $114\frac{3}{8}$?

5. A bankrupt's estate being worth £1756, he pays his creditors 7*s.* $1\frac{1}{2}$*d.* in the £; how much do they lose?

6. Arrange in order of magnitude $3\frac{7}{8}$: $\frac{3}{8}$ of $\frac{7}{8}$ of $11\frac{1}{4}$: 1·8 of 2·1 : 87·1 of ·04 : 26·3 of $\cdot 1\dot{2}$.

7. Show that $\frac{1}{3}$ of $1\frac{1}{4}$ of $9\frac{5}{8}$ miles is to $\cdot 0\dot{5}$ of 5·5 furlongs as 350 : 3; and find $\sqrt[3]{\cdot 027}$ and of $1\frac{61}{64}$.

8. Subtract 1fl. 1c. 8m. from 2fl. 3m., and show that 5 times the remainder is = 8s. 6d.

9. If there be a cistern whose dimensions are 6, 5, and 4 feet, find the sides of another to contain 3 times as much, and whose sides will be proportional to those of the first one.

10. Multiply duodecimally 2ft. 10in. 11pts. by 3ft. 7in. 5pts, and express the result in square measure.

11. What income will a person have who invests £4700 in the 3 per cent. consols, at $95\frac{3}{8}$, and what rate of interest will it give?

12. The capital of a railway company is £3700000. It has borrowed £2300000, at $4\frac{1}{4}$ per cent. per annum. Its receipts for the year are £635000. Its working expenses 45 per cent. of the receipts. What dividend will the shareholders receive for the year if £11000 be retained as a reserve fund?

K.

1. If £3 are equal to 20 thalers: 25 thalers equal to 93 francs: 27 francs to 5 scudi: 62 scudi equal to 135 gulden. How many gulden in £2?

2. Find the true present worth of a bill of £567 11*s.* 3*d.* due 73 days hence, at 4½ per cent. per annnm.

3. A cistern 12 feet long, 8ft. 10in. broad, and 6ft. 6in. deep, is filled with water, and after a time 4000 galls. are drawn off, leaving a sediment which is sold for £2 3s. 11½d per cubic foot. Find what it brings, a gallon being = 277·279 cubic inches.

4. A man buys 100 animals for £100: of these, calves cost £5 each; lambs, £1 each; and goats, 5s. each. How many of each were there?

5. A person buys 30lbs. of tobacco, at 3*s.* 8*d.* per lb., 40 lbs., at 3*s.* 6*d.*, and 50 lbs. 3*s.* 3*d.*, and mixes them: how much will he gain per cent. by selling the mixture at 3½*d.* per ounce?

6. If the silver coin of England is made of 37 parts of pure silver and 3 parts of copper, and a lb. troy of this metal is coined into 66*s.*: find the weight of pure silver in a shilling.

7. Supposing that the duty on spirits was 6*s.* 3*d.* per gallon, and that 10740000 galls. were consumed. Find how the revenue would be affected if the duty had increased 30 per cent., whilst the consumption was reduced 3 per cent.

8. A bag contains a certain number of sovereigns, 3 times as many shillings, and 4 times as many pence, and the whole sum is £280: how many are there of each?

9. The length, breadth, and depth of a rectangular wooden box are 16,12 and 8 inches, respectively, and the thickness of the wood ½ an inch: when the box is empty it weighs 3 lbs., when filled with sand it weighs 25 lbs Compare the weight of wood and sand.

10. A person borrowed sufficient money to pay for 13 hhds. of sugar, each weighing 12 cwts. 3 qrs. 7 lbs., at £2 1s. 6d. per cwt., he keeps the sugar for 8 months, and sells the whole quantity for £420. How much has he left after paying the borrowed money with interest at 5 per cent. per annum?

11. A person invests £1,250 in the new 5 per cent. Austrian loan issued at 80 per cent., at 1⅞ discount. How much stock will he have, and what rate per cent. of interest will the investment give?

12. A Ship Company whose capital is £330000, borrows on debentures £90000, at 6 per cent., undertaking to insure their vessels for £340000, which is effected, at 3¾ per cent. The whole profits of the company before paying the insurance premium were £36200 for the year. What dividend can it pay the shareholders after placing £2000 to the reserve fund?

L.

1. If the carriage of 3 cwts. 1 qr. 14 lbs. for 64 miles cost 12*s*. 6*d*., what will be charged for carrying 4½ tons. 24 miles ?

2. The number of passengers in a train were 270, and the 1st, 2nd, and 3rd classes were in the proportion of 4, 7, and 16. The miles travelled by each class were 40, 35, and 25, and the rates were 2d., 1½d., and 1d. per mile : find the total earning of the train.

3. A chain is taken along the line A D, (920 links) in the six-sided field, A, B, C, D, E, F. The distance of the points B, C, E, F, from A D, are 182, 250, 190, 136 links, meeting it at points 120, 560, 750, 95 links from A respectively. Find the acreage of the field.

4. A young man's salary increases ⅓ every year : his expenses are ⅓ of his salary ; at the end of 4 years he has saved £259$\frac{7}{27}$: find his first year's salary.

5. Divide £21 among 5 men, 12 women, and 18 children, giving each man twice as much as a woman. and the 12 women twice as much as the 18 children.

6. A telegraph company finds its political, commercial, and domestic messages are in number 14, 57, 29. The average number of words in the three classes 130, 14, 36, respectively: how much does the company receive for 100 average messages, at the charge of 3*s*. 9*d*. for 20 words ?

7. Required the edge of a cubical mound of earth which contains 320 cubic chains 13504 links ?

8. How much must I invest in the 3 per ccnt. stock at 90, in order that when I sell £1000 stock at 93½, and the remainder at 84¼, I may gain £6 5s : and if I invest the proceeds in the 4 per cents. at par, find the difference in my income.

9. Four persons enter into partnership, the 2nd has twice as large a capital as the first, and the third as much as half the sum of the other two, and the fourth has a sum equal to the capital of all the others. The total profits are £5000, how much should each receive ?

10. If I spend 6 guineas per week, what must be my income to allow me to lay up £300 per annum ?

11. Three men start together from the same point, and walk continually in the same direction round a circle ¾ of a mile in circumference in 10, 18, 90 minutes, respectively. Find the least time in which they will be together, and number of miles each travels.

12. A train 88 yards long overtakes a man walking along the line at the rate of 4 miles an hour and passes him in 10 seconds, 20 minutes after it overtakes another man and passes him in 9 seconds; where will the train be when the last man overtakes the first?

M.

1. Two lines are 41·06328 and ·0438 of an inch long, respectively. How many lines as long as the latter can be cut off from the former? What will be the length of the remaining line?

2. A person's average annual income from 1830 to 1850 (inclusive) was £374 9*s.* 8*d.* In 1830 his income was £369 18*s.* 10*d,* and in 1851 his income was £360 1*s.* 1*d.* What was his annual average income from 1831 to 1851 (inclusive.)

3. A rectangular field whose length is 3 times its breadth, contains 6 acres 900 yards: find its length and breadth.

4. A bankrupt can only pay 6s. 10½d. in the pound: how much per cent. will his creditors lose?

5. A man buying goods by means of false scales defrauds to the extent of 15 per cent., and 15 per cent. in selling: find his whole gain per cent.

6. What will be the amount of £2,400 at 10 per cent. paid half-yearly in 1½ years at compound interest; and at what rate of simple interest will the same principal amount to the same sum in the same time?

7. A railway is constructed 35 miles at an estimated expense or £6,000 per mile, and £50 shares are selling at 4 premium: but it is found to cost £60,000 more; what should now be the price of the shares?

8. At what price must a person buy shares, which pay £3 5s. per cent. per annum dividend, to have 5 per cent. for his money? A person having bought 20 of them at this price sells them, when they have risen £7 each, and invests in 3¼ per cent. stock at 90 find the change in his income.

9. Three soldiers A, B, and C, divide 1155 cartridges among them; as often as A takes 4, B takes 3, and as often A takes 6, C. takes 7: find how many each has?

10. An army in a defeat loses ⅙ of its number, and 8,000 prisoners: after being reinforced by 6,000 men, it again loses ¼ of its number in retreat, and 36000, left. What was the original force?

11. Multiply, and also divide $4\cdot\dot{2}8571\dot{4}$ by $4\cdot\dot{0}\dot{9}$.

12. The road between A and B, distant 15 miles, goes over a hill whose summit is 3 miles from A. Two men set out at the same time from A and B, the former going 4 miles per hour up hill and 5¼ down, the latter 3⅓ up and 4½ down. How far will the slower have to go when the faster reaches B?

N.—*Open Competition*, 1871 (*Time 3 hours*).

1. A man has £576 6s. 8d. per annum after paying income tax at the rate of 4d. in the pound. What is his income?

2. Find the price of 3 acres 1 rood 7 perches and 88 square yards at £161 6s. 6d. per acre.

3. Find the edge of a cube which contains 3,605 feet 64 inches.

4. Multiply by duodecimals 8 ft. 7 inches 2 pts., by 5 ft. 7 inches 11 pts., and the product by 3 ft. 7 inches. What does the product become when expressed in cubic feet, cubic inches, and a fraction of a cubic inch?

5. A person in London owes another in Petersburgh 460 rubles, which must be remitted through Paris. He pays the requisite sum to his broker when the exchange between London and Paris is 23 frs. for £1, and between Paris and Petersburgh 2 frs. for 1 ruble. The remittance is delayed until the rates of exchange are 24 frs. for £1 and 3 frs. for 2 rubles. What does the broker gain or lose by the delay?

6. What advantage does a man gain, who, owing £1000 at the present time, is allowed to extend the settlement of his debt over the next twelve months, by making four equal quarterly payments, simple interest to be calculated at the rate of 5 per cent. per annum?

7. Prove, without using the common rule for the extraction of the square root, that 2·76 is the square root of 7·654.

8. Find the difference between interest and discount on £7747 5s. for 6 months, reckoning 5 per cent. simple interest.

9. A person who has £1010 stock in a railway company guaranteed to pay 5 per cent. per annum, receives for his half-yearly dividend the guaranteed interest and a further bonus of 5s. 3d. per cent. for the half-year; find the amount of his dividend when an income-tax of 5d. in the pound has been deducted from it.

10. The mean temperature from the 9th to the 16th Jan. 1871, both days inclusive, was 34·6°, and from the 10th to the 17th it was 39·8°; the mean temp. on the 9th was 30·5°; what was it on the 17th?

11. A man leaves one-third of his property to his widow and ⅕ to his son, who gets £326 more than either of his four sisters, between whom the residue is equally divided; what is the whole sum left?

12. The railway fare from London to Dover (70 miles) being 14s. 7d. and that from Paris to Boulogne (240 kilometres) being 36·16 frs.; compare the cost of travelling in England and France, the French metre being equal to 3 feet 3·37068 inches and £1 being worth 25·44 francs.

13. If a piece of work can be finished in 45 days by 35 men, and if the men drop off by 7 at a time at the end of every 15 days, how long will it be before the work is finished?

14. A room is 16 feet 7 inches long, 13 feet 5 inches wide, and 10 feet 6 inches high. Find the cost of papering it with paper 1 foot 8 inches wide at 10d. per yard, and of carpeting it with carpet ¾ yard wide at 4s. 2d. per yard.

15. The gross receipts of a railway company in a certain year are apportioned as follows: 41 per cent. to pay working expenses, 56 per cent. to give the shareholders a dividend at the rate of 3½ per cent. on their shares, and the remainder, £15,000, is reserved. Find the paid up capital of the company.

O.

1. A crew row down a stream in 2 hours, and row back again in 3 hours: compare the rate of rowing and the rate of the stream.

2. Simplify

$$1\tfrac{1}{2} \text{ of } 2\tfrac{4}{5} + 6\tfrac{7}{8} \div 2\tfrac{3}{4} - \left(5\tfrac{1}{2} + \frac{\cdot 24 + \cdot 5\dot{3}}{2\tfrac{1}{5} - \cdot \dot{6}\dot{4}}\right)$$

3. Find by duodecimals the height of a parallelopiped which contains 659 ft. 1248 in. and whose base is 26 ft. 6 in.

4. The content of a rectangular field whose sides are as 3 to 5 is 40 acres: find its diagonal in yards.

5. At a game of billiards A can give B 5 points in 60; B can give C 4 points in 44; and C can give D 4 points in 40: how many points can A give D in 72?

6. A farm is let for £96 per annum and a certain number of quarters of wheat; but when wheat is 38*s.* per quarter, the whole rent is 15 per cent. less than when wheat is 56*s.* per quarter: find the number of quarters of wheat included in the rent.

7. In what cases will a recurring decimal have a recurring decimal for its square root? find $\sqrt{\cdot \dot{1}9753086\dot{4}}$.

8. If gold can be beaten out so thin that a grain will form a leaf 56 square inches, how many of these will make an inch thick; the weight of a cubic foot of gold being 10 cwt. 95 lbs.?

9. A watch which is 10 minutes too fast at 12 o'clock on Monday gains 3 minutes and 10 seconds per day: what will be the time by it at 10 hrs. 15 m. on the following Saturday morning?

10. Find by reducing to their prime factors the L. C. M. of 2431, 2717, 3553, and 4199 and add the fractions $\frac{7}{2431}$, $\frac{8}{2717}$, $\frac{9}{3553}$, and $\frac{10}{4199}$.

11. Explain the difference between a decimal and a vulgar fraction; and simplify

$$\frac{2 \times \sqrt{1+\frac{1}{3}} \div \sqrt{1-\frac{1}{5}}}{5 \times \sqrt{1+\frac{1}{2}} \times \sqrt{1-\frac{1}{3}}} + \frac{\cdot \dot{3}}{\cdot 45} \times \tfrac{5}{11} \div 1\tfrac{17}{22} + \cdot \dot{8}1196\dot{5}.$$

12. Bronze contains 91 per cent. of copper, 6 per cent. of zinc, and 3 per cent. of tin. A mass of bell-metal (consisting of copper and tin only) and bronze fused together is found to contain 88 per cent. of copper, 4·875 per cent. of zinc, and 7·125 of tin. Find the proportion of copper and tin in bell-metal.

P.

1. A and B are out shooting snipe; A fires 5 shots to B's 3, but A kills only 1 in 3 while B kills 1 in 2: when B has missed 36 shots, how many birds has A killed?

2. A pound troy of British standard gold $\frac{11}{12}$ fine is worth £46 12*s*. 6*d*.: find the value of a gold coin weighing 7 dwts. 11 grs. of which the percentage of pure gold is 92·5.

3. Add $\frac{5}{11}$ of a pound troy to $\frac{5}{11}$ of a pound avoirdupois, and give the result (*a*) in troy weight, (*b*) in avoirdupois weight.

4. A merchant buys 2520 quarters of corn, $\frac{1}{5}$ of which he sells at a gain of 8 per cent. and the remainder at a gain of 5 per cent. If he had sold the whole at a gain of 6 per cent., he would have made £63 more. What was the cost price per quarter?

5. On the top of the Crystal Palace there was a cistern $13\frac{3}{8}$ feet long, 6 feet broad; it contained 294·25 cubic feet; find its depth; find also the weight of water it contains, a cubic inch of water weighing 252·5 grains.

6. There are two vessels, one containing 12 gallons of water, the other 6 gallons of wine; if a gallon be taken out of each and poured into the other, and if this be repeated four times, find how much wine and water will then be in each.

7. A cube contains $2\cdot\dot{3}7\dot{0}$ yards: find (1) its edge; (2) its diagonal; (3) the cost of painting its outside at $1\cdot\dot{4}$*s*. per square foot.

8. Two parallel walls are 10 feet apart; a man whose eye is 5 ft. 6 in. high has to walk 20 ft. back from the lower wall before he can see the top of the higher: given that the lower is 8 ft. high, find the height of the higher.

9. The value of a certain length of material A is $\frac{54}{55}$ of the value of the same length of another material B, and the weight of 17 yds. 2 ft. of A is $\frac{53}{51}$ of the weight of 15 yds. 1 ft. 9 in. of B. If the value of 3 cwts. 27 lbs. of A be £35 2*s*. what is the value of 1 cwt. 2 qrs. 12 lbs. of B?

10. If the actual weight of a cubic foot of water be 62·35 lbs. avoirdupois, find the error in calculating the weight of 1000 cubic ft. on the approximate assumption that a cubic fathom weighs 6 tons.

11. A man running at the rate of ten miles an hour towards a spot where minute guns are discharging, finds that 18 minutes 45 seconds elapse between his hearing the first and twentieth reports; find the velocity of sound.

12. In the centigrade thermometer the boiling point is 100° and the freezing 0°, in the common thermometer (Fahrenheit's) the boiling point is 212° and the freezing 32°. Find the marks on the common thermometer corresponding to 24°, −8°, and −54° on centigrade, respectively.

Q.

1. A person can row 9 miles an hour on still water, and he finds it takes him just 3 times as long to row up as to row down the river: find the rate of the stream.

2. Simplify

$$\frac{\dfrac{816}{166463} - \dfrac{1}{1393}}{1 + \dfrac{1}{1393} \times \dfrac{816}{166463}}$$

3. Find the dimensions of a rectangular cistern, whose base is a square and height equal to half the length of one side of the base, and which contains 53000 gallons, each gallon being equal to 277·274 cubic inches.

4. If the diameter of a guinea be to that of a sovereign as 8 : 7, find the proportion of their thicknesses.

5. Find the expense of lining the bottom and sides of a rectangular cistern 12 ft. 9 in. long, 8 ft. 3 in. broad, and 6 ft. 6 in. deep, with lead which costs £1 8*s.* per cwt., and weighs 1 lb. to 18 square inches.

6. A and B can do a piece of work in 12 days, A and C can do the same amount of work in 15, and B and C in 18 days: in what time could each do it separately?

7. In a certain Post Office the proportion of unpaid letters to the whole number posted was 9 per cent.; and of the paid letters 61 per cent. were stamped: the whole number posted being 650,000, how many were stamped?

8. Find the square root of the sum of the squares of 10·4 and 14·31; find also to four places of decimals the value of (*a*) $\frac{2\sqrt{2}}{\sqrt{10}}$ (*b*) $\frac{1}{1-\frac{1}{5}\sqrt{20}}$.

9. If 25 per cent. be gained by a furnace which consumes its own smoke, how much per annum will be saved by a man who before used 7 tons of coals per week at 23*s.* per ton?

10. The average price of wheat in 1836 was 62*s.* 3*d.* and from 1836 to 1842 inclusive was 63*s.*, and from 1837 to 1843 inclusive it was 64*s.* 2*d.*; find the average price of wheat in 1843.

11. A can mow $2\frac{1}{2}$ acres in $4\frac{2}{3}$ days, and B $2\frac{1}{6}$ acres in $3\frac{1}{2}$ days; they mow together a field of 10 acres: how long have they been engaged at it, and what share will each do?

12. Find in inches and the decimal of an inch the length at 60° Fahrenheit of a column of mercury which at 32° is ·76 of a mètre in length, supposing the cross section of the column to be invariable, and mercury to expand ·00018 of its bulk for each degree centigrade; a mètre being 39·37079 inches, and 5° centigrade = 9° Fahrenheit.

R.

1. If a person invests £9040 16*s.* in the $3\frac{1}{2}$ per cents. at 94, what income will he have, brokerage $\frac{1}{8}$, and other expenses $\frac{1}{20}$ per cent. ?

2. If I invest in the 3 per cents. at 72 and on their rising to 80 sell out and invest in the 4 per cents. at 96 and gain £43 10*s.* per annum: find the sum at first invested, and the income I will now have:

3. A person having to pay £1085 two years hence, invests a certain sum in the 3 per cents. at 73, to accumulate interest until the debt is due, and the same sum the next year. He sells out the same price and just pays the debt, what sum did he invest?

4. How much 4 per cent. stock at $93\frac{1}{2}$ must be sold out to pay a bill of £954 17*s.* 6*d.*, 9 months before it is due, discount at the rate of 5 per cent. per annum ?

5. Find to the nearest $\frac{1}{10}$th the rate per cent. of compound interest at which money will double itself in 6 years.

6. A and B have the same capital: A purchases an equal amount of stock in the 3 per cents. at 91 and in the $3\frac{1}{4}$ at $97\frac{1}{2}$. B invests equally in both stocks, and his income is 1*s.* less than A's, what sum had each.

7. The capital of a railway company is £5000000, and the net 60 per cent. of the gross receipts: find its gross receipts, the shareholders receiving 5 per cent. per annum free of income-tax at 5*d.* in the pound ?

8. A person sells out 8000 consols in the 3 per cents. at $89\frac{1}{4}$, and invests half the proceeds in Dutch 4 per cents. at $87\frac{7}{16}$ and the other half in Dutch $2\frac{1}{2}$ per cents. at $54\frac{15}{16}$. Find the alteration in his income.

9. One company guarantees to pay 5 per cent. on shares of £100 each; another $4\frac{5}{8}$ per cent. on shares of £7 10*s.* each; the price of the former is £$124\frac{1}{2}$, of the latter £8 10*s.*; compare the rates of interest they would give to a purchaser.

10. A person expends £3500 in the purchasing Great Nugget shares of £5 when they are at 2 premium, and £500 in purchasing Agua Frias of £2 when at $\frac{3}{4}$ discount; he sells out when the Nuggets fall to par and the Agua Frias rise to 3 premium. Find his gain or loss after paying his broker $\frac{1}{8}$ per cent. on all the money that passed through his hands.

11. By selling out of the 3 per cents. at 96 and investing in railway 5 per cent. stock a person increases his income 25 per cent. Find the price of the railway stock.

12. At what price must a person invest £5210 in a $4\frac{1}{8}$ per cent. stock so as to have the same income as if he invested in the 3 per cent. consols at $93\frac{7}{8}$, $\frac{1}{8}$ per cent. brokerage, and 1 per cent. brokerage on the stock purchased in the former case; and what income will he have?

S.

India Forest. (*Time 2½ hours.*)

1. What would be the cost of painting the 4 walls of a room whose length is 24 feet 8½ inches, breadth 16 feet 3½ inches, and height 11 feet 6 inches, at 4s. a square foot.

2. Reduce to a simple fraction—

$$\frac{2\frac{1}{2} + 1\frac{1}{4} - 2\frac{1}{8}}{3\frac{3}{4} + 2\frac{1}{3} - 4\frac{1}{4}} \times \frac{\frac{9}{49} - \frac{4}{25}}{\frac{75}{121} - \frac{16}{27}} \div \frac{\frac{3}{7} + \frac{2}{5}}{\frac{6}{11} + \frac{4}{5}}.$$

3. By selling a horse for £82 5s. a person lost 6½ per cent; how much per cent. would be lost or gained if he had sold the horse for £90 10s.?

4. Find within an inch the diagonal of a square field containing 2 acres.

5. Multiply by duodecimals 10 feet 5 inches 3 parts by 9 feet 7 inches 10 parts, and express the product in square inches and the fraction of a square inch.

6. A person invests £2500 in the 9 per cent. Japanese loan, which was issued at 98 per cent. at 1½ discount; how much stock will he have and what rate of interest will the investment give?

7. Find the difference between the interest and the discount on £2598 15s. for 3 years at 5 per cent.

8. Express 4 lbs. 3 ozs. 12 dwts. 6 grains troy weight as the decimal of a pound avoirdupois.

9. Reduce $\cdot 267187\dot{5}$ to a vulgar fraction, and if the unit be £3 reduce the fraction to shillings, pence, and decimals of a penny.

10. A wall 5 times as high as it is broad and 8 times as long as it is high contains 18,225 cubic feet. Find the breadth of the wall.

11. A person had £6900 stock paying 3 per cent. in a company, and was offered the choice of being paid off at par or of receiving £110 of a new 2½ per cent. stock for every £100. He chose the former alternative, and invested his money in the 3 per cent. consols. at 92. Find the amount of his stock in consols. and the excess of his income above what it would have been if he had agreed to the proposed conversion.

12. Find the prime factors of 111540, 296352, 404352; and thence write down all the numbers which will divide them all without a remainder, and the smallest number they will all divide without a remainder.

T.

Constabulary Cadets.

1. A person buys £700 stock in 3 per cent. consols. at 94½, and invests £585 in Russian 5 per cent. stock at 97½. How much stock has he standing in his name; and if he sell out the first at 95 and out of the second at 96¼, find his gain or loss?

2. If 5½ per cent be lost by selling goods at £5 5s. 6d. per cwt., what per cent. will be gained or lost by selling at 3s. 2d. per pound?

3. A person invests £1638 in the 3 per cents. at 91; he sells £1200 stock at $93\frac{1}{8}$ and the remainder at 85, how much does he gain or lose.

4. How many lbs. of tea at 2s. 8d. per lb. must be mixed with 495 lbs. at 2s. 9d. per lb. so that a profit of 10 per cent. be realized by selling at 3s. per lb.?

5. A person has 1260 qrs. of wheat; he sells $\frac{1}{5}$ at a gain of 5 per cent., $\frac{1}{3}$ at a gain of 8 per cent., and the remainder at a gain of 12 per cent. Had he sold the whole at a profit of 10 per cent. he would have made £23 2s. more than he did. Find the price paid per quarter.

U.

Competitive Examination for Clerkships in the Bank (Time 2 hours.)

1. Find (by Practice) the dividend on £1731 10s., at 13s. 3d. in the pound.

2. Find (by Practice) the price of 3 tons 15 cwt. 1 qr. 7 lbs., at £3 per ton.

3. Add together $2\frac{1}{3}$, $\frac{3}{16}$, $1\frac{7}{24}$ and $\frac{1}{2}$.

4. Divide 4·67 by 2·643 to five places of decimals.

5. Convert the following sums into pounds and decimal parts of a pound, to three places: add the results, and compare with the tot obtained in the ordinary way:—£34 5s. 7d., £21 6s. 4d., £3 2s. 6d., £9 5s. 10d., £61 2s. 4d. Place the two workings side by side.

6. Find the simple interest on £7426 15s. 6d. at $3\frac{1}{2}$ per cent. per annum, for 14 months.

7. Find the simple interest on £516 3s. 7d. for 97 days, at $4\frac{1}{4}$ per cent. per annum.

8. Find the amount of £7428 in three years, at 4 per cent. compound interest.

9. A bill at three months, drawn 11th April, for £198 17s. 6d., is discounted 7th May, at $5\frac{1}{2}$ per cent.; calculate the proceeds.

10. The following sums of money are invested:—£35,000 at 4 per cent.; £126,000, at $4\frac{1}{2}$ per cent.; £40,000 at $3\frac{1}{2}$ per cent.; and £56,000 at 3 per cent. Calculate the average interest on the whole amount invested.

11. A sum of money, £500, is lodged on the 12th April to a deposit account, when $2\frac{1}{2}$ per cent. interest is allowed; on the 27th May the interest falls to 2 per cent.; on the 17th August it is advanced to 3 per cent.; and on the 30th September the deposit is called in. Calculate how much the depositor will get.

12. The following sums bearing interest are lodged:—£360, 13th March, at $3\frac{1}{2}$ per cent.; £420, 17th June, at 4 per cent.; £1000, 5th July, at 4 per cent.; £500, 7th September, at $4\frac{1}{2}$ per cent.; and £1200, 28th November, at 5 per cent. Calculate the interest due on 31st December.

ANSWERS.

I.

(1) 590. (2) 1403. (3) 2077. (4) 4570. (5) 8009. (6) 9097. (7) 461001. (8) 2005087. (9) 27000706. (10) 840020. (11) 10004008. (12) 1102070050. (13) 90040. (14) 304040004. (15) 603050. (16) 7003040. (17) 8000001002. (18) 500401. (19) 30007040. (20) 506000020. (21) 4000505. (22) 8000000600. (23) 10020001. (24) 400004040. (25) 9050060004. (26) 805000000099.

II.

(1) 218721. (2) 3634130. (3) 38203510748. (4) 2159758. (5) 2073488. (6) 1143459. (7) 1423010. (8) 1608991. (9) 3292. (10) 1967324. (11) 176888441. (12.) 12413. (13) 304125276. (14) 2348414. (15) 305852878. (16) 567036010. (17) 12733136. (18) 504605285. (19) 4985971533. (20.) 4568592. (21) 9598141817. (22) 122356701416. (23) 5113599. (24) 11014405709. (25) 293771741. (26) 1996644. (27) 426683748. (28) 1824386339. (29) 1198207. (30) 324213593. (31) 27378073. (32) 17018867. (33) 11257491. (34) 14145339. (35) 13439199. (36) 12914265. (37) 297459646. (38) 97989889. (39) 985619428.

III.

(1) 61333. (2) 66641. (3) 217049. (4) 21107. (5) 96444. (6) 2165. (7) 92514. (8) 662733. (9) 47693. (10) 1618. (11) 6779495701. (12) 6097700810072. (13) 593684253. (14) 4228908880. (15) 4230399880. (16) 7977100909213. (17) 27909498. (18) 875308643. (19) 163161134. (20) 500710920089. (21) 96106639364.

IV.

(1) 690739. (2) 702621. (3) 947089. (4) 1171740. (5) 566952. (6) 8690693. (7) 199617 : 243012. (8) 3694992 : 4310824. (9) 7278538 : 8475010. (10) 81745410 : 87768756. (11) 109344206 : 116232030. (12) 54949721 : 62052737. (13) 43606640 : 76155882. (14) 68604840 : 72808200. (15) 9003456000 : 851576880. (16) 3996915833 : 80737410. (17) 4342886402400. (18) 24620422875300. (19) 97152093486360. (20) 49631586380800. (21) 9108568931220. (22) 751680000 : 146880000. (23) 151240000 : 129350000. (24) 5551104 : 6244992. (25) 10611337 : 11576004. (26) 5024659561. (27) 46129276014848. (28) 5690578298. (29) 51734631391872. (30) 3592212. (31) 1486958060. (32) 3131672. (33) 1000362150. (34) 153288487686. (35) 250075455045. (36) 69162164100652. (37) 49062139937803. (38) 9523916224416.

V.

(1) 32885 + 1*; 19731 + 1. (2) 11899 + 2 : 10411 + 7. (3) 1038816 : 779112. (4) 3071 : 2943 + 1. (5) 3338 : 2616 + 10. (6) 2966 + 21 : 2559 + 72. (7) 9159 + 26 : 8468. (8) 15423 + 148 : 12325 + 65. (9) 11878 : 11136 + 508. (10) 953 + 368 : 917 + 8. (11) 932746. (12) 829107 + 5. (13) 62084251446 + 4. (14) 8866443377. (15) 120874856968. (16) 967427210 + 61. (17) 908382685 + 366. (18) 25484 + 44. (19) 457878404 + 311. (20) 12086960 + 59. (21) 30000 + 56. (22) 55655242 + 37.

* Remainder.

(23) 11593 : 6011 + 15. (24) 7282 : 6499 + 77. (25) 14556 : 116751 + 12. (26) 12247 + 441 : 161969 + 16. (27) 10979 + 319 : 80515 + 67. (28) 488 + 1179 : 684 + 379. (29) 4697 + 86 : 74 + 3586. (30) 5800 + 400 : 5301 + 540. (31) 38745 + 20 : 27675 + 20. (32) 14964 + 55 : 7183 + 20. (33) 39147 + 10 : 21748 + 25. (34) 39873 + 21 : 22152 + 6. (35) 1904916 : 3565612. (36) 3996370 : 7495894. (37) 9155 + 3½ : 3410 + 21. (38) 11805 + ½ : 2724 + 7¼.

VI.

(1) 1452507303. (2) 117288146. (3) 1527 + 7. (4) 74813. (5) 788910. (6) 13707360. (7) 102901. (8) 999. (9) 10670 : 8536 : 10246401 : 9. (10) 1067. (11) 23887872. (12) 17030. (13) 152 days. (14) 2397 days. (15) £14400. (16) 66. (17) 477120. (18) 385d. (19) 11. (20) 14s. and 11s. (21) 30. (22) 1038832. (23) 3623. (24) 4620. (25) 145881529087.

VII.

a. (1)...(2) 9017059096; seven thousand one hundred and six millions five hundred and ninety-seven thousand and forty-eight. (3) 662 and 473. (4) 31036. (5) Sixty-two millions eight thousand six hundred and seventy and one-half pounds. (6) 1053173490.

b. (1)...(2) 52 years. (3) £334292471. (4) See p. 26. (5) Six hundred and thirty-eight thousand five hundred and ninety-four. (6) 104 weeks.

c. (1) 4083. (2) Three hundred millions thirty thousand and thirty : nine hundred and ninety-nine thousand nine hundred. (3) 187870 + 27 rem. (4) 23 days. (5) 99 : 119 : 118. (6) 186000 miles.

d. (1) 334997947676. (2) 6062513. (3) 233 and 204. (4) 24725¼. (5) 258694½ miles. (6) 224433 fr. 86146 sovereigns.

e. (1) 3891144444627440. (2) 297. (3) £1. (4) 197127. (5) 808920 times. (6) 4236 + 26. rem.

f. (1) 2482896964595. (2) (*a*) 74. (*b*) 81. (*c*) 56. (*d*) 195. (*e*) 79, and final result 10. (3) 275 acres ; £1306¼. (4) 106689. (5) 2268. (6) 850.

VIII.

(1) 15120f. (2) 81120 feet. (3) 76886 oz. (4) 42281 lbs. (5) 2703294 in. (6) 124614 oz. (7) 44387 oz. (8) 582205 oz. (9) 865170 in. (10) 2689542 in. (11) 40704 qrts. (12) 200944 grs. (13) 136240 sq. ft. (14) 99528 grs. (15) 91214 grs. (16) 137036 in. (17) 2181240 secs. (18) 17875 sq. in. (19) 643104 sq. in. (20) 99204 ft. (21) 101095 grs. (22) 1574624 drs. (23) 9591f. (24) 78256 grs. (25) 1358880 drs. (26) 506722f. (27) 673328 oz. (28) 58191 ft. (29) 85931 grs. (30) 25432235 secs. (31) 3055709 secs.

IX.

(1) £25 14s. 1d. (2) 428 tons 4 cwts. 13 lbs. 11 oz. (3) 90 lbs. 11 oz. 3 dwts. 17 grs. (4) 13 acs. 12 pls. 17 yds. 5 ft. (5) 48 wks. 3 days 13 hrs. 8 m. 55 secs. (6) 19 lbs. 4 oz. 3 dwts. 22 grs. (7) 5 mls. 3 fur. 38 pls. 2 yds. 8 in. (8) 204 tons 2 cwts. 2 qrs. 1 lb.

(9) 1 wk. 10 hrs. 25 secs. (10) 25 loads. (11) 1041 gross 8 doz. (12) 7 ac. 26 pls. 10½ yds. (13) 4 lbs. 7 dwts. 13 grs. (14) 18 cwts. 8 lbs. 7 oz. (15) 91 mls. 3 fur. 15 pls. 5 yds. 1 ft. 8 in. (16) 3 mls. 7 fur. 9 pls. 2 yds. 1 ft. 5 in. (17) 46 lbs. 3 oz. 2 dwts. 2 grs. (18) 14 cub. yds. 2 ft. 1278 in. (19) 2 cub. yds. 26 ft. 57 in. (20) 49 mls. 2 fur. 31 pls. 3 yds. 1 ft. 11 in. (21) 2 tons 8 cwts. 1 qr. 18 lbs. 2 oz. (22) 21 cubic yds. 4 ft. 189 in. (23) 113067. (24) 9880 crowns. (25) 16048 half sovs. + 2*s*. (26) 11280 nls. (27) 43815 francs. (28) £4 17*s*. 7*d*. (29) 294.

X.

(1) £153 13*s*. 5*d*. (2) £232 17*s*. 10*d*. (3) £218 19*s*. 9*d*. (4) £235 15*s*. 11*d*. (5) £247 6*s*. 7¾*d*. (6) £234 8*s*. 8½*d*. (7) £11068 4*s*. (8) £176 19*s*. 6¾*d*. (9) £3867 8*s*. 11¾*d*. (10) £11100 6*s*. 4¼*d*. (11) £8590 13*s*. 4¾*d*. (12) £23440 5*s*. 2*d*. (13) £35856 4*s*. 10*d*. (14) £18251 12*s*. 5¾*d*. (15) £10845 16*s*. 9¼*d*. (16) £13867 18*s*. 5¾*d*. (17) £3269 1*s*. 10¾*d*. (18) £3297 4*s*. 5*d*. (19) £71516 14*s*. 7*d*. (20) £42610 15*s*. 8¼*d*. (21) £54353 18*s*. 8¼*d*. (22) £2851 2*s*. 0¼*d*. (23) £4942 7*s*. 3¼*d*. (24) £2886 0*s*. 3*d*. (25) £3712 4*s*. 11½*d*. (26) £4771 7*s*. 10*d*. (27) £5206 13*s*. 9¾*d*. (28) £5670 13*s*. 7¾*d*. (29) £11898 14*s*. 6½*d*. (30) £21986 13*s*. 0¼*d*. (31) £114528 19*s*. 11¼*d*. (32) £18837 11*s*. 3½*d*. (33) £11558 15*s*. 3¼*d*. (34) £4384 12*s*. 0½*d* (35) £2677 14*s*. 11*d*. (36) £2324 6*s*. 10¼*d*. (37) 2152 cwts. 2 qrs. 16 lbs. (38) 297 cwts. 1 qr. 13 lbs. (39) 321 lbs. 4 oz. 19 dwts. (40) 374 lbs. 6 ozs. 16 dwts. (41) 90 acs. 23 pls. (42) 158 acs. 3 rds. 39 pls. (43) 279 acs. 1 r. 7 pls. (44) 144 acs. 24 pls. 17 yds. 2¼ ft. (45) 656° 48′ 10″. (46) 3064° 55′ 36″. (47) 106 gals. 3 nags. (48) 131 gals. 2 qts. (49) 163 tons 9 cwts. 14 lbs. (50) 53 yds. 2 ft. 11½ in. (51) 165 hhds. 50 gals. 2 qts. (52) 148 mls. 4 fls. 11 pls. 1 ft. 6 in. (53) 138 hrs. 56′ 20″. (54) 340 yrs. 3 m. 2 wks. 4 days. (55) £231509 14*s*. 11*d*. (56) £109771 0*s*. 6*d*. (57) £217749 11*s*. 5*d*.

XI.

(1) £89 8*s*. 7¾*d*. (2) £102 18*s*. 10¾*d*. (3) £966 16*s*. 9¾*d*. (4) £1821 1*s*. 7¾*d*. (5) £87 13*s*. 8¼*d*. (6) 103 16*s*. 0¾*d*. (7) £1895 17*s*. 6¾*d*. (8) £2693 0*s*. 8¾*d*. (9) £1012 2*s*. 9¼*d*. (10) 86 cwts. 3 qrs. 2 lbs. (11) 179 cwts. 2 qrs. 25 lbs. (12) 4 cwts. 2 qrs. 24 lbs. (13) 36 ac. 3rds. 34 pls. (14) 77 ac. 14 pls. (15) 186 ac. 3 rds. 31 pls. (16) 110 ac. 2 rds. 15 pls. (17) 13 lbs. 14 oz. 15 drs. (18) 61 lbs. 14 oz. 12 drs. (19) 681 lbs. 15 oz. 15 drs. (20) 73 lbs. 2 oz. 13 drs. (21) 70 yrs. 11 m. 1 wk. (22) 15 yrs. 10 m. 3 wks. (23) 3 days 12 hrs. 52 min. (24) 42 days 21 hrs. 54 m. (25) 79 oz. 17 dwts. 21 grs. (26) 56 lbs. 10 oz. 14 dwts. (27) 7 lbs. 4 oz. 15 dwts. (28) 1 oz. 16 dwts. 21 grs. (29) 2 yds. 2 ft. 11 in. (30) 6 mls. 7 fur. 34 pls. (31) 2 sq. yds. 7 ft. 114 in. (32) 19 sq. pls. 25 yds. 2 ft. 16 in. (33) 97 yds. 3 qrs. 3 nls. (34) 47 ells 3 qrs. 2 nls. 2 in. (35) £53 6*s*. 6*d*. (36) £44 13*s*. 7½*d*. (37) £13 4*s*. 9*d*. (38) £147 10*s*. 9¾*d*. (39) £2 8*s*. 6*d*. (40) £247 11*s*. 7¾d. (41) £966 2*s*. 2½*d*. (42) £288 4*s*. 3¾*d*. (43) £740 7*s*. 9¼*d*. (44) £577696 14*s*. 10½*d*. (45) 202 tons 18 cwts. 2 qrs. 20 lbs. (46) 53 yds. 1 ft. 2 in. (47) £80898 10*s* 9½*d*. (48) £557862 12*s*. 6½*d*. (49) £480259 12*s*. 6¾*d*. (50) £319251 7*s*. 7½*d*. (51) £310205 5*s*. 10¾*d*. (52) 18 mls. 6 fur. 38 pls. 4 yds. (53) 10 mls. 6 fur. 38 pls. 4 yds. 5 in.

XII.

(1) £175 17*s*. 11*d*. (2) £201 7*s*. 9¼*d*. (3) £271 18*s*. 9*d*. (4) £556 19*s*. 10½*d*. (5) £336 9*s*. 7*d*. (6) £508 2*s*. 10½*d*. (7) £17039 11*s*.; £19169 9*s*. 10½*d*. (8) £28343 19*s*.; £29524 18*s*. 11½*d*. (9) £46079 5*s*. 4*d*.

£51839 3s. 6d. (10) £64695 11s. 3d.; £69008 12s. (11) £51345 17s. 6d.; £51773 15s. 1¾d. (12) £42830 19s. 6d.; £46724 14s. (13) £39225 19s. 11¼d.: 42068 9s. 2½d. (14) £32004 16s. 1¾d.; £33015 9s. 8½d. (15) £49454 5s. 9¼d.; £53786 1s. 0¼d. (16) £30324 4s. 1½d.; £48985 5s. 1½d. (17) 2930 ac. 3 rds. 10 pls. (18) 12212 ac. 8 pls. (19) 2188 cwts. 1 qr. 16 lbs. (20) 12021 cwts. 3 qrs. 27 lbs. (21) 14800 yds. 1 qr. 2 nls. (22) 27909 yds. 2 qrs. 1 nl. (23) £2502 14s. 7d. (24) 447 tons 3 cwt. 2 qrs. 15 lbs. 13 oz. 15 drs. (25) £757 0s. 4¼d.; £447 2s. 6d. (26) £587 10s. 4½d.; £2859 16s. 1½d. (27) £564 5s. 9d.; £1758 5s. 2d. (28) £1409 3s. 6d.; £3588 18s. 2½d. (29) £46935 18s. 11½d.; £10631 5s. (30) £4139 3s. 8½d.; £32214 0s. 6d. (31) £5371 8s. 9d.; £8702 17s. 1¼d. (32) £17268 7s. 0½d.; £7073 9s. 2½d. (33) £37483 15s. 5d.; £11097 8s. 4½d. (34) £37746 13s. 7d.; £18705 13s. 8¼d. (35) £60589 18s. 0½d.; £161804 7s. 6½d. (36) £100376 19s. 0¾d.; £11699 0s. 6d. (37) £1046104 19s. 0¾d.; £174792 18s. 5¼d. (38) £4363 16s.; £6354 1s. 6d. (39) £20030 2s. 8½d.: £41562 5s. 7½d. (40) £37686 4s. 1½d.; £17580 10s. 8¾d. (41) £58593 7s. 1½d.; £15255 3s. 9d.

XIII.

(1) £13 19s. 2d. (2) £11 16s. 4½d. (3) £12 12s. 9½d. (4) £13 18s. 6¾d. (5) £11 11s. 10½d. (6) £12 18s. 6¾d. (7) £142 9s. 5¼d.; £116 6s. 0¾d.+5¼d.* (8) £112 0s. 8¼d.; £99 11s. 8½d.+1s. 1½d. (9) £21 18s. 10½d.; £18 5s. 8¾d. (10) £10 17s. 9d.+8¼d.; £17 5s. 6¾d.+6¾d. (11) £34 18s. 3¼d.+7¾d.; £29 7s. 10d. (12) £22 11s. 11¼d. (13) 75 cwts. 6 lbs. 7 oz. 7 drs.+3 oz. 8 drs. (14) 43 cwts. 3 qrs. 17 lbs. 10 oz. 10 drs.+8 oz. 4 drs. (15) 1 cwt. 2 qrs. 25 lbs. 2 oz. 3 drs.+7 drs. (16) 3 acres 1 rd. 14 pls. (17) 7 acres 3 roods 29 pls. 28 yards 5 feet+2 feet 72 inches. (18) 495 (19) 1100. (20) 1465. (21) 1195. (22) 1175 + 7 lbs. (23) 965. (24) £71 15s. 11d. + 1s. 10½d. (25) £2 6s. 6½d. + 15s. (26) £17 11s. 10½d. + 10s. 6d. (27) £6 11s. 9½d. + £1 15s. (28) £1 3s. 8¼d. + 6d. (29) 4s. 8¾d. + 2¼d. (30) £10845 13s. 1d. (31) £981 6s. 4d. (32) £20270 4s. 11d. (33) £103899 0s. 3d. (34) £409 8s. 11d. (35) £239 9s. 9d. (36) £148805 3s. 1d. (37) 37200 11s. 3¼d. (38) £3246 9s. 7d.; £811 12s. 4¾. (39) £79695 19s. 11¾d.: (40) £39 11s. 9¼d. (41) £39 11s. 9¼d. (42) 7994. (43) 1197. (44) 4s. 9½d. (45) 17 cwts. 2 qrs. 12 lbs. (46) £8269 6s. 4½d. (47) £6206 13s. 8¾d. (48) 234567 grs.: 40 lbs. 8 oz. 13 dwts. 15 grs. (49) 144 sq. mls.: 92160 acrs. (50) £2959 2s. 1d.: £32463 19s. 2d. (51) 1680 h. crns. 400 h. guins. (52) 6¼d. (53) 516168 pints. (54) £4 7s. 3¼d. (55)

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* Remainder.

2760 qrs. 5 bus. 3 pks. (56) 71874 in. (57) 93168 grs. (58) 9842796 sq. in. (59) 131 lbs. 7 oz. 18 dwts. 20 grs. (60) 5377200 oz. (61) 110 tons 2 cwt. 3 qrs. 20 lbs. 4 oz. (62) 639 qrs. 4 bus. 2 pks. 2 qts. 1 pint. (63) £21 12s. 8½d. (64) 12707 ft. (65) 203 acs. 2 rds. 13 pls. 10 yds. 7 ft. 108 in. (66) 168 lbs. 11 oz. 17 dwts. 8 grs. (67) 512 mls. 4 fur. 33 pls. 2 yds. 1 ft. 3 in. (68) 62857296 in. (69) 18561 qrs. 7 bus. 1 pk. 1 gall. 1 qrt. 1 pint. (70) 2 ac. 3 rds. 2 pls. 18 yds. 8 ft. 72 in. (71) 5 wks. 10 hrs. 12′ 32″. (72) £96 8s. 7½d. (73) 633 tons 5 cwt. (74) £64 15s. (75) 203 mls. 5 fur. 15 pls. 2 ft. 6 in. (76) 1 ton 2 cwt. 2 qrs. 18 lbs. 15 ozs. (77) 59 ac. 1 rd. 7 yds. 1 ft. 57 in. (78) 3297 pints. (79) 7s. 9½d; £43 0s. 11¾d. (80) 6¼d. (81) 3 ac. 1 rd. 2 pls. (82) 192. (83) £1905 15s. (84) 92 tons 18 cwts. 1 qr. 2 lbs. 6 oz. (85) 128; 4 in. (86) 725. (87) £1 18s. 4d. (88) £9 4s. 10½d. (89) 3s. 11d. (90) £1449. (91) 168 c. y. 15 ft. 1247 in. (92) 134 tons 17 cwts. 3 qrs. 27 lbs. 8 oz. 7 drs. (93) 224003877″. (94) £5062 10s. (95) 5d. (96) 4744 mls. 4 fur. 36 pls. 2 yds. (97) 2722. (98) £88 1s. 2½d. (99) £658 12s. (100) 18¾ mls.

XIV.

(1) £750 2s. : £62 10s. 2d. : (2) £15 6s. 5¾d. : £4 8s. 1½d. (3) 1½d. (4) £986 0s. 9¾d. : £51 17s. 11¼d. (5) £52 6s. 10½d. ; £47 13s. 1½. (6) £5 10s. 3d. (7) 1 qr. 25 lbs. 8 oz. (8 3 yrs. 30 wks. (9) £56 16s. 5¾d. (10) 13 ac. 3 rds. 24 pls (11) 7 cwts. 3 qrs. 26 lbs. (12) £17 12s. 10½d. (13) 146 dollars. (14) 5 tons 9 cwts. 3 qrs. 23 lbs. 7 oz. 6 drs. + 3 oz. (15) £110 5s. (16) £300. (17) 20 shirts. (18) 25 sts. 2 lbs. 8 oz. (19) 500 dollars. (20) 49 yds. 3 qrs. 2 nls. (21) £8 13s. 4. (22) £175 15s. 8¼d.; £125 3s. 2¼d. (23) £5372 4s. 7¾d. (24) 313632000 sq. inches. (25) 570240 inches. (26) 41847 farthings. (27) 662832 oz. (28) £25 5s. 1¼d. (29) £58 15s. (30) £257 0s. 4¼d. (31) 224461½ feet. (32) 18 lbs. 2 oz. 18 dwts. 3 grs. (33) 93 tuns 2 hhds. 17 gals. 3 qrts. $1\frac{2}{5}$ pt. (34) £5506 5s.; £4958 15s. (35) 2 miles an hour. (36) £1 7s. (37) 4s. 7½d. British money. (38) £1 13s. 1½d. (39) $107224\frac{8}{13}$ times. (40) 3 cub. yds. 20 ft. 1455 in. (41) 179685 sq. feet. (42) 2 mls. 5 fur. 39 pls. 1 yd. 4 in. (43) 39 lbs. 8 ozs. 17 dwts. 20 grs. (44) 278172 inches. (45) 9 tons 2 cwts. 2 qrs. 26 lbs. 15 oz. (46) 4221 feet. (47) 16 hrs. 20 mins. 18 secs. (48) 514752 ozs. (49) 59 mls. 5 furs. 34 pls. 1 ft. 9 in. (50) 229168 ozs. (51) ½ hour. (52) 27 mls. 1 fur. 15 pls. 5 yds. 3 in. (53) 3 yds. 1 ft. (54) 35101½ feet. (55) 592 sq. yds. 4 ft. 106 in. (56) 192669 feet. (57) 1820752 drs. (58) 85931 grs. (59) 27 cwts. 1 qr. 23 lbs. 7 ozs. (60) 615187 cubic inches. (61) 49602 feet. (62) 37½ lbs. (63) 29 mls. 1 fur. 11 pls. 2 yds. 2 ft. 4 in. (64) 17526. (65) 266450 grs. (66) 632592 oz. (67) 8 mls. 3 fur. 32 pls. 3 yds. 2 in. (68) 1028 tons 1 cwt. 3 lbs. 11 oz. (69) 115 tons 17 cwts. 2 qrs. 6 lbs. (70) 2 wks. 6 days 5 hrs. 52 m. 14 secs. (71) 18561 qrs. 7 bus.

1 pk. 1 gal. 1 qrt, 1 pt. (72) 2 ac. 3 rds. 2 pls. 18 yds. 8 ft. 72 in. (73) $83668686\frac{3}{4}$ sq. feet. (74) £15 17s. 3d. (75) £147 0s. $5\frac{1}{4}$d. (76) £5062 10s. (77) 80 ozs. 12 dwts. 12 grs. (78) 1715 yards; £380 3s. 2d. (79) 798765439 drs.; £4992283 19s. $10\frac{1}{2}$d. (80) 1920. (81) £4 7s. 6d.; $7\frac{1}{4}$d. (82) 701 lbs. 6 ozs. 6 dwts. 5 grs. (83) 45724 persons. (84) £72 13s. $10\frac{1}{2}$d. (85) 34940585 sq. in. (86) 74907 and 92 rem. (87) £7562 18s. 4d. (88) £8 12s. $8\frac{3}{4}$d. (89) £3 10s. $11\frac{1}{2}$d. gain. (90) £8 13s. $4\frac{1}{2}$d.; £92 19s. 2d. (91) A. £350; B. £450; C. £720. (92) 17 tons 14 cwts. 3 qrs. 21 lbs. 2 ozs. (93) £232 15s. $3\frac{1}{2}$d. (94) 484. (95) £4724 6s. $1\frac{1}{2}$d. (96) 791083425 secs. (97) £296 11s. $11\frac{3}{4}$d. (98) 99. (99) £2 17s. 11d. (100) white hats; 1s. 6d. (101) 4° 24′ 6″. (102) 13267583 times. (103) £762 16s. 8d. (104) 6632 acres 1 rd. 1 pl. 6 yds. 4 ft. 140 in.; 206 ac. 1 rd. 16 pls. 7 yds. 8 ft. $30\frac{1}{4}$ in. (105) 28800 times; 5 hrs. $18\frac{3}{4}$ m. (106) 330; £309 14s. $4\frac{1}{2}$d. (107) 44 mls. 960 yds. (108) £2 5s. (109) £106 16s. 9d. (110) 1° 55′ 23″; 23° 21′ $10\frac{3}{4}$″. (111) £4 14s. $10\frac{1}{2}$d. (112) 20942352 sq. ft. (113) 10945 mls. 7 fur. 37 pls. (114) 765 ac. 3 rds. 9 poles. (115) 4s. 3d. (116) A. £39 7s. $7\frac{1}{2}$d.; B. £65 12s. $8\frac{1}{2}$d.; C. £105 0s. 4d.; D. £118 2s. $10\frac{1}{2}$d. (117) 4 yrs. 45 days 15 hrs. 45 m. (118) £42 8s.; £7 2s. (119) 95 lbs. 1 oz. 2 dwts. 22 grs. (120) £998 18s. $7\frac{3}{4}$d. (121) $26\frac{2}{3}$ gals. (122) 33. (123) 2s. 3d. (124) 18s. 4d. (125) 209 boys. (126) £2 18s. 4d. (127) 1 bushel. (128) £3 10s. (129) 60 miles. (130) 29 lbs. 6 oz. (131) 77 shares. (132) £1 17s. (133) 1 hr. 40 min. (134) 226 ac. 3 rds. 20 pls. (135) 16 lbs. 14 ozs. (136) £3 11s. 8d.; £10 15s.; £1 15s. 10d. (137) 23850 gross. (138) £13 11s. $11\frac{3}{4}$d. (139) 4d. (140) Loss £30 3s. $5\frac{1}{2}$d.

XV.

(1) $7\frac{2}{3} : 4\frac{1}{7} : 1\frac{16}{17}$. (2) $54\frac{12}{25} : 212\frac{29}{33} : 72\frac{9}{11}$. (3) $14\frac{29}{144} : 103\frac{13}{19} : 111\frac{16}{21}$. (4) $9\frac{1}{2} : 21\frac{8}{17} : 13\frac{38}{47}$. (5) $\frac{64}{9} : \frac{38}{5} : \frac{129}{7}$. (6) $\frac{5171}{40} : \frac{387}{23} : \frac{59}{9}$. (7) $\frac{316}{27} : \frac{351}{1} : \frac{704}{11}$. (8) $\frac{289}{9} : \frac{199}{12} : \frac{13504}{119}$. (9) $\frac{209}{11} : \frac{170}{17}\ \frac{870}{29}$. (10) $\frac{1053}{89} : \frac{1794}{46} : \frac{9405}{57}$. (11) 22 : 39. (12) 9 : 4. (13) $5 : 4 : \frac{1}{48}$. (14) 6 : 19. (15) 12 : 8. (16) 14 : 8. (17) $\frac{3}{7} : \frac{3}{10} : \frac{2}{11}$. (18) $\frac{7}{103} : \frac{4}{5} : \frac{9}{11}$. (19) $\frac{9}{13} : \frac{4}{9} : \frac{5}{7}$. (20) $\frac{1}{6} : \frac{5}{9} : \frac{11}{13}$. (21) $\frac{20}{21}$. (22) $\frac{7}{8}$. (23) $\frac{5355}{7144}$. (24) $\frac{12,\ 16,\ 20,\ 21}{24}$. (25) $\frac{30,\ 28,\ 15,\ 42}{48}$. (26) $\frac{90,\ 255,\ 264,\ 190}{330}$. (27) $\frac{6545,\ 6120,\ 7293,\ 4455}{8415}$. (28) $\frac{360,\ 350,\ 385,\ 546}{630}$. (29) $\frac{7}{8} : \frac{63}{80} : \frac{3}{32}$. (30) $\frac{11}{80} : \frac{13}{160} : \frac{37}{80}$. (31) $\frac{3}{4} : \frac{5}{8} : \frac{3}{8}$. (32) $\frac{5}{8} : \frac{7}{8} : \frac{5}{16}$. (33) $\frac{5}{8} : \frac{13}{16} : \frac{7}{16} : \frac{29}{32}$. (34) $\frac{3}{7}$. (35) $\frac{7}{11}$. (36) $\frac{11}{13}$. (37) 95.

XVI.

(1) $\frac{1}{4}$; $2\frac{4}{7}$. (2) 2; $2\frac{3}{13}$. (3) $1\frac{17}{60}$. (4) $1\frac{73}{84}$. (5) $1\frac{29}{36}$. (6) $1\frac{17}{28}$. (7) $1\frac{2341}{3120}$. (8) $1\frac{35}{36}$. (9) $2\frac{31}{36}$. (10) $1\frac{8}{21}$. (11) $2\frac{37}{63\text{[illegible]}}$.

(12) $4\frac{14}{15}$; $6\frac{3}{44}$. (13) $3\frac{65}{72}$; $9\frac{11}{16}$. (14) $5\frac{71}{168}$; $24\frac{19}{70}$. (15) $7\frac{8}{9}$; $13\frac{2}{3}$. (16) $2\frac{151}{385}$; $4\frac{142}{357}$. (17) $10\frac{29}{45}$; $18\frac{3}{16}$. (18) $8\frac{11}{18}$; $12\frac{7}{10}$. (19) $13\frac{61}{120}$; $44\frac{7}{10}$. (20) $5\frac{89}{120}$; $46\frac{49}{184}$. (21) $5\frac{31}{40}$; $40\frac{4}{5}$. (22) $6\frac{7}{12}$; $103\frac{157}{264}$. (23) $8\frac{5}{42}$; $44\frac{23}{60}$. (24) $12\frac{25}{84}$; $19\frac{7}{26}$. (25) £140 12s. $2\frac{1}{4}$d. (26)£258 6s. 6d. (27) £317 7s. $2\frac{1}{26}$. (28) $60\frac{67}{96}$; $9\frac{67}{87}$. (29) $85\frac{71}{72}$; $8\frac{6}{19}$. (30) $105\frac{313}{360}$. (31) $40\frac{155}{252}$. (32) $36\frac{144}{247}$.

XVII.

(1) $\frac{1}{4}$; $\frac{1}{3}$; $\frac{3}{7}$. (2) $\frac{1}{7}$; $\frac{3}{19}$; $\frac{3}{8}$. (3) $\frac{47}{110}$. (4) $\frac{7}{40}$. (5) $2\frac{4}{35}$; $6\frac{13}{56}$; $33\frac{5}{11}$. (6) $4\frac{31}{120}$; $2\frac{1}{90}$; $2\frac{23}{30}$. (7) $\frac{41}{100}$; $17\frac{14}{15}$; $11\frac{13}{14}$. (8) $2\frac{1}{5}$; $1\frac{17}{24}$; $8\frac{11}{15}$. (9) $2\frac{22}{35}$; $\frac{83}{84}$; $3\frac{23}{24}$. (10) $33\frac{3}{26}$; $\frac{61}{72}$; $15\frac{15}{34}$. (11) $10\frac{5}{21}$; $\frac{5}{72}$; $2\frac{1}{308}$. (12) $8\frac{1}{3}$; $17\frac{38}{39}$; $26\frac{41}{76}$. (13) $12\frac{7}{8}$; $4\frac{1}{10}$; $3\frac{1}{35}$. (14) $3\frac{35}{36}$; $21\frac{89}{138}$; $16\frac{1}{56}$. (15) $42\frac{136}{143}$; $47\frac{1}{36}$; $99\frac{4}{17}$. (16) $3\frac{179}{180}$; $17\frac{11}{15}$; $\frac{5}{14}$. (17) $2\frac{19}{60}$; $12\frac{46}{63}$; $\frac{1}{3}$. (18) $21\frac{1}{20}$; $\frac{121}{240}$; $1\frac{661}{1001}$. (19) $\frac{3}{14}$; $\frac{113}{315}$; $\frac{141}{280}$.

XVIII.

(1) $\frac{12}{55}$; $123\frac{171}{200}$; $\frac{4}{7}$. (2) $\frac{1}{6}$; $2\frac{2}{3}$; $\frac{5}{132}$. (3) $\frac{20}{63}$; $\frac{15}{16}$; 18. (4) $16\frac{31}{84}$; $\frac{39}{64}$; $7\frac{7}{44}$. (5) $\frac{15}{224}$; $26\frac{1}{4}$; $8\frac{8}{9}$. (6) $5\frac{29}{32}$; $17\frac{1}{10}$; 18. (7) $\frac{34}{315}$; 2; $7\frac{7}{44}$. (8) $97\frac{3}{5}$; $1\frac{9}{40}$; $\frac{5}{48}$. (9) $443\frac{3}{32}$. (10) 84. (11) $99\frac{1}{12}$. (12) $253\frac{15}{28}$. (13) 30. (14) $40\frac{71}{84}$. (15) $96\frac{3}{35}$. (16) 91. (17) $15\frac{7}{9}$. (18) 56. (19) 2s. 11d. (20) £3 12s 2d. (21) 8s. 7d. (22) 63.

XIX.

(1) $\frac{10}{63} : \frac{3}{4}$. (2) $\frac{20}{81} : 13\frac{3}{4}$. (3) $1\frac{31}{36} : 1\frac{79}{92}$. (4) £8 6s. $4\frac{4}{11}$d. (5) £21 8s. $6\frac{6}{7}$d. (6) 23 cwts. 1 qr. $2\frac{4}{5}$ lbs. (7) 5 ac. 1 rd. 17 pls. 15 yds. 1 ft. 18 in. (8) $1\frac{2}{3} : \frac{21}{80} : 13\frac{1}{2}$. (9) $\frac{5}{32} : 1\frac{3}{13} : 32\frac{2}{3}$. (10) $\frac{7}{20}$. (11) $\frac{9}{10}$. (12) $\frac{1}{98}$. (13) $3\frac{6}{7}$. (14) $1\frac{2}{5}$. (15) $\frac{15}{18}$. (16) $\frac{1}{2}$. (17) $\frac{41}{189}$. (18) $\frac{1}{8}$. (19) $2\frac{11}{272}$. (20) $\frac{169}{306}$; 4 : $\frac{49}{75}$. (21) $1\frac{224}{351} : 3\frac{3}{19} : \frac{21}{110}$. (22) $\frac{34}{65} : 13\frac{1}{3} : 2\frac{5}{6}$. (23) $\frac{87}{115} : \frac{3}{19} : \frac{2}{865}$. (24) $1\frac{38}{45} : \frac{45}{76}$: 92. (25) $3\frac{5}{9} : 2\frac{12}{29} : 13\frac{1}{3}$. (26) $1\frac{9}{22} : \frac{1}{6} : 7\frac{4}{11}$. (27) $\frac{16}{27}$. (28) $\frac{13}{23}$. (29) $10\frac{1}{25}$. (30) $\frac{59}{120}$. (31) 15*s.* 10*d.* (32) $5\frac{1}{3}$ inches.

XX.

(1) ·095. (2) ·039. (3) 12·068. (4) ·609. (5) 50·06. (6) 29·009. (7) 47·4. (8) 69·607064. (9) 120·014019. (10) 55·6494 (11) 60·06006. (12) 17·017017.

XXI.

(1) 425·9573. (2) 757·499. (3) 47·0709. (4) 19·6338. (5) 6404·8502. (6) 448·8074. (7) 183·0475. (8) 564·282692. (9) 779·1686. (10) 580·491902. (11) 7999·77021. (12) 515·92373. (13) 12988·7753.

XXII.

(1) 118·114. (2) 975305·1258. (3) 109997342·809099. (4) 11·7537. (5) 12·561. (6) 446·352. (7) 115·2839. (8) 2·109. (9) 3011·5026. (10) 1126·849608. (11) 29738·94371. (12) 1637·269485. (13) 33·06886.

XXIII.

(1) 510·3. (2) 375·54564. (3) 146·4561. (4) 341·508486. (5) ·0037777. (6) 1·69194. (7) ·37368892. (8) 2460168. (9) 2·691066. (10) 11961·232. (11) 636·83564. (12) ·035594. (13) ·7625 : ·07392. (14) ·2760494. (15) 5424·8324832. (16) ·01977048. (17) 32·5819809. (18) ·3154032.

XXIV.

(1) 12·96. (2) 7·8125. (3) 185000. (4) 12·932... (5) ·021... (6) 7 tons 18 cwts. 3 qrs. 10 lbs. 8 oz. 7·168 drs. (7) £160 14s. 5½d.* (8) 24106·25. (9) 3·271... (10) ·15. (11) 66·993... (12) 686·769... (13) ·09672. (14) 74·782... (15) $12054\cdot\dot{0}5\dot{4}$. (16) 40000. (17) $399016\cdot1\dot{4}\dot{8}$. (18) ·6586... (19) 50. (20) $32\cdot9\dot{4}155251\dot{1}$. (21) ·013 : ·0013 : 130. (22) ·01026... (23) 1230·769... (24) $\frac{1}{80}$. (25) $\frac{7}{4000}$. (26) $2\frac{88}{2000}:\frac{9}{1000}$. (27) $23\frac{81}{1000}:\frac{11}{10000}$. (28) ·5 : ·2 : ·875 ; ·8 ; ·76. (29) ·4375 : ·44 : ·40625 : ·734375, : ·1552. (30) ·790625£ : ·8875 : ·03125 : ·94375. (31) 40625£ $\cdot469791\dot{6}$. ·210416 : ·0675. (32) $\cdot08\dot{3}$: ·25 : $\cdot5208\dot{3}$: ·125. (33) : $\cdot14\dot{8}80952\dot{3}$: $2\dot{9}86507\dot{7}$. (34) ·615625 : ·396875 : ·00625. (35) ·615625 : ·5625 : ·21875 : ·296875. (36) ·09375. (37) $\cdot\dot{0}58823529411764\dot{7}$ (38) ·078125 : $\cdot2\dot{1}4285\dot{7}$. (39) ·084 : $\cdot00008680\dot{5}$ (40) ·875£ : ·68125£. (41) ·840625£ : ·975£. (42) ·890625£ : ·396875£. (43) ·54375£ : ·21875£. (44) 15s. : 4·536d. : 5·58d. (45) 23 hrs. 25 mins. 26·4 secs. (46) 4s. 2·72d. (47) 3 rds. 7 pls. 9 yds. 8 ft. 42·912 in. (48) 13s. 1¾d. : 19s. 6¾d. (49) £5 10s. 6d. (50) £1 6s. 11·856d. (51) 15s. (52) 5s. (53) 14s. 7½d. (54) 6d. (55) 5¼d.

XXV.

(1) $\frac{7}{9}:\frac{2}{3}$. (2) $\frac{5}{11}:\frac{5}{37}$. (3) $\frac{5}{18}:\frac{13}{22}$. (4) $\frac{7}{12}:\frac{9}{13}$. (5) $\frac{11}{15}:\frac{5}{6}$. (6) $\frac{2}{11}:\frac{2}{27}$. (7) $\frac{31}{74}:\frac{11}{140}$. (8) $\frac{21}{111}:\frac{3}{52}$. (9) $\cdot100\dot{9}5\dot{6}$; $\cdot022\dot{2}$.

XXV(*a*).

(1) $4\frac{1}{5}$. (2) ·0325201. (3) $\frac{7}{675}$. (4) £667 13s. $10\frac{41}{312}$d. : $\cdot621385\dot{7}3\dot{2}$. (5) $10\frac{14623}{63180}$. (6) $\cdot\dot{0}\dot{3}$. (7) 5. (8) 1 : 1. (9) $2\cdot\dot{5}934\dot{9}$. (10) $33\frac{4900572971}{21505112815}$. (11) $\cdot63\dot{6}3\dot{3}$. (12) $1:7\frac{1}{3}$. (13) $\frac{5}{168}$. (14) £38 12s. $6\frac{1}{3}$d. (15) $2024\frac{69007587}{78291600}$. (16) $2\frac{2432}{4335}$. (17) $\frac{2}{9}$. (18) $\frac{183}{11205}$. (19) $2\frac{3901}{5247}$. (20) $\frac{4913}{26241}$. (21) (*a*) $13\frac{52}{129}$. (*b*) £28 7s.

* Accurately $5\cdot5489\dot{7}81021\dot{8}$.

XXVI.

(1) $12{\cdot}27\dot{4}\dot{2}$., (2) 7·3475. (3) $\frac{17}{800}$. (4) $\cdot0\dot{1}141552\dot{5}$. (5) $\cdot6109\dot{1}2698\dot{4}$ (6) 10 : ·276 is nearer by $\cdot0000\dot{1}$. (7) £3 18s. 1d. (8) £6 18s. $1\frac{1}{16}$d. (9) £255 6s. 0·96d.; 9 lbs. 12 oz. 4·80768 drs.; 9s. 4d. (10) $2\frac{23}{48}$; $7\frac{33}{80}$. (11) 14·49 : £1738 16s. (12) ·3215. (13) $\cdot\dot{8}4615\dot{3}$; $\cdot\dot{2}6027397$; $\cdot\dot{3}7\dot{8}$; $\cdot\dot{3}720930\dot{2}$. (14) £3465. (15) $\frac{184}{945}$. (16) $9\frac{4}{5}$d. (17) $\frac{1785}{3672}$. (18) $\frac{3}{5}$. (19) ·034102562. (20) $\frac{677}{1731}$. (21) 1. (22) 17s. 8d. (23) $\cdot\dot{0}7692\dot{3}$. (24) $\frac{36}{275}$. (25) 13400 : ·00134. (26) ·09375. (27) 3 qrs. 10 lbs. 14·3616 oz. (28) 1433·43717. (29) $2\frac{23}{30}$. (30) 9s. $1\frac{1}{2}$d. (31) 39. (32) £4 2s. $1\frac{33}{35}$d. (33) $34\dot{4}{\cdot}32994103$. (34) ·0190752. (35) ·015625. (36) $\cdot1041\dot{6}$. (37) $\frac{403}{3330}$. (38) $\cdot38541\dot{6}$£. (39) 8·14968d. (40) $\frac{1}{4}$. (41) 14·79816. (42) $\frac{1333}{5525}$. (43) £192 8s. 1d. (44) $\cdot008\dot{3}$. (45) 1s. 6d. (46) 1375·00000125. (47) 2s. $7\frac{1}{2}$d. (48) $17888\frac{1}{16}$. (49) £2 17s. $5\frac{339}{361}$d. (50) ·005634765625.

XXVII.

(1) £2 12s. $10\frac{1}{2}$d. (2) £2 4s. (3) £33 8s. 3d. (4) £93 2s. $0\frac{3}{4}$d. (5) £36 14s. 3d. (6) £5378 19s. $10\frac{1}{2}$d. (7) £1544 3s. $10\frac{1}{4}$d. (8) £738 13s. (9) £17289 0s. $6\frac{3}{8}$d. (10) £9643 6s. $0\frac{3}{4}$d. (11) £1912 14s. (12) £179 17s. $10\frac{3}{4}$d. (13) £258 17s. 1d. (14) £367 3s. $10\frac{1}{2}$d. (15) £1060 6s. $7\frac{1}{4}$d. (16) £625 19s. $0\frac{3}{8}$d. (17) £184 14s. 8d. (18) £67 8s. 6d. (19) £742 5s. 5d. (20) £34596 9s. $8\frac{1}{4}$d. (21) £91949 1s. $10\frac{11}{16}$d. (22) £78 6s. (23) £34 9s. $9\frac{1}{2}$d. (24) £1030 3s. 9d. (25) £31907 9s. 11d. (26) £10757 19s. 9d. (27) £669 2s. $9\frac{3}{4}$d. (28) £49 14s. $1\frac{1}{2}$d. (29) £44 1s. $11\frac{31}{80}$d. (30) £100 15s. $7\frac{1}{2}$d. (31) £60 12s. $3\frac{3}{16}$d. (32) £311 8s. (33) £87 0s. $7\frac{1}{2}$d. (34) £83 6s. $5\frac{1}{4}$d. (35) £10 2s. $0\frac{1}{4}$d. (36) £909 9s. 3d. (37) £25557 18s. $1\frac{1}{2}$d. (38) £641 14s. $4\frac{1}{2}$d. (39) £3282 2s. $8\frac{1}{4}$d. (40) £99 18s. (41) £1381 15s. $0\frac{3}{4}$d. (42) £664 0s. $7\frac{1}{2}$d. (43) £1073 11s. $11\frac{3}{4}$d. (44) £50 2s. 3d. (45) 18 cwts. $4\frac{11}{50}$. lbs. (46) 128 cwts. 1 qr. $10\frac{2}{25}$ lbs. (47) £12 15s. (48) 140 yds. (49) £4502 1s. $9\frac{1}{4}$. (50) £2 5s. (51) 18s.

XXVIII.

(1) £223 11s. 3d. (2) £1912 14s. (3) £10299 1s. 8d. (4) £138 14s. 4d. (5) £1204 17s. 6d. (6) £2976 16s. 8d. (7) £1828 18s. 8d. (8) £1727 4s. $6\frac{3}{4}$d. (9) £245 5s. 5d. (10) £37 16s. 3d. (11) £216 19s. $5\frac{7}{16}$d. (12) £417 3s. 9d. (13) £55 19s. $2\frac{3}{4}$d. (14) £1581 19s. $0\frac{1}{4}$d. (15) £63 6s. $5\frac{3}{4}$. (16) £158 16s. 6d. (17) £190 9s. $1\frac{1}{2}$d. (18) £966 19s. $7\frac{1}{2}$d.

(19) £411 17s. 10d.
(20) £20618 11s. 2d.
(21) £91942 17s. $6\frac{9}{16}$d.
(22) £9571 15s. $0\frac{9}{56}$d.
(23) £377 0s. $6\frac{3}{28}$d.
(24) £1375 13s. $4\frac{3}{4}$d.
(25) £82 0s. $7\frac{1}{2}$d.
(26) £327 17s. $6\frac{15}{16}$d.
(27) £9 6s. 8d.
(28) £7 4s. $4\frac{13}{16}$d.
(29) £360 12s. 6d.
(30) £419 3s. $4\frac{1}{2}$d.*
(31) £2238 16s. $9\frac{1}{4}$d.
(32) £25 5s. $9\frac{5}{8}$d.
(33) £10 18s. $2\frac{3}{4}$d.
(34) £51 1s. $8\frac{1}{2}$d.
(35) £241 10s.
(36) £2210 12s. $3\frac{1}{2}$d.
(37) £431 13s. $7\frac{1}{2}$d.
(38) £269 17s. 8·9d.
(39) £910 14s. $3\frac{1}{2}$d.
(40) £93 14s. $4\frac{3}{4}$d.
(41) £370703 2s. 6d.
(42) £2025 9s. $11\frac{1}{2}$d.
(43) £1073 11s. $11\frac{3}{4}$d.
(44) £11586 2s. $7\frac{1}{2}$d.
(45) £24 15s. $1\frac{27}{32}$d.
(46) £2210 12s. $3\frac{1}{2}$d.
(47) £227 14s.
(48) £897 0s. $11\frac{1}{2}$d.
(49) £4724 6s. $1\frac{1}{2}$d.
(50) £343 12s. $4\frac{3}{4}$d.
(51) £570 8s. $5\frac{3}{4}$d.
(52) £36 10s. $2\frac{1}{4}$d.
(53) £154 10s. 9d.
(54) £21 6s. $0\frac{1}{2}$d.
(55) £189 19s. $6\frac{17}{48}$d.
(56) £3933 2s. 6d.
(57) £6804 10s. $9\frac{1}{2}$d.
(58) £77 10s. $1\frac{1}{14}$d.

XXVIII (*a*).

(1) 11 yds. 1 ft. $7\frac{1}{2}$ in.
(2) 3 sq. yds. 5 ft. 90 in.
(3) 2 c. ft. 1457 in.
(4) 3 sq. yds. 4 ft. 96 in.
(5) 24 ft. 10 in.
(6) 282 c. ft. 891·023624 in.
(7) $10124\frac{31}{36}$ sq. in.
(8) 9 ft. 2′ 7″ 2‴ 5⁗
(9) 277 sq. yds. 7 ft. 11 in.
(10) 1 ft. $9\frac{1}{3}$ in.
(11) 15 in.
(12) $13\frac{5}{48}$ sq. ft.
(13) 2374826666666 yds. 18 c. ft. : 736 feet.
(14) $7\frac{1}{2}$ feet.

XXIX.

(1) 450 ac.
(2) 1 wk. 1 day 9 hrs. 36 mins.
(3) £133 6s. 8d.
(4) 400 men.

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* A month is here taken as 30 days.

5. 45 days.
6. 13$\frac{1}{2}$ weeks.
7. 10 hrs.
8. £15 18*s.* 11$\frac{52831}{131867}$*d.*
9. 165 days.
10. £7 7*s.* 6$\frac{66}{73}$*d.*
11. £11 15*s.* 9*d*; 2*s.* 6$\frac{3}{4}$*d.*
12. 11$\frac{1}{4}$ hrs.
14. £30400.
13. £496 14*s.* 3$\frac{3}{4}$*d.*
15. 3 ac. 1 rd. 7 pls. 88 yds.
16. £5583 6*s.* 8*d.*
17. £2380 10*s.*
18. £30 14*s.* 3*d.*
19. £100 9*s.* 4$\frac{164}{449}$*d.*
20. £423 18*s.* 6*d.*
21. £137 2*s.* 10$\frac{7}{1880}$*d.*
22. £12 11*s.* 10$\frac{66}{425}$*d.*
23. 21 cwts. 3 qrs. 21 lbs.
24. £57 6*s.* 8$\frac{1}{4}$*d.*
25. £12 2*s.* 8*d.*
26. 35 cows.
27. 77 cwts. 1 qr. 22$\frac{10}{11}$ lbs.
28. £66 10*s.*
29. 460 ac. 3 rds. 24 pls.
30. 82$\frac{1}{2}$ yds.
31. £189 14*s.* 11*d.*
32. £83 11*s.* 3*d.*
33. 5 cwts. 2 qrs. 14 lbs.
34. £31 6*s.* 9$\frac{3}{7}$*d.*
35. £3 10*s.*
36. £75.
37. £90.
38. 10·4318*d.*
39. 1*s.* 8*d.*
40. £1 8*s.* 9*d.* : £1 3*s.* : 5*s.* 9*d.*
41. £19 9*s.* 2*d.* : £25 18*s.* 10$\frac{2}{3}$*d.* : £32 8*s.* 7$\frac{1}{3}$*d.*
42. £144 and £128.
44. 10$\frac{13}{14}$ hours.
43. £11 0*s.* 10*d.* : £8 16*s.* 8*d.*
45. 36 men.

XXX.

1. £2 5*s.* 8$\frac{1}{3}$*d.*
2. 126 acres 1 rd. 5$\frac{5}{8}$ pls.
3. £5.
4. £239 16*s.* 4$\frac{4}{5}$*d.*
5. 16 days.
6. 2340 lbs.
7. £100.
8. 10 hrs.
9. 6440 men.
10. 13$\frac{1}{2}$ days.
11. 30 days.
12. 36$\frac{2}{3}$ lbs.

XXXI.

1. £60 12*s.* 3$\frac{3}{16}$*d.*
2. £35 0*s.* 9$\frac{204}{349}$*d.*
3. £8 18*s.* 6$\frac{3}{4}$*d.*
4. £25 17*s.* 1$\frac{11625}{11681}$*d.*
5. 22 days.
6. £12 16*s.* 3$\frac{61859}{63748}$*d.*
7. 3 oz.
8. 52$\frac{1}{2}$ days.
9. 160 men.
10. £7198.
11. £299 5*s.*
12. 66 men.
13. 18 days.
14. 3 lbs. 4$\frac{1}{2}$ oz.
15. 6*s.*
16. £767.
17. 46$\frac{29}{40}$ sovereigns.
18. 5*s.*
19. 6000 pages.
20. £4000.

21. £3.
22. £18 18*s.*
23. £14 3*s.* 4*d.*
24. £8 15*s.*
25. 2894 francs 3 centimes.
26. 11 cwts. 1 qr.
27. £1736 4*s.* $7\frac{1}{8}$*d.*
28. £502 0*s.* $11\frac{1}{8}$*d.*
29. $7\frac{1}{2}$ days.
30. 64 yards.
31. £2 10*s.* $10\frac{1}{2}$*d.*
32. £2881 16*s.* 8*d.*
33. £3848 12*s.* $11\frac{3}{8}$*d.*
34. £2143 8*s.* $10\frac{1}{4}$*d.*
35. £25 5*s.* $9\frac{3}{8}$*d.*
36. 6440 men.
37. $73\frac{1}{44}$.
38. 12·527.
39. 18·7 lbs.
40. $5\frac{1}{4}$*d.*
41. £242 9*s.* $4\frac{3}{4}$*d.*
42. £3.
43. 16 days.
44. 2 cwts. 3 qrs. 14 lbs.
45. 4 oz. $13\frac{1}{2}$ drs.
46. £75 5*s.* $5\frac{47}{243}$*d.*
47. 2*s.* $2\frac{4687}{9056}$*d.*
48. £7.
49. $58\frac{5}{22}$.
50. £27 8*s.* $6\frac{3}{4}$*d.*
51. £2417.
52. 9044 lbs. 6 oz. 7 dwts. $4\frac{4}{13}$ grs.
53. £42 9*s.* $4\frac{23}{28}$*d.*
54. £174 12*s.*
55. $6\frac{1}{4}$*d.*
56. £27 0*s.* $2\frac{2367}{4312}$*d.*
57. £28 11*s.* $6\frac{3}{4}$*d.*
58. £1849 11*s.* 6*d.*
59. $12\frac{5}{8}$ hours.
60. 132 men.

XXXII.

1. $981\frac{3}{5}$ thalers.
2. £11 5s.
3. $14\frac{1}{6}$ marks for £1.
4. 135 lines.
5. £14 4s. $6\frac{20}{41}$d.
6. 25·2 francs.
7. 30 gals.
8. 7 dozen.
9. $5\frac{1}{3}$ lbs.

XXXIII.

1. £600.
2. A, £60 : B, £72 : C, £78.
3. A, £13 4*s.*, B, £19 16*s.*, C, £26 8*s.*, D, £21 9*s.*

XXXIV.

1. £65 19*s.* 6*d.*
2. £289 5*s.* $7\frac{4}{5}$*d.*
3. £75 6*s.* 8*d.*
4. £36 18*s.* 1*d.*
5. £687 2*s.* $10\frac{1}{5}$*d.*
6. £304 13*s.* 9*d.*
7. £307 7*s.* $3\frac{3}{80}$*d.*
8. £7 14*s.* $1\frac{47}{73}$*d.*
9. £38 14*s.* $11\frac{17}{20}$*d.*
10. £438 8*s.* 0·165*d.*

11. £17 11*s.* $4\frac{41}{125}$*d.*
12. 3 per cent.
13. $4\frac{1}{4}$ per cent.
14. 4 per cent.
15. $4\frac{3}{4}$ per cent.
16. 8 years.
17. 4 yrs. 6 months.
18. £42 5*s.* 10*d.*
19. £1411 9*s.* 2*d.*
20. £277 7*s.* 6·3*d.*
21. £173 2*s.* 9*d.*
22. £1604 5*s.*
23. £34 17*s.* $5\frac{1}{2}$*d.*
24. $2\frac{1}{4}$ per cent.
25. £214 7*s.* $1\frac{5}{11}$*d.*
26. $2\frac{14848}{15850}$.
27. £95 12*s.* 1*d.*
28. 3 years.
29. £195 1*s.* $10\frac{1}{2}$*d.*
30. $3\frac{3}{8}$ per cent.

XXXV.

1. £13 10*s.* : £730.
2. 13*s.* $5\frac{112}{331}$*d.*
3. 16*s.* 8*d.*
4. £500.*
5. £2350.
6. £43 15*s.* 11*d.*
7. £21 19*s.* $1\frac{53}{89}$*d.*
8. £173 17*s.* $5\frac{48}{113}$*d.*
9. £260 16*s.* $2\frac{21}{326}$*d.*
10. £424 6*s.* $10\frac{3}{13}$*d.*
11. £315.
12. £32. (13) 183 days ; 7483 : 7300

XXXVI.

1. £390.
2. £29 14*s.* $8\frac{23}{50}$*d.*
3. £57 11*s.* $7\frac{1}{8}$*d.*
4. £5 12*s.* 6*d.*
5. £4 17*s.* $8\frac{1}{4}$*d.*
6. £20 10*s.* 6*d.*
7. £57 18*s.* $3\frac{3}{8}$*d.*
8. £4736 16*s.* $10\frac{2}{13}$*d.*
9. £1900.
10. £52 7*s.* 9*d.*
11. £76 3*s.* 2·65*d.*

XXXVII.

1. £581 3*s.* $9\frac{5}{7}$*d.*
2. $7\frac{7}{18}$ months.
3. $8\frac{3}{7}$ months.
4. $4\frac{1}{3}$ months.
5. $12\frac{1}{2}$ months.

XXXVIII.

1. £8950.
2. £7200.
3. £6400.
4. 99 shares.
5. 160 shares.
6. £1611 16*s.* 3*d.*
7. £1111 10*s.*
8. £1168 6*s.* 3*d.*
9. £16873 17*s.* 6*d.*
10. £5223 2*s.* 6*d.*

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* The explanation in such answers, for obvious reasons, is omitted.

(11) £106 10s. (12) £97 10s. (13) £160 6s. 8d. (14) £197 10s. (15) £98 2s. 6d. (16) 3$\frac{4}{7}$; £4 8s. 10$\frac{2}{3}$d. (17) £3 19s. 6$\frac{6}{11}$d. : £4 3s. 4d. (18) £4 14s. 8$\frac{16}{169}$d. (19) £4. (20) £600 16s. 8$\frac{200}{719}$d. (21) Increase of £8 8s. 9$\frac{3}{5}$d. (22) £587 15s. 1$\frac{11}{49}$d. (23) £75. (24) 60 per cent. (25) Decreased £34 7s. 6d. (26) Increase of £5 18s. 4d. (27) Increased £16 6s. 2$\frac{82}{187}$d. (28) £24960. (29) £26 13s. 4d. (30) Increase of £115 4s. (31) £124 4s. 3$\frac{13}{17}$d. (32) 4$\frac{652}{737}$. (33) 2$\frac{17}{19}$. (34) £15 2s. 10d. nearly. (35) £170 10s. 5d. (36) £40000 : £450000. (37) £413135 11s. 10$\frac{22}{59}$d. (38) £5948$\frac{172}{311}$. (39) Decrease £446 3s. 0$\frac{12}{13}$d. (40) £100. (41) Capital £1000000 : Receipts £100000. (42) 821 : 1195 ; 189 : 190.

XXXIX.

(1) £4 5s. 10$\frac{2}{25}$d. per cwt. (2) 6s. 6$\frac{6}{23}$d. (3) 15$\frac{1}{2}$ per cent. (4) £156 12s (5) £75 : loss £24 (6) 19s. 8d. (7) £65 9s. 11d. (8) £476 6s. 8d. (9) 272 cwts. 2 qrs. 14 lbs. (10) 4s. 8d. and 5s. 3d. (11) 10s. 6d. and £8 5s. 4$\frac{32}{127}$d. (12) £18 15s. per cent. gain. (13) 1s. 1$\frac{4}{5}$d. (14) £14 11s. 8d. (12)*a* 10s. 7$\frac{223}{361}$d.

XL.

(1) 19s. 2d. (2) 44. (3) 1541. (4) £58 1s. 4d. (5) 25313·48. (6) 63$\frac{7}{11}$. (7) 40·6. (8) 12 per cent. (9) 18s. 9d. (10) £250 5s. 9$\frac{3}{4}$d. (11) 10$\frac{1}{12}$d. (12) 120000 (13) 3·129896477... (14) £2107 10s. (15) 1420·479... : 4326·5186... :5705·042.... (16) 12·403 : 37·779... : 49·816... (17) 71$\frac{3}{7}$. (18) 1000.

XLI.

(1) 10648 : 900. (2) ·00000256 : 12·25. (3) 429981696 : 1·2762815625. (4) 85184 : $\frac{8}{27}$. (5) 46656 : 8000. (6) 134689 : 130321. (7) $2\cdot\dot{3}7\dot{0}$: $\frac{81}{14641}$: $6\cdot162\dot{0}3\dot{7}$. (8) $\frac{121}{169}$: $\frac{125}{343}$: $\frac{2197}{4913}$. (9) $2\cdot\dot{0}86419753\dot{}$: 9$\frac{128}{169}$. (10) 1120[illegible].

XLII.

1. 53·43.
2. 637·4.
3. $16\frac{1}{5}$.
4. $2\frac{4}{15}$: ·0169.
5. $5\frac{4}{5}$: ·12 : $\cdot\dot{3}$.
6. $22\frac{11}{18}$: 49·07.
7. 7047 : 19·45.
8. 2·6463 : $6{\cdot}48\dot{3}$.
9. 1187·4023 : $\cdot\dot{7}$.

XLIII.

1. 7093.
2. 97·09.
3. 8767.
4. $17\frac{2}{7}$.
5. 2·924.
6. 512.
7. 23.
8. 611.
9. 79.
10. 1061.
11. 6·16 : 1·848.
12. $66\frac{2}{3}$ square ft. £4 16*s.* $3\frac{1}{3}$*d.*
13. $4\frac{1}{2}$.
14. $1\frac{33}{140}$.

XLIV.

1. 165.
2. 6.
3. 54.
4. $\frac{3}{4}$.
5. $\frac{4}{5}$.
6. 172·5.
7. 99.
8. $\dot{2}85714$.
9. $193\frac{5}{16}$.
10. $46\frac{7}{8}$.
11. 5460.
12. 1024.
13. $40{\cdot}\dot{4}8\dot{1}$.
14. 1.

XLV.

1. £270 2*s.* 5·307456*d.*
2. £668 10*s.* 6·825*d.*
3. £334 5*s.* 3·4125*d.*
4. £9091 16*s.* 1·782075*d.*
5. £5264 4*s.* 2·15625*d.*
6. £10653 17*s.* 10·07889*d.*
7. £1229 9*s.* 4·6875*d.*
8. £138 15*s.* 8·0832*d.*
9. £562 16*s.* 6·774912*d.*
10. £525 12*s.* 7·884375*d.*
11. £333 6*s.* 8*d.*
12. £1001 5*s.*
13. £204 0*s.* 9·648096*d.*
14. £21 5*s.* 2·26992*d.*

XLVI.

(1) 28 dys. 4 hrs. 23 mins. 29 sec. (2) £11 15s. (3) £111 7s. (4) £5001 3s. 4d. (5) $28\frac{2}{9}$. (6) $31\frac{991}{1000}$. (7) $64\frac{16}{33}$. (8) $8\frac{8}{55}$. (9) 105·306196. (10) 98429·98. (11) 6·1082649. (12) ·0076 nearly. (13) 270 secs. (14) 49 mls, 5 fur. 31 pls. 2 yds, 2 ft. 5 in. (15) $3\frac{1}{6}$ hrs. (16) £14069 10s. $7\frac{1}{2}$d. (17) £5705 16s. 7d. (18) $27\frac{11}{30}$. (19) $10\frac{103}{108}$. (20) 4. (21) $\frac{25}{117}$. (22) 9350050219·861379. (23) 1·005. (24) ·269255. (25) 6·25875. (26) ·084. (27) 941860 cubic in. (28) £3682 10s. (29) £2424 16s. $10\frac{1}{2}$d. (30) 1 $2\frac{1}{3}$ years. (31) $1132\frac{13}{24}$. (32) $69\frac{25}{36}$. (33) 48. (34) $4\frac{1}{3}$. (35) 504780·97398097. (36) 8·997997. (37) ·4727103. (38) ·00029. (39) 1·25.

XLVII.

(1) 695 lbs. 4 oz. 3 dwts. 9 grs. (2) 1s. $10\frac{1}{2}$d. (3) £8 14s. 2d. (4) £250 2s. 6d. (5) $\frac{52}{63}$. (6) $1\frac{11}{27}$. (7) $\frac{5}{17}$. (8) $\frac{7}{150}$. (9) 407·755812. (10) 33·934899. (11) 160504·9226. (12) ·1144. (13) £16 0s. 6d. (14) 71 mls. 4 fur. 14 pls. 5 yds. 1 ft. 2 in. (15) 144 ft. (16) £26 17s. $8\frac{1}{4}$d. (17) £11 18s. 6d. (18) $16\frac{283}{330}$. (19) $\frac{131}{180}$. (20) $\frac{365}{6255}$. (21) 6. (22) 8004·709485. (23) 11·57294. (24) 1460·15448. (25) 102·204. (26) ·002$\dot{7}$. (27) $77591\frac{1}{4}$ sq. ft. (28) $176\frac{1}{4}$ acres. (29) £5 1s. (30) 8 yrs. (31) $19\frac{1}{12}$. (32) $7\frac{31}{39}$. (33) 1. (34) $20\frac{1}{40}$. (35) 20135·2652434. (36) ·005899. (37) 21·812076. (38) 13762844·$\dot{4}$. (39) 1·43$\dot{1}\dot{8}$.

XLVIII.

(1) 27 tons 4 cwts. 2 qrs. 16 lbs. (2) 1012 lbs. (3) £1467 17s. $2\frac{1}{4}$d. (4) £22 7s. 2d. (5) $1\frac{79}{120}$. (6) $7\frac{3}{8}$. (7) $9\frac{1}{3}$. (8) $4\frac{1}{4}$. (9) 70·621588. (10) 48·112. (11) 256·5148683. (12) 6676. (13) 35164·8 grains. (14) 107183736 sq. inches. (15) 3 lbs. (16) £3 14s 9d. (17) £1670 8s. $5\frac{1}{3}$d. (18) $3\frac{5}{8}$. (19) $1\frac{74}{117}$. (20) $\frac{1}{2}$) (21) $\frac{6}{13}$. (22) 945·70522. (23) 29·2996. (24) 34·342295. (25) ·0991 nearly. (26) £3 1s. 8d. (27) 3779 c. yds. 7 ft. 561 in. (28) 16 days. (29) £4 17s. $10\frac{1}{2}$d. (30) 7 years. (31) $1\frac{11}{20}$. (32) $3\frac{5}{24}$. (33) $\frac{5}{36}$. (34) 9. (35) 500·815913. (36) 2·206. (37) 6·586216. (38) ·0862633, &c. (39) ·001041$\dot{6}$.

XLIX.

(1) 117488 oz. (2) £21 17s. 6d. (3) £115 4s. $3\frac{1}{2}$d. (4) £221 8s. $4\frac{1}{2}$d. (5) $3\frac{257}{1122}$. (6) $26\frac{25}{28}$. (7) 35. (8) $\frac{243}{296}$. (9) 1138·3579114. (10) 161·300048. (11) 222·2927084. (12) 6·53. (13) ·0088541$\dot{6}$. (14) 969 m. 7 pls. 2 ft. 8 inches. (15) £86 1s. $10\frac{1}{2}$d. (16) £13 19s. $1\frac{1}{2}$d. (17) £3106 17s. 5d. (18) $17\frac{29}{315}$. (19) $2\frac{103}{171}$. (20) $\frac{1}{8}$. (21) $4\frac{134}{187}$. (22) 828·422458. (23) 2345·87648. (24) 1·87640928. (25) 14834·6. (26) 15·239. (27) 13008 grs. (28) $18\frac{3}{4}$ days. (29) £120 17s. $3\frac{1}{2}$d. (30) $3\frac{3}{4}$. (31) $15\frac{101}{132}$. (32) $1\frac{1437}{1480}$. (33) $8\frac{3}{8}$. (34) $1\frac{183}{384}$. (35) 536·292872. (36) 131·67779. (37) ·063489950244. (38) ·00728. (39) ·0375.

L.

(1) 431700 ozs. (2) £2 5s. 10d. (3) £579 11s. (4) £10 5s. $1\frac{1}{4}$d. (5) $16\frac{117}{350}$. (6) $4\frac{41}{44}$. (7) $2\frac{1}{4}$. (8) $\frac{1}{35}$. (9) 1018·8086. (10) 123·3034072. (11) 4·304463. (12) ·1435. (13) 133·584 ft. (14) 1 lb. 12 dwts. 7 grs. (15) 1s. 10d. (16) £86 4s. 6d. (17) £1811 18s. 6d. (18) $8\frac{117}{160}$. (19) $\frac{447}{455}$. (20) $2\frac{4}{5}$. (21) $9\frac{3}{10}$. (22) 4366·437556. (23) 14·5652781. (24) 4873·7880. (25) 740·9. (26) ·01875. (27) 43 ac. 2 rds. 25 p. 12 y. 3 ft. 109 in. (28) $28\frac{1}{2}$ m. (29) 3s. $7\frac{3}{4}$d. (30) $9\frac{1}{2}$. (31) $18\frac{7}{15}$. (32) $10\frac{373}{385}$. (33) $24\frac{34}{55}$. (34) $1\frac{1}{5}$. (35) 2115·546234. (36) ·9987. (37) ·00136596977. (38) 99·6317. (39) 423·84.

LI.

(1) 7 tons 3 cwts. 3 qrs. 5 lbs. 11 ozs. (2) £17 6s. $1\frac{1}{2}$d. (3) £8090 10s. 8d. (4) £259 4s. (5) $7\frac{88}{165}$. (6) $5\frac{7}{24}$. (7) 18. (8) $\frac{193}{385}$. (9) 800·056901. (10) 21·06. (11) 3005·10652. (12) ·0057. (13) 24410·88 grs. (14) 10065 ft. (15) 105. (16) £502 6s. $10\frac{1}{2}$d. (17) £7315 16s. 2d. (18) $10\frac{13}{44}$. (19) $\frac{31}{35}$. (20) $\frac{33}{64}$. (21) $\frac{8}{27}$. (22) 217·6224563. (23) 1·9075. (24) 64·0736192. (25) ·2095. (26) 4·25. (27) 18792 grs. (28) 4 days. (29) £558. (30) 10. (31) $12\frac{101}{180}$. (32) $\frac{13}{20}$. (33) $7\frac{7}{44}$. (34) $3\frac{58}{165}$. (35) 813·987803. (36) 109·353. (37) 443·30018719. (38) 16·4918. (39) ·0228125.

LII.

(1) 180829 oz. (2) £9 4s. $10\frac{1}{2}$d. (3) £30613 5s: 6d. (4) £109 15s. $6\frac{3}{5}$d. (5) $5\frac{173}{180}$. (6) $2\frac{23}{40}$. (7) $142\frac{7}{8}$. (8) $\frac{725}{776}$. (9) 439·8067. (10) 7·37966. (11) 680·33892. (12) 6·5451, &c. (13) 6s. $8\frac{1}{4}$d. (14) 512 m. 4 fur. 33 pls. 2 yds. 1 ft. 3 in. (15) 10s. (16) £36 10s. $2\frac{1}{4}$d. (17) £491 5s. $10\frac{1}{2}$d. (18) $6\frac{34}{63}$. (19) $5\frac{13}{81}$. (20) 2. (21) $13\frac{29}{37}$. (22) 4255·13424. (23) 61·90348. (24) ·0488976. (25) ·00149. (26) 1·91163194. (27) 62,857,296. (28) 1320. (29) £34 10s. $7\frac{1}{2}$d. (30) $3\frac{1}{2}$ yrs. (31) $15\frac{6}{112}$ (32) $1\frac{13}{60}$. (33) $\frac{23}{410}$. (34) $\frac{33}{37}$. (35) 375·1096. (36) 84·2307. (37) 43·0782. (38) ·04375. (39) ·5972.

LIII.

(1) 15 days 15 hrs. 2 m. 53 secs. (2) £13 19s. 6d. (3) £902 9s. $10\frac{1}{10}$d. (4) £702 4s. 6d. (5) $8\frac{61}{90}$. (6) $3\frac{5}{6}$. (7) $1\frac{3}{5}$. (8) $\frac{28}{297}$. (9) 420·8249. (10) 25·977. (11) 70·52626. (12) 1·401 nearly. (13) 3 cwts. 2 qrs. 7 lbs. (14) 80 mls. 1 fur. 8 pls. 1 yd. 1 ft. (15) $10\frac{1}{11}$ miles. (16) £12 17s. $1\frac{1}{2}$d. (17) £206 5s. 7·96659732d. (18) $19\frac{33}{50}$. (19) $13\frac{41}{60}$. (20) $\frac{5}{13}$. (21) $\frac{1}{6}$. (22) 27308·092. (23) 8296·5. (24) ·46802632. (25) 4·452. (26) ·07875. (27) 264627 sq. ft. (28) $3\frac{1}{2}$ years. (29) £71 15s. (30) £408. (31) $10\frac{17}{80}$. (32) $53\frac{9}{16}$. (33) 24. (34) $73\frac{62}{63}$. (35) 1061·235529. (36) 84299.89191. (37) ·142398. (38) 17·648. (39) 5·952380.

LIV.

(1) 125 lbs. 3 oz. 5 dwts. 12 grs. (2) 48 lbs. (3) £953 14s. 8d. (4) £24 10s. $2\frac{5}{8}$d. (5) $15\frac{3}{7}$. (6) $4\frac{17}{18}$. (7) 39. (8) 10. (9) 12988·76693. (10) 446·352. (11) 11961·232. (12) ·0406. (13) £2 12s. 6d. (14) 92 mls. 4 fur. 18 pls. 4 yds. 2 ft. 10 in. (15) $9\frac{1}{2}$ miles. (16) £20 13s. (17) £3098 6s. 2d. (18) $15\frac{1}{12}$. (19) $3\frac{3}{4}$. (20) 25. (21) $\frac{7}{44}$. (22) 51206·082. (23) 55·0124. (24) 318·906015. (25) 245·53846... (26) 8·625. (27) $221339\frac{1}{4}$ square feet. (28) $8\frac{7}{8}$ guineas (£9 6s. $4\frac{1}{2}$d.) (29) £113 17s. $3\frac{17}{18}$d. (30) 16 years 8 months. (31) $9\frac{11}{12}$. (32) $3\frac{17}{44}$. (33) $142\frac{1}{2}$. (34) $2\frac{1}{3}$. (35) 2000·83672. (36) 865·49885. (37) 6861·90045. (38) ·00071... (39) ·091954...

LIV.*

(1) 40 lbs. 8 oz. 13 dwts. 11 grs. (2) £2 17*s.* 6*d.* (3) £122 3*s.* 0*d.* (4) £257 5*s.* 3*d.* (5) $8\frac{11}{19}$. (6) $\frac{43}{72}$. (7) $1\frac{10}{11}$. (8) $2\frac{2}{11}$. (9) 5511·14976. (10) 31·463839. (11) 9·844635. (12) 60·917. (13) ·16953125. (14) 1686 pints. (15) 3*s.* 4*d.* (16) £30 13*s.* 6*d.* (17) £4216 12*s.* 2*d.* (18) $7\frac{7}{16}$. (19) $\frac{139}{330}$. (20) $5\frac{5}{8}$. (21) $1\frac{5}{14}$. (22) 212·32874. (23) 1·4419. (24) 340·8646095. (25) 111. (26) 81·84 yds. (27) 1779426 sq. ft. (28) 10 men. (29) £21579 15*s.* (30) 2 years. (31) $8\frac{25}{36}$. (32) $\frac{17}{144}$. (33) $17\frac{17}{20}$. (34) $\frac{8}{309}$. (35) 3535·62 minutes. (36) 1256·64 yards. (37) ·000009588. (38) ·072. (39) 699·84 cubic inches.

LV.

(1) $11\frac{67}{415}$. (2) $1\frac{11}{35}$. (3) $13\frac{1}{2}$. (4) $8\frac{8}{45}$. (5) 899·24467. (6) 701·1381. (7) 3·585207. (8) 48·295. (9) ·009375. (10) $7\frac{77}{210}$. (11) $\frac{113}{360}$. (12) 112. (13) $\frac{84}{109}$. (14) 48·416. (15) 2 ozs. 6 dwts. 4·32 grs. (16) 26638·694. (17) ·00329375. (18) 66·30172. (19) 185680. (20) 1 gall. (21) £72071 3s. $3\frac{3}{4}$d. (22) 8 yrs. (23) 14 mls. 7 f. 19 pls. 2 yds. 2 ft. 7 in. (24) 90. (25) £38 16s. 9d. (26) £4371 0s. 7d. (27) 90·909; ·074. (28) 6336000; $176785\frac{5}{7}$ tons. (29) 120 days: one 11 hrs. 46′; the other 12 hrs. 14′. (30) $1\frac{1}{19}$ hrs.

LVI.

(1) $11\frac{13}{20}$. (2) $7\frac{381}{500}$. (3) 92. (4) $\frac{48}{155}$. (5) 539·58519. (6) 102·910604. (7) 1·6086255. (8) ·455. (9) 110 d. (10) 42256 oz. (11) $43\frac{1}{2}$ ac. (12) £185 14*s.* 9*d.* (13) $14\frac{79}{144}$. (14) $25\frac{251}{260}$. (15) $84\frac{3}{4}$. (16) $1\frac{1}{22}$. (17) $9\frac{3}{4}$ d. (18) ·27 pints. (19) ·03374679. (20) 1240. (21) $\cdot 0\dot{2}3809\dot{5}$. (22) 40 lbs. 8 oz. 7 grs. (23) £3 15*s.* (24) £945. (25) $3\frac{3}{4}$. (26) £4 9*s.* $0\frac{3}{8}$*d.* (27) £17 14*s.*

LVII.

(1) $11\frac{29}{360}$. (2) $\frac{1}{8}$. (3) $2\frac{23}{28}$. (4) $\cdot 0\dot{0}3174\dot{6}$. (5) 712113·92. (6) 15·125. (7) £4 2*s.* 2*d.* (8) $41\frac{4}{5}$. (9) $\frac{19}{1115360}$. (10) 7 pls. 1 ft. 3·17 in. (11) 16*s.* $6\frac{3}{4}$*d.* (12) £81. (13) 17 cwts. 2 qrs. 5 lbs.; £32 16*s.* 4*d.* (14) $18\frac{3}{4}$ gain per cent. (15) 230·45. (16) A £90, B £75, C £105,

LVIII.

1. £121 10*s*. 2. £17 14*s*. 10·89792*d*.
3. £606 11*s*. $1\frac{4}{5}$*d*. 4. 3·6142857.
5. £4836 13*s*. 4*d*. 6. 95·136 lbs.
7. 3 cwts. 2 qrs. 23 lbs. 8. £85.

LIX.

1. £5 11*s*. $4\frac{1}{2}$*d*. 2. £3 4*s*. 11*d*.
3. £11 14*s*. $1\frac{1}{13}$*d*.; £7 9*s*. $2\frac{7}{13}$*d*. 4. £28 19*s*. $1\frac{5}{13}$*d*.
5. £6 15*s*. $10\frac{4}{11}$*d*.; £8 18*s*. $11\frac{133}{316}$*d*. 6. 1200 men.
7. 9 men. 8. 21 hrs. $15\frac{303}{308}$ m.
9. 90 men. 10. $123\frac{1}{3}$ galls.
11. $50\frac{210}{281}$ minutes past 3. 12. £13 5*s*. $5\frac{5}{33}$*d*.

LX.

1. 26 ac. 3 rds. 3 pls. 29 yds. 2 ft. 2. 45 men.
3. 10 hrs. 4. $1086157\frac{1}{7}$.
5. £4 2*s*. 6. $12\frac{47}{48}$*d*.
7. $7\frac{161}{333}$ in. 8. 7 miles.
9. $3\frac{13}{27}$. 10. 1000.
11. 7 per cent. 12. $12\frac{1}{2}$ per cent.

LXI.

(1) 1837; 560. (2) $\frac{119}{240}$. (3) $40\frac{397}{3456}$ ozs. (4) 500 books. (5) £9 6*s*. 9*d*.; ·35571428. (6) $10\frac{59}{400}$*d*. (7) 134190. (8) 80 ozs. (9) $20\frac{10}{27}$ minutes. (10) 10 yards 11 inches. (11) £540; and 17*d*. (12) £3 15*s*. 10*d*. (13) (14) (15) 72 gallons. (16) £345. (17) £5 0*s*. 9*d*. (18) 14 boys. (19) £36 1*s*. $10\frac{1}{2}$*d*. (20) 4. (21) $\frac{1}{3}$. (22) Half-an-hour. (23) 108; 14826240. (24) ·0316; ·7071; ·01; ·0009. (25) 3 days. (26) 5 miles.

LXII.

(1) 300030018. (2) 1798120070. (3) Thirty millions six hundred thousand one hundred and one. (4) 26447199. (5) £1456 0s. 5d. (6) 18789697. (7) £1142 19s. $8\frac{3}{4}$d. (8) 35839202872. (9) $240443678\frac{1}{3}$. (10) 1111373374. (11) $4053\frac{801}{837}$. (12) £1705869 5s. $5\frac{1}{2}$d. (13) 297366 + 1581 rem. (14) £11026623 7s. $7\frac{1}{4}$d. (15) £3016 8s. $3\frac{1}{4}$d. (16) £539135 3s. 9d. (17) £24563 13s. $7\frac{1}{4}$d. (18) 1 ton 9 cwts. 3 qrs. 19 lbs. 2 oz. (19) 156 cwts. (20) £1720. (21) $10\frac{3}{8}$. (22) $2\frac{167}{168}$. (23) $7\frac{7}{44}$. (24) $\frac{3}{26}$. (25) 741·3534521. (26) 1347·36841. (27) ·571456. (28) 490000. (29) 3600 grains. (30) £9 16s. 8d. (31) 3s. 4d. (32) £14 8s. 9d. (33) $574\frac{14}{15}$ yds. (34) 10 per cent. (35) 8 hours. (36) $26\frac{13}{16}$ miles from London. (37) Friday, 26 June, 1761. (38) ——— (39) 7 ft. a day. (40) £36 5s. (41) ·0074; $1\frac{3}{14}$. (42) £621 13s. 4d. (43) $94\frac{3}{8}$. (44)

LXIII.

(1) £3 3*s.* $10{\cdot}27\dot{6}\dot{3}$*d.* (2) £529 0*s.* $7\frac{1}{2}$*d.*; £489 1*s.* $0\frac{1}{2}$*d.* (3) £885 12*s.* 7·93125*d.*; £5 2*s.* 11*d.* (4) £858 18*s.* 2·7721875*d.* (5) $16\frac{181}{252}$: $129\frac{23}{180}$. (6) 469·5102 : 444·144; $1{\cdot}12738\dot{6}$. (7) $\cdot4017\dot{8}5714\dot{2}$. (8) 4*s.* 0·18996*d.*; ·19123. (9) 838·42. (10) 25 poles 2 yards $6\frac{4}{7}$ inches. (11) £14 3*s.* 4*d.*; £21 18*s.* $3\frac{3}{8}$*d.* (12) 18 lbs. 8 oz. (13) $\frac{11}{12}$ of an inch. (14) Increases it 25 per cent. (15) $2\frac{22}{31}$*d.* : $2\frac{1}{31}$*d.* : and $1\frac{18}{31}$*d.* (16) £10330 18*s.* $2\frac{3}{4}$*d.*; £590 6*s.* $10\frac{1}{4}$*d.* (17) £2700. (18) £57 12*s.* (19) £36. (20) £919 12*s.* 8·110521*d.* (21) £99 1*s.* $11\frac{4}{73}$*d.* (22) £1415 1*s.* $6\frac{23}{56}$*d.*; £1431 3*s.* $2\frac{487}{504}$*d.*; £16 1*s.* $8\frac{25}{63}$*d.* (23) £128 1*s.* 0·860025*d.* (24) 11268 men. (25) 11*s.* $7\frac{23}{43}$*d.* (26) $\cdot2565814\dot{3}\dot{9}$. (27) A 2*s.* 6*d.*, B 3*s.*, C 3*s.* 4*d.* (28) $\frac{889}{900}$; £5835. (29) £12308 7*s.* 10*d.* (30) $10\frac{1}{4}$*d.* (31) 59·25; 35·75; 5. (32) $2\frac{4}{7}$ days. (33) £5 5*s.* $10\frac{1}{2}$*d.* (34) 3 ft. 9 in. (35) 385. (36) 1 hr. $7\frac{1}{2}$ m. (37) 216 acres. (38) $265\frac{12}{17}$; 132·5. (39) 25. (40) $1{\cdot}\dot{3}\dot{6}$; ·013774, &c. (41) Gain £5 13*s.* $9\frac{35}{41}$*d.* (42) Decrease ·03283... (43) 16 ft. : 12 ft. (44) $82\frac{1}{2}$. (45) 16*s.* 3*d.* (46) 12·58... (47) 7 minutes 33·6 seconds. (48) £65 10*s.* 4·8*d.* (49) 5 per cent. (50) $2{\cdot}\dot{3}\dot{6}$. (51) 729 : $5\frac{3}{8}$. (52) $89\frac{1}{4}$. (53) £547 7*s.* $3\frac{3}{5}$*d.* (54) £297 15*s.* 3*d.* (55) £3 5*s.* $9\frac{47}{77}$*d.* : 1*s.* $9\frac{13}{231}$*d.** (56) £16206 15*s.* (57) £2394 14*s.* $9\frac{123}{821}$*d.*; £2390 13*s.* $2\frac{194}{257}$*d.* (58) $10915745{\cdot}2196382428940568475\dot{4}5219638\dot{8}$. (59) 564. (60) $\frac{395}{3111}$. (61) 9,5,1,6, or 5,9,6,1. (62) 70 yards. (63) 1 pole 1 ft. $3\frac{1}{5}$ inches. (64) £24 2*s.* $3\frac{11}{47}$*d.* (65) 982. (66) 6*s.* $9\frac{21}{53}$*d.* (67) $5\frac{3}{4}$ per cent. gain. (68) 400. (69) 9·5538... : 8·637... (70) £3 4*s.* $9\frac{81}{247}$*d.* : £5 5*s.* $9\frac{207}{397}$*d.* (71) 10 hrs. 30 mins. (72) Decrease 3·8374713. (73) 4 per cent. (74) 19 hrs. 20 mins. (75) 5 per cent. gain. (76) $1\frac{13}{27}$ lbs., $1\frac{1}{9}$ lbs., $\frac{20}{27}$ lbs. (77) 4·395... (78) $2\frac{1}{2}$. (79) £3,212,000. (80) $\frac{60}{77}$ galls. (81) 914 yds. 1 ft. 7 in. (82) $2\frac{7}{86}$. (83) $1\frac{1}{2}$ hour. (84) 10 hrs. $48\frac{72}{721}$ mins. (85) 1·25 lbs. (86) 9009. (87) 19 ft. 3′ 7″ 8‴ 2⁗ 10′′′′′. (88) 19 cubic feet $519\frac{17}{72}$ inches. (89) 1. £22 1*s.* $2\frac{2}{17}$*d.* : 2. £44 2*s.* $4\frac{4}{17}$*d.* : and 3. £7 7*s.* $0\frac{13}{17}$*d.* (90) If its denominator is 2 or 5, or made up of twos or fives, see **101.** (91) See **100.** (92) Less than the units in denominator. See **102.** (93) 1. £62 4*s.* 7*d.*; 2. £46 13*s.* $5\frac{1}{4}$*d.*; 3. £37 6*s.* 9*d.*; 4. £31 2*s.* $3\frac{1}{2}$*d.* (94) $28{\cdot}6\dot{5}7894736842105263\dot{1}$. (95) 1 year $163\frac{63}{69}$ days. (96) 2*s.* 4*d.* (97) 4*s.*, and 4*d.* (98) $23464{\cdot}50\dot{3}$. (99) 12 per cent. (100) 736 men. (101) 8·738*d.* (102) Increased $63\frac{1}{3}$ per cent. (103) £3 2*s.* $10\frac{698}{763}$*d.* : £4 16*s.* $9\frac{9}{31}$*d.* (104) 50 per cent. (105) £30 1*s.* $6\frac{3}{4}$*d.* (105 *a*) £80. per cent. (106) A, £245 : B, £175 : C, £210. (107) £113 1*s.* $10\frac{6}{7}$*d.* (108) $13\frac{11}{13}$ minutes. (109) £343 : £45 15*s.* (110) £40. (111) $1079\frac{2}{3}$ Austrian lire. (112) $1' \, 4{\cdot}28\dot{1}9047\dot{6}''$. (113) 32 men. (114) £25 19*s.* $6\frac{163}{1168}$*d.* (115) 105. (116) $481\frac{43}{95}$ acres. (117) £4 10*s.* (118) £8508 10*s.* $10\frac{1}{2}$*d.* (119) £181 1*s.* $5\frac{601}{2407}$*d.* (120) £274 13*s.* 9*d.* (121) 21 days. (122) Nitre, 30 cwts. 1 qr. $16\frac{4}{5}$ lbs.; sulphur, 3 cwts. 3 qrs. $5\frac{3}{5}$ lbs.; charcoal, 5 cwts. 3 qrs. $5\frac{3}{5}$ lbs. (123) £10020. (124) $37{\cdot}58\dot{3}$ yards. (125) £1305 18*s.* $0\frac{1}{4}$*d.* (126) 37 lbs. (127) Train goes $2\frac{1}{11}$ times as fast (23 : 11). (128) £248 5*s.* 4*d.* (129) 1 lb. 2 oz. 11 dwts. 16 grs. (130) $\frac{4}{15}$: $\frac{8243}{9900}$: $\frac{19}{73}$. (131) $685\frac{5}{7}$. (132) 3 days. (133) £2700. (134) £6 9*s.* 10*d.* (135) £4173 16*s.* 3·624525... (136) $6\frac{3}{8}$. (137) £15 0*s.* $11\frac{1}{2}$*d.* each. (138) £1000. (139) £4 16*s.* (140) $2\frac{3}{17}$. (141) ·8014... (142) The first; $2212\frac{4}{7}$ yards. (143) £1040. (144) £36 13*s.* 1·420905*d.* (145) Increased

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* In discounter's favour.

£99 5*s.* $6\frac{213}{262}$*d.* : Gain per cent. £44 2*s.* $5\frac{81}{131}$*d.* (146) £62 10*s.* (147) £350 5*s.* (148) 101 shares. (149) 2 hrs. $3\frac{4}{7}$ mins. (150) 4 per cent. (151) 86. (152) 3620. (153) 837. (154) ·5. (155) £100 2*s.* $6\frac{90}{157}$*d.* (156) $287466\frac{2}{3}$ solid yards. (157) 34 miles. (158) £565 13*s.* $7\frac{2908}{3125}$*d.* (159) 22 and 12. (160) 30 minutes. (161) $4\frac{1}{2}$ miles per hour. (162) £557 5*s.* 6*d.* gain. (163) 24 hrs. 3 mins. 53·55....secs. (164) $85\frac{5}{27}$. (165) £18630. (166) $38\frac{2}{3}$ hrs. (167) First, $2\frac{1}{2}$; second, $1\frac{1}{2}$ yrs. (168) 6*s.* $8\frac{1}{2}$*d.* nearly. (169) 40 per cent. (170) 27 inches. (171) $82\frac{2}{3}$ yds. (172) $2297\frac{101}{467}$ cubic ft. (173) 111104 mètres. (174) £125. (175) $57\frac{3}{5}$*s.* (176) $1020\frac{1}{17}$. (177) 706·1. (178) 20 c. ft. $7'\ 9''\ 0'''\ 3''''\ 4'''''$. (179) 20 c. ft. $1116\frac{5}{18}$ c. in. (180) 2 hrs. (181) ·7182819. (182) $13\frac{1}{3}°$;—$9\frac{7}{9}°$;—$22\frac{2}{9}°$. (183) £3 18*s.* $5\frac{1}{2}$*d.* (184) $4\frac{14}{87}$ in.* (185) 2 hrs. 40′. (186) 620 thalers. (187) 22·38149. (188) $8071\frac{7}{12}$ sq. in. (189) $1\frac{29}{35}$ days. (190) $10\frac{8}{21}$ years. (191) £22 10*s.* $2\frac{728}{8011}$*d.* (192) 122 florins. (193) 10 ft. (194) £1 11*s.* 8·2199074*d.* (195) 1061; $3\frac{2}{7}$. (196) £80. (197) 27 fr. (198) 4 fl. 2 c. 5 m. : $\frac{1}{11}$. (199) $2490\frac{5}{12}$ sq. in. (200) 35259000. (201) 63·5102.... (202) 28 yds. 4 in. (203) 84. (204) £3377 10*s.* : $96\frac{1}{2}$. (205) $\frac{216}{4715}$*d.* (206) £1 5*s.* 9*d.* (207) $61\frac{17}{27}$ yds. : £13 17*s.* 4*d.* (208) 2·718281. (209) 162000 secs. ; ·00000049603174; ·005859375. (210) £2340 : 8·6. (211) £2 4*s.* 9*d.*; £36 3*s.* $1\frac{31}{56}$*d.* (212) £22 10*s.* increase. (213) 754. (214) $1691\frac{7}{12}$ square inches. (215) 132·1142857. (216) $1\frac{257}{343}$ days. (217) See **100.** (218) $57\frac{28427}{224039}$. (219) £4 3*s.* $10\frac{422}{763}$*d.* (220) £2707 2*s.* 10·02337684806*d.* (221) $3\frac{1719}{3427}$*d.* (222) £8000. (223) £4763 15*s.* $8\frac{4}{7}$*d.* (224) 6,600. (225) $36\frac{108}{197}$ hours. (226) 28 miles, and 52 miles an hour. (227) £37 17*s.* 5·7*d.*

LXIV.

(1) 26·1509125. (2) 18s. 9d. per gallon. (3) $22\frac{1}{8}$ years. (4) $5\frac{85}{123}$ = 5·69105 gain. (5) 18495000. (6) £52 10s. increase. (7) $88\frac{6}{13}$. (8) $283\frac{5}{9}$.

LXV.

(1) $128\frac{10}{13}$. (2) Decreased $18\frac{74}{377}$ per cent. (3) £$7092\frac{28}{141}$, and £8 10s. $2\frac{26}{47}$d. (4) $82\frac{1}{7}$. (5) See the Question. (6) 10 gulden. (7) £1 19s. $4\frac{56}{127}$d. (8) £2 3s. $2\frac{52774}{85007}$d. gain. (9) 1797. (10) 10 miles.

LXVI.

(1) Train $1\frac{7}{8}$ times as fast (8 to 15). (2) $50\frac{3}{19}$. (3) 1·794... (4) 12 ft. $9'\ 8''''\ 11'''''$. (5) 12 c. ft. $1296\frac{107}{144}$ in. (6) 28·63. (7) 26·5435416. (8) Increased £29 16s. $7\frac{63}{263}$d. (9) 50 per cent. (10) £2 15s. 3d. (11) $66\frac{2}{3}$ men. (12) $17\frac{1}{9}$ days. (13) 3 mins. 46·8 secs. (14) 55 mins. past 10 o'clock. (15) $\frac{1}{7}$ of the work.

LXVII.

(1) 27720. (2) $\frac{4336}{58881}$. (3) 666; ·00067; ·6324... (4) 62·45614...; ·707106... (5) £792 15s. (6) The three and a

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* In question read 17 instead of 7.

half per cents. (7) He sees 30 yds. and travels twice as fast. (8) 5 dwts. $14\frac{12934}{70399}$ grs. (9) ·1. (10) $65\frac{5}{27}$ yds. (11) $10\frac{1}{2}$d. per lb. (12) ·28; 18000. (13) $\cdot 04938271\dot{6}$; $\cdot 8\dot{3}$. (14) £50000; £7000. (15) 10 per cent. (16) $6\frac{4}{81}$ days. (17) $\cdot 0101273\dot{1}4\dot{8}$. (18) 24s. $6\frac{2}{5}$d. (19) A, £128 3s.: B, £598 0s. 8d.: C, £170 17s. 4d. or $\frac{4}{21}$. (20) $7\frac{1}{2}$ yrs. (21) Inversely proportional to the price per gallon. (22) 176 tons 18 cwts. 2 qrs. $15\frac{3}{11}$ lbs. (23) £2124 15s. 9d. (24) 5 dwts. 10 grs. (25) $1\frac{21}{42}$. (26) 7s. $4\frac{1}{2}$d. (27) 736. (28) 64 days. (29) 8 horses. (30) 2·828; ·014; ·039592. (31) 4 lbs. 8 oz. 12 dwts. $19\frac{901}{1177}$ grs. (32) £1750. (33) $\cdot 488541\dot{6}$. (34) $1\frac{1}{2}$; $\frac{2}{5}$; $2\frac{1}{26}$. (35) $\frac{7}{25}$. (36) £33 1s. $2\frac{2}{5}$d. (37) 1250; ·125. (38) $7\frac{1}{4}$. (39) 2·1544345... (40) (1) $\frac{3}{100}$: (2) $\frac{231}{8000}$. (41) £325 10s. (42) $3\frac{9}{13}$ days. (43) $74\frac{2}{27}$ hrs.; $1333\frac{1}{3}$ hrs.; 2000 hrs. (44) 14 yds. 1 ft. (45) £56 8s. 4d. (46) £1499 15s. 7·2d. (47) 15 days. (48) Income £900, and rate 5d. (49) 11 : 2. (50) £3 10s. (51) £900. (52) 237500 francs.

LXVIII.

(1) $\cdot 62138573\dot{2}$: ·914378... (2) 370 ac. 2 r. 27·49...pls. (3) 183267·735. (4) £125 4s. nearly. (5) 24855 mls. 2 f. 16 pls. 5 yds. 1 ft. (6) $25\frac{100225}{1980311}$ yards. (7) 9·73807455...francs.

LXIX.

(1) £653 4s. $4\frac{52}{59}$d. (2) £937 13s. 2d. (3) 29·7 yds. (4) £375. (5) £2 9s. $3\frac{1}{9}$d. (6) £3 5s. 4d. (7) 184 yds. 2 in. (8) 125 sq. ft. $75\frac{125}{144}$ in. (9) £31 1s. $7\frac{463}{743}$d. (10) 25 ft. 4 in. (11) 8 ft.; 13·856; 64 ft. (12) $8\frac{443}{576}$ m. past 10. (13) 26th March 1790. (14) $10\frac{9}{16}$. (15) 1609·306; £1093 1s. 11d.

LXX.

(1) $\cdot 0955\dot{7}5324\dot{6}$. (2) Income £120; rate of Interest £$3\frac{363}{679}$. (3) £11 15s. $2\frac{2}{5}$d. (4) 750 men. (5) 56 hrs. (6) 68 c. yds. 26 ft. 1164 in. (7) £16 2s. $10\frac{93}{400}$d. (8) $\cdot 8\dot{3}$ ·04. (9) $2\cdot 223\dot{4}$. (10) 10 in. (11) 6·339 in. (12) $38\frac{2}{11}$ min. (13) $25\frac{3}{4}$ yds.; $30\frac{9}{10}$ yds. (14) £125 principal. (15) ·24.

LXXI.

(1) ·01849; and ·62996... (2) 360360. (3) $20\frac{12}{73}$. (4) 1250 men. (5) $-31\frac{31}{39}$; and $\frac{12}{29}$. (6) £600. (7) £12. (8) $6\frac{86}{345}$ dys. (9) A=£23 5s. 9d.; B=£30 14s. 3d. (10) 73 c. yds. 8 ft. 98 in. (11) $11\frac{2}{3}$ days. (12) 124 fathoms. (13) $66\frac{2}{3}$ yds. from A's corner; A passed his corner thrice, B his twice (14) 102 S. bonds. (15) A=£18400; B=£$12266\frac{2}{3}$.

LXXII.

(1) $7\frac{1}{180}$. (2) $2\frac{13}{54}$. (3) 27. (4) $4\frac{1}{2}$. (5) 1669·94885. (6) 164·3248. (7) 31·0539544. (8) 6196. (9) ·05. (10) $10\frac{11}{28}$. (11) $2\frac{169}{180}$. (12) $14\frac{5}{8}$. (13) $3\frac{2}{7}$. (14) 2603·04″. (15) 3 oz. 17·97 grs. (16) 1356·048. (17) ·000104. (18) 3·4992. (19) 6086 pints. (20) 43 cwts. 3 qrs. 14 lbs. (21) £3791 11s. $3\frac{1}{4}$d. (22) £660. (23) 23 mls. 1 fur. 18 pls. 4 yds. 2 ft. 3 in. (24) 21s. 4d. (25) £25 2s. $3\frac{1}{2}$d. (26) £4466 6s. 2·4d. (27) $1\frac{13}{47}$ days; A $\frac{20}{47}$, B $\frac{15}{47}$, C $\frac{12}{47}$. (28) £900, £1120, £750. (29) $1633\frac{1}{2}$ tons. (30) ·06561; 3·43. (31) 80 per cent.

LXXIII.

(1) £36 *5s.* (2) 6256·9; 307. (3) £1299 *5s.* $0\frac{60}{107}$*d.*; 2 yrs. (4) 62·6. (5) $14\frac{7}{12}$; 4 : 7 by weight 7 : 5. (6) $5\frac{3}{7}$ days. (7) £12,000. (8) 45 miles. (9) 1200. (10) £242,914$\frac{242}{247}$. (11) 30 and 20 miles an hour. (12) $\frac{1}{3}$. (13) 538020.

LXXIV.

(1) 3522178; 3998936616. (2) G. C. M = 14 days 7 hrs. 11′ 17″; L. C. M = 2757 days 8 hrs. 10′ 9″. (3) $\frac{11}{26}$. (4) 3738. (5) 1100 ft. per sec. (6) 44·$\dot{5}$789473684210526З. (7) 4374. (8) 11 c. ft. $1207\frac{569}{576}$ in. (9) 34 ft. 6 in. (10) $2\frac{1}{2}$ per cent. (11) £10. (12) $77\frac{1}{2}$. (13) 3 men. (14) B, 50 yards, $4\frac{3}{4}$ m.

A. (1) $16\frac{47}{1609}$ decrease. (2) $3\frac{1}{2}$. (3) 15 days. (4) $71\frac{29}{81}$. (5) $275\frac{1}{2}$. (6) A, 7d., B, 1d. (7) $83\frac{1}{3}$. (8) £26 16s. (9) $36\frac{1}{8}$. (10) $23\frac{3}{8}$, and $20\frac{7}{9}$. (11) 5. (12) 26 c. ft. $727\frac{5}{144}$ c. inches.

B. (1) . . . (2) £6. (3) $106\frac{1}{4}$. (4) 70 days. (5) £12483 6s. 8d.; £55 9s. $7\frac{5}{9}$d. (6) £12600000. (7) 12m. $15\frac{5}{7}$ secs. (8) $6\frac{1}{2}$ inches. (9) $5\frac{345}{511}$. (10) 11th June, 10 o'clock, p.m. (11) $5\frac{1}{2}$ feet. (12) 2d.

C. (1) $2019{\cdot}70\dot{2}8571\dot{4}$. (2) £16 18s. $11\frac{14}{23}$d. (3) 3s. (4) £33000 increased. (5) £131 15s. $4\frac{4}{5}$d.; £65 17s. $8\frac{2}{5}$d.; £43 18s. $5\frac{3}{5}$d.; £417 5s. $5\frac{1}{5}$d. (6) £20 8s. $0\frac{4}{9}$d. (7) $3970\frac{7}{12}$. (8) £127 12s. 3d. (9) 6·183. (10) £1772 6s. $1\frac{1}{13}$. (11) 11:7. (12) $13\frac{229}{242}$ secs.

D. (1) 678; 19 yds. 2·94 ft. (2) 1605. (3) £50 15s. decrease. (4) $11{\cdot}28592\dot{8}$ ft. (5) 28 c. ft. 3′ 5″ 10‴ 1⁗. (6) $48886\frac{1}{144}$ in. (7) $6\frac{14}{81}$. (8) 5s. 10d. (9) $151\frac{1}{89}$ oz. (10) £24 6 f. 3 c. 3·984375 ml. (11) £503 4s. 7·1712d. (12) 864; 432; 3348; 27.

E. (1) 225. (2) $\frac{51}{65}$. (3) 46 c. ft. $298\frac{25}{48}$ in. (4) £$9\frac{1}{11}$. (5) 45. (6) $8\frac{3337}{8550}$d. (7) Thursday; Wednesday; Monday. (8) 800000. (9) $78\frac{2}{3}$. (10) 74s 3d. (11) 18s. $5\frac{7}{13}$d.; 2s. $6\frac{10}{13}$d. (12) 15 ft.

F. (1) 0. (2) £140; £105; £84; £70; £60. (3) 69·282 in. (4) £23 12s. 6d. (5) 0. (6) $10\frac{6}{19}$. (7) 34 c. ft. $1108\frac{31}{72}$ in. (8)

$112{\cdot}88250\dot{9}476\dot{1}$. (9) The Fives; £13125. (10) $5\frac{1}{3}$ days. (11) $32\frac{8}{11}$ m.; $7\frac{1}{11}$, and $57\frac{9}{11}$ m. past 6. (12) 25 miles; 100 miles.

G. (1) £$786\frac{4}{29}$. (2) 0. (3) 20 ft. 10′ 8″ 6‴ = $3008\frac{1}{2}$ sq. in. (4) 95. (5) £196 13s. $6\frac{3}{4}$d. (6) £3 5s. $2\frac{4}{7}$d. increase. (7) 90 days; 900 540, and 360 miles. (8) $73\frac{1}{3}$ s. (9) $30\frac{30}{459}$ m. past 12. (10) $\cdot8\dot{3}$. (11) £3339 1s. 0·49962d. (12) $1\frac{1}{3}$ hours.

H. (1) 4 per cent. (2) 291 c. ft. $1099\frac{5}{9}$ in. (3) 556800. (4) $36\frac{3}{13}$. (5) £10. (6) $26\frac{8}{27}$ mls. from London. (7) $12\frac{9}{14}$ m. (8) $1\frac{2}{27}$ hours. (9) $1{\cdot}\dot{4}\dot{6}$. (10) $89\frac{1}{11}$; $3\frac{1}{48}$. (11) $69{\cdot}\dot{3}2\dot{1}$. (12) $26\frac{62}{63}$.

I. (1) £35 0s. $7\frac{19}{35}$d. (2) 441. (3) 6000600. (4) £2412 10s; $96\frac{1}{2}$. (5) 15136 : 92365. (6) £9782 12s. $2\frac{3}{35}$d. (7) $91\frac{3}{7}$. (8) 10s. $10\frac{2}{3}$d. (9) £166 16s. 9d. (10) $1\frac{5}{16}$ hours. (11) $5\frac{1}{7}$. (12) 61·705...

J. (1) £1302 1s. 8d. (2) 22·2071625. (3) £50010. (4) £176 13s. 4d. (5) £$3173\frac{1}{57}$. (6) 26·3 of ·12, $\frac{1}{3}$ of $\frac{1}{8}$ of $11\frac{1}{4}$, &c. (7) ·3; $1\frac{1}{4}$. (8) £·085 × 5 = 8s. 6d. (9) 8·653...; 7·211; 5·76... (10) 10 sq. ft. $75\frac{139}{144}$ in. (11) £147 16s. $8\frac{709}{763}$d.; £$3\frac{111}{763}$. (12) $6\frac{1}{3}$ per cent.

K. (1) 20. (2) £562 10s, (3) £103 12s. $7\frac{181}{384}$d. (4) 3 calves 81 lambs, 16 goats. (5) $35\frac{35}{53}$. (6) 3 dwts. $8\frac{4}{11}$ grs. (7) £875981 5s. increased. (8) 240; 720; 960. (9) 105:254. (10) £62 17s. $2\frac{15}{16}$d. (11) £1600; $6\frac{2}{3}$. (12) $4\frac{19}{33}$.

L. (1) £6 5s. (2) £45 6s. 3d. (3) 2 ac. 3 rd. 8·536 poles (4) £60. (5) £7 10s.; £9; £4 10s. (6) £34 6s. $7\frac{1}{2}$d. (7) 684 links. (8) £1350; gain £9 5s. (9) £555 11s. $1\frac{1}{3}$d.; £1111 2s. $2\frac{2}{3}$d.; £833 6s. 8d.; £2500. (10) £627 12s. (11) $22\frac{1}{2}$ minutes; $1\frac{1}{18}$, $\frac{11}{18}$, and $\frac{3}{16}$ miles. (12) 60 miles,

M. (1) 937 lines; and ·02268 inches. (2) £374 0s. 3d. (3) 299·6997; 99·8999. (4) $65\frac{5}{8}$. (5) $32\frac{1}{4}$. (6) £2778 6s.; and £10 10s. 2d. (7) £42. (8) £65; he loses £13. (9) 396; 297; 462 (10) 60000. (11) $17{\cdot}\dot{5}3246\dot{7}$; $1{\cdot}\dot{0}4761\dot{9}$. (12) $4\frac{4}{5}$ miles.

N. (1) £586 2s. $0\frac{4}{5}$d. (2) £584 5s. $11\frac{361}{880}$d. (3) 15 ft. 4 in. (4) 174 c. ft. $617\frac{7}{8}$ in. (5) £11 15s. (6) £31 5s. (7) See *Key*. (8) £4 14s. $5\frac{9}{31}$. (9) £27 6s. $4\frac{241}{250}$. (10) 72·1°. (11) £711 5s. $5\frac{5}{11}$d. (12) 33089 : 31680. (13) 75 days. (14) £5 5s.; and £6 17s. $4\frac{1141}{1235}$d. (15) £8000000.

O. (1) Rate of rowing: rate of stream : : 5 : 1. (2) $\frac{133}{175}$. (3) 25 feet 4 inches. (4) 662 yards. (5) 18 points. (6) 30 quarters. (7) ·4. (8) 275625 grains. (9) 40 m. $36\frac{7}{43}$ secs. past 10. (10) L.C.M. = 46189; sum of fractions = $\frac{498}{46189}$. (11) $1\frac{1}{2}$. (12) 3 : 1.

P. (1) 40. (2) £1 9*s.* $2\frac{6379}{7040}$*d.* (3) *b.* 13 oz. $4\frac{12}{35}$ drs.; *a.* 1 lb. 1 dwt. 16 grs. (4) £6 5*s.* (5) 3 ft. 8 in.: 8 tons 3 cwts. 3 qrs. $1\frac{4}{175}$ lbs. (6) $9\frac{17}{84}$ galls. of water, and $2\frac{47}{84}$ galls. of wine; and in the second vessel $3\frac{17}{84}$ wine, and $2\frac{47}{84}$ water. (7) Edge 4 ft.; diagonal 6·928 ft.; cost of painting £6 18*s.* 8*d.* (8) $9\frac{1}{4}$ feet. (9) £16 5*s.* (10) $133\frac{1}{3}$ lbs. (11) 1100 ft. per second. (12) $75\frac{1}{5}$°; $17\frac{3}{5}$°; and $-65\frac{1}{5}$°.

Q. (1) $4\frac{1}{2}$ miles per hour. (2) $\frac{1}{289}$. (3) Length 25·4 feet, depth 12·7 feet. (4) 1280 : 1029. (5) £37 16*s.* $4\frac{1}{2}$*d.* (6) A $21\frac{3}{17}$ days; B $27\frac{9}{13}$ days; C $51\frac{3}{7}$ days. (7) 360815. (8) 17·69; (a) ·8944 (b) 9·469. (9) £104 13*s.* (10) 70*s.* 5*d.* (11) $8\frac{64}{97}$ days; A mows $4\frac{62}{97}$, and B $5\frac{35}{97}$ acres. (12) 30·00558144112 inches.

R. (1) £336. (2) £9396; income £435. (3) £511. (4) £984 6*s.* $10\frac{7818}{15521}$*d.* (5) 12·2 per cent. (6) £7917. (7) £425531 18*s.* $3\frac{27}{47}$*d.* (8) £85 16*s.* $1\frac{53589}{409907}$*d.* gain. (9) 6800 : 7221. (10) £489 7*s.* 6*d.* (11) £128. (12) $129\frac{1}{4}$: £165.

S. (1) £188 12s. (2) $\frac{111}{140}$. (3) $2\frac{222}{325}$ gain. (4) 139 yards 5 in. (5) $14508\frac{1}{5}$ sq. in. (6) £$2590\frac{130}{193}$; $9\frac{62}{193}$. (7) £50 16s. $10\frac{12}{25}$d. (8) $3{\cdot}539\dot{1}4285\dot{7}$. (9) 16s. $0\frac{19411}{77775}$d. (10) $4\frac{1}{2}$ ft. (11) £7500; £35 5s. (12) G. C. M. 12, L. C. M. 1289148981120.

T. (1) £1300; loss £4. (2) Gain $217\frac{145}{211}$. (3) £6 loss. (4) $185\frac{5}{8}$ lbs. (5) £2 10s.

U. (1) £1147 2s. $4\frac{1}{2}$d. (2) £11 5s. $11\frac{1}{4}$d. (3) $4\frac{5}{16}$. (4) 1·7669. (5) £129 2s. $6\frac{1}{2}$d.; and £129 2s. 7d. (6) £303 5s. $2\frac{19}{200}$. (7) £5 16s. $7\frac{38081}{1250000}$d. (8) £8355 9s. 9·55008d. (9) £2 0s. $4\frac{3002}{18437}$ discount. (10) $3\frac{244}{357}$. (11) £5 11s. $11\frac{1}{75}$d. (12) £51 6s. $2\frac{112}{365}$d.

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